KBO Orientability

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Reference

JAR 2009 Harald Zankl, Nao Hirokawa, and Aart Middeldorp
KBO Orientability
Journal of Automated Reasoning 43(2), pp. 173-201, 2009

Term Rewriting

Definition

- pair of terms $l \to r$ is rewrite rule if $l \notin \mathcal{V} \wedge \mathcal{V}\mathsf{ar}(r) \subseteq \mathcal{V}\mathsf{ar}(l)$
- term rewrite system (TRS) is set of rewrite rules
- (rewrite relation) $s \to_{\mathcal{R}} t$ if $\exists l \to r \in \mathcal{R},$ context $C,$ substitution $\sigma.$
  $s = C[l\sigma] \ \wedge \ t = C[r\sigma]$

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EXAMPLE

TRS $\mathcal{R}$

$$x + 0 \to x \qquad x + \mathsf{s}(y) \to \mathsf{s}(x + y)$$

$$x \times 0 \to 0 \qquad x \times \mathsf{s}(y) \to x \times y + x$$

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rewriting

$$\begin{aligned} \mathsf{s}(0) \times \mathsf{s}(0) &\to_{\mathcal{R}} \mathsf{s}(0) \times 0 + \mathsf{s}(0) \\ &\to_{\mathcal{R}} 0 + \mathsf{s}(0) \\ &\to_{\mathcal{R}} \mathsf{s}(0 + 0) \\ &\to_{\mathcal{R}} \mathsf{s}(0) \end{aligned}$$

terminated

Termination

Definition

TRS $\mathcal{R}$ is terminating if there is no infinite rewrite sequence $t_1 \to_{\mathcal{R}} t_2 \to_{\mathcal{R}} \cdots$

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- introduced by Knuth and Bendix, 1970
- best studied termination methods
- great success in theorem provers (Waldmeister, Vampire, ...)

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- weight of term t is

$$w(t) = \begin{cases} w_0 & \text{if } t \in \mathcal{V} \\ w(f) + w(t_1) + \cdots + w(t_n) & \text{if } t = f(t_1, \ldots, t_n) \end{cases}$$

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- weight function (w, w_0) is admissible for precedence $>$ if

$$w(f) > 0 \qquad \text{or} \qquad f \geqslant g$$

for all unary functions f and all functions g

Definition

Knuth-Bendix order $>_{\mathrm{kbo}}$ on terms $\mathcal{T}(\mathcal{F},\mathcal{V})$:
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  - $s = f(s_1, \ldots, s_{i-1}, s_i, \ldots, s_n)$, $t = f(s_1, \ldots, s_{i-1}, t_i, \ldots, t_n)$, and $s_i >_{\mathrm{kbo}} t_i$; or

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  - $s = f(s_1, \ldots, s_n)$, $t = g(t_1, \ldots, t_m)$, and $f > g$

Definition

let $X \subseteq \mathbb{R}_{\geqslant 0}$. TRS $\mathcal{R}$ is KBO$_X$ terminating if

- $\exists$ precedence $>$
- $\exists$ admissible weight function $(w, w_0) \in X^{\mathcal{F}} \times X$

such that $l >_{\text{kbo}} r$ for all $l \to r \in \mathcal{R}$

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Theorem Knuth and Bendix, 1970

TRS is terminating if it is KBO$_{\mathbb{N}}$ terminating

Quiz

Example I

$$\mathsf{a}(\mathsf{a}(x)) \to \mathsf{b}(\mathsf{b}(\mathsf{b}(x))) \qquad \mathsf{b}(\mathsf{b}(\mathsf{b}(\mathsf{b}(\mathsf{b}(x))))) \to \mathsf{a}(\mathsf{a}(\mathsf{a}(x)))$$

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is $\mathrm{KBO}_{\mathbb{N}}$ terminating ?

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is $\mathrm{KBO}_{\mathbb{N}}$ terminating ? — yes

PROOF

take precedence $\mathsf{a} > \mathsf{b}$ and weight function

$$w(\mathsf{a}) = \boxed{?} \qquad w(\mathsf{b}) = \boxed{?} \qquad w_0 = 1$$

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take precedence $\mathsf{a} > \mathsf{b}$ and weight function

$$w(\mathsf{a}) = 3 \qquad w(\mathsf{b}) = 2 \qquad w_0 = 1$$

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another solution: take precedence $> = \varnothing$ and weight function

$$w(\mathsf{a}) = 13 \qquad w(\mathsf{b}) = 8 \qquad w_0 = 1$$

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Example II

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is $\mathrm{KBO}_{\mathbb{N}}$ terminating ? — no

History of KBO

THEOREM

Knuth and Bendix, 1970

TRS is terminating if it is $KBO_{\mathbb{N}}$ *terminating*

Theorem — Knuth and Bendix, 1970

TRS is terminating if it is $KBO_{\mathbb{N}}$ terminating

Theorem — Dershowitz, 1979

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$KBO_{\mathbb{R}_{\geqslant 0}}$ termination is decidable within exponential time

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$\mathit{KBO}_{\mathbb{R}_{\geqslant 0}}$ *termination is decidable within exponential time*

THEOREM — Korovin and Voronkov, 2001, 2003

- *TRS is* $\mathit{KBO}_{\mathbb{N}}$ *terminating* $\iff$ *it is* $\mathit{KBO}_{\mathbb{R}_{\geqslant 0}}$ *terminating*
- $\mathit{KBO}_{\mathbb{R}_{\geqslant 0}}$ *termination is decidable within polynomial time*

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- *$KBO_{\mathbb{R}_{\geqslant 0}}$ termination is decidable within polynomial time*

THEOREM — Zankl and Middeldorp, 2007

$KBO_{\{0,1,\ldots,B\}}$ termination ($B \in \mathbb{N}$) can reduce to SAT and PBC

This Talk

MAIN RESULT

for every TRS $\mathcal{R}$ and $N = \sum_{l \to r \in \mathcal{R}} (|l| + |r|) + 1$

$$\mathcal{R} \text{ is } \mathrm{KBO}_{\mathbb{N}} \text{ terminating} \iff \mathcal{R} \text{ is } \mathrm{KBO}_{\{0,1,\ldots,B\}} \text{ terminating}$$

where, $B = N^{4^{N+1}}$

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OVERVIEW OF PROOF

- Principal Solutions

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- Bound by Norm

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given precedence, $\text{KBO}_{\mathbb{N}}$ problem reduces to linear integral constraints

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Principal Solutions

Gale, 1960; Dick, Kalmus and Martin, 1990; Korovin and Voronkov, 2003

set I of KBO inequalities are of form

$$\{ \ \vec{a}_i \cdot \vec{x}_i \ \ R_i \ \ 0 \ \}_i \qquad \text{with} \quad R_i \in \{\geqslant, >\}, \vec{a}_i \in \mathbb{Z}^n, \vec{x} \in \mathbb{N}$$

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Theorem

- *principal solution of A always exists*
- $\forall$*principal solution* $\vec{x}$: $\quad$ (I *is solvable* $\iff$ $\vec{x}$ *satisfies* I)

$$I = \left\{ \begin{array}{rcl} 2 \cdot w(\mathsf{a}) - 3 \cdot w(\mathsf{b}) & > & 0 \\ -3 \cdot w(\mathsf{a}) + 5 \cdot w(\mathsf{b}) & > & 0 \end{array} \right\} \qquad A_I = \begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$$

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since

$$A_I \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad A_I \begin{pmatrix} 13 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

- $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is not principal solution of A_I

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- $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is not principal solution of A_I
- $\begin{pmatrix} 13 \\ 8 \end{pmatrix}$ is principal solution of A_I and satisfies I

  hence I is solvable

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MCD: Method of Complete Description

MCD

Dick, Kalmus and Martin, 1990

- $(a_1, \ldots, a_n)^\kappa = (\vec{e}_i \mid a_i \geqslant 0) +\!\!+ (a_j\vec{e}_i - a_i\vec{e}_j \mid a_i < 0, a_j > 0)$

MCD

Dick, Kalmus and Martin, 1990

- $(a_1, \ldots, a_n)^\kappa = (\vec{e}_i \mid a_i \geqslant 0) + (a_j\vec{e}_i - a_i\vec{e}_j \mid a_i < 0, a_j > 0)$

- for $m \times n$ matrix A and $0 \leqslant i \leqslant m$

$$S_i^A = \begin{cases} E_n & \text{if } i = 0 \\ S_{i-1}^A(\vec{a}_i S_{i-1}^A)^\kappa & \text{otherwise} \end{cases}$$

MCD

Dick, Kalmus and Martin, 1990

- $(a_1, \ldots, a_n)^\kappa = (\vec{e}_i \mid a_i \geqslant 0) +\!\!+ (a_j \vec{e}_i - a_i \vec{e}_j \mid a_i < 0, a_j > 0)$

- for $m \times n$ matrix A and $0 \leqslant i \leqslant m$

$$S_i^A = \begin{cases} E_n & \text{if } i = 0 \\ S_{i-1}^A (\vec{a}_i S_{i-1}^A)^\kappa & \text{otherwise} \end{cases}$$

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THEOREM

$\vec{s}^A$ *is principal solution of* A

Example

$$I = \left\{ \begin{array}{rcl} 2 \cdot w(\mathsf{a}) - 3 \cdot w(\mathsf{b}) & > & 0 \\ -3 \cdot w(\mathsf{a}) + 5 \cdot w(\mathsf{b}) & > & 0 \end{array} \right\} \qquad A_I = \begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$$

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which satisfies I. hence I is solvable

Bound by Norm

MCD

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GOAL

bound $\vec{s}^A \in \{0, 1, \ldots, B\}^n$

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APPROACH

define norm $\| \cdot \|$ to calculate $\|\vec{s}^{A_I}\|$ with $B = \|\vec{s}^{A_I}\|$

Norm

Definition

$\|A\| = \max_{i,j} |a_{ij}|$ for $m \times n$ matrix $A = (a_{ij})_{ij}$

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- $\|a^\kappa\| = \|a\|$
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- $(a_1, \ldots, a_n)^\kappa$ *is* $n \times k$ *matrix with* $k \leqslant n^2$.
- S_i^A *is* $m \times k$ *matrix with* $k \leqslant n^{2^i}$.

Norm

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LEMMA

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- S_i^A *is* $m \times k$ *matrix with* $k \leqslant n^{2^i}$.
- $\|S_i^A\| \leqslant (2m\|A\|)^{2^i - 1}$
- $\|\vec{s}^A\| \leqslant n^{2^m} \cdot (2m\|A\|)^{2^m - 1}$

Lemma

let $\mathcal{R}$ *be TRS of size* N
I *be set of inequalities induced by KBO with fixed precedence*
A_I *be of size* $m \times n$

- $m \leqslant N$
- $n \leqslant N$
- $\|A_I\| \leqslant N$

therefore

$$\|\vec{s}^{A_I}\| \leqslant N^{4^{N+1}} := B$$

thus,

$$\vec{s}^{A_I} \in \{0, 1, \ldots, B\}^n$$

Main Result

THEOREM

$\mathcal{R}$ *is* $KBO_{\mathbb{N}}$ *terminating* $\iff$ $\mathcal{R}$ *is* $KBO_{\{0,1,\dots,B\}}$ *terminating*
where, $B = N^{4^{N+1}}$

Main Result

Theorem

$\mathcal{R}$ is KBO$_{\mathbb{N}}$ terminating $\iff$ $\mathcal{R}$ is KBO$_{\{0,1,\ldots,B\}}$ terminating
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Corollary

Zankl and Middeldorp's SAT and PBC encodings are complete for this B

Summary

let $\mathcal{R}$ be TRS, $N = \sum_{l \to r \in \mathcal{R}} (|l| + |r|) + 1$, and $B = N^{4^{N+1}}$

$\mathcal{R}$ is $\text{KBO}_{\mathbb{R}_{\geqslant 0}}$ terminating

$\iff$ $\mathcal{R}$ is $\text{KBO}_{\mathbb{N}}$ terminating — Korovin and Voronkov

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- theoretical interest of decidability issue is more or less closed
- how about automation? ☞

Automation

Theorem — Dick, Kalmus, and Martin, 1990

$KBO_{\mathbb{R}_{\geqslant 0}}$ termination is decidable by MCD

THEOREM Dick, Kalmus, and Martin, 1990

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THEOREM Korovin and Voronkov, 2001, 2003

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$KBO_{\{0,1,\ldots,B\}}$ termination ($B \in \mathbb{N}$) is decidable via SAT/PBC encodings

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$KBO_{\{0,1,\ldots,B\}}$ termination ($B \in \mathbb{N}$) is decidable via SAT/PBC encodings

Theorem

$KBO_{\mathbb{R}_{\geqslant 0}}$ termination is decidable via SMT (linear arithmetic) encoding

Experiments

- test-bed: 1381 TRSs from Termination Problem Data Base 4.0

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SMT	26	107	0

Conclusion

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CONCLUSION

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FUTURE WORK

find optimal B

Thank You for Your Attention