Workshop on Linear Logic

Geometry of Interaction, Traced Monoidal Categories and Implicit Complexity

Date: **7th November 2011 – 11th November 2011**
Venue: **Room 110, Faculty of Science Building No.3, Kyoto University**
Social dinner: **Izakaya Anji Fuchomae (Annex), 9th November (Wednesday) 18.30–**
(Non-students 5.000¥**, Students** 3.000¥**)**

Program

7th November (Monday)

10:00-10:15 Opening

10:15–11:00 Masahito Hasegawa (Kyoto Univ.) *A Model of Braided Linear Logic*

We give a model of braided linear logic based on crossed G-sets for a given group G. This model is derived as a ribbon category of modules of a ribbon Hopf algebra in the compact closed category of sets and binary relations, where the ribbon Hopf algebra is obtained using the quantum double construction of Drinfel'd. This ribbon category serves as a non-trivial model of braided propositional linear logic equipped with a linear fixed-point operator.

11:00–11:30 Shin'ya Katsumata (Kyoto Univ.) *Relating Computational Effects by TT-Lifting*

We consider the problem of establishing a relationship between two monadic semantics of the λ_c-calculus extended with algebraic operations. We show that two monadic semantics of any program are related if the relation includes value relations and is closed under the algebraic operations in the λ_c-calculus. If time permits, I introduce an ongoing work on the semantics of effect systems.

11:30–13:00 Lunch Break

13:00–14:30 Jean-Yves Girard (IML, Marseille) *Transcendental Syntax (1) System F*

14:30–14:45 Coffee Break

14:45–15:30 Masahiro Hamano (PRESTO, JST) *A Geometry of Interaction for Polarized Linear Logic*

We present a Geometry of Interaction (GoI) model for the multiplicative fragment (without structural rules) of Olivier Laurent's polarized linear logic. Our polarized GoI consists of the following two parts, both of which are motivated by the study of how polarity is captured in the original spirit of Girard's GoI 1 ('89). (i) A polarized GoI situation is presented by adding multi-points to a traced monoidal category. A key ingredient for this technique is uniformity of traces for multi-points. This method of interpreting polarized proofs is a minimal categorical condition necessary to interpret polarized GoI in the same spirit as the usual untyped GoI situations (Abramsky-Haghverdi-Scott '02). (ii) The Int construction (Joyal-Street-Verity '96) is separately investigated in the presence of categorical pullbacks and multipoints. The construction yields a compact denotational model of the polarized logic in the sense of (Hamano-Scott '07). The presence of multipoints forces a kind of "bidirectional dataflow" to the Int construction in the polarized case.

We show how running execution formulas derived from (i) and (ii) characterizes not only the dynamics of cut-elimination of proofs (the original aim of GoI) but also the focusing property, peculiar to polarized proof theory.

(This is joint work with Phil Scott.)

15:30–15:45 Coffee Break

15:45–16:30 Thomas Seiller (IML, Marseille) *Graphs of Interaction*

Generalizing the first geometry of interaction based on permutations introduced in the article Multiplicatives, I will explain how one can construct a similar framework where permutations are replaced by directed weighted graphs. In this framework, it is possible to define a notion of orthogonality by measuring cycles and obtain a geometry of interaction for multiplicative additive linear logic (MALL). Based on this localized construction, one can define both a categorical model - by working up to delocations - and a notion of truth. Eventually, I will show how this framework can be related to Girard's geometry of interaction in the hyperfinite factor. By associating an operator to a graph, one can show that the operations we defined (tensor product, application, etc.) are the same as those introduced by Girard in GoI5. In particular, it shows that orthogonality in GoI5 - defined using the determinant - corresponds to a particular way of measuring cycles.

16:30–17:15 Jonas Frey (PPS, Paris) *Characterizing Realizability Toposes*

Peter Johnstone repeatedly bemoaned the lack of system in the theory of realizability toposes – while we know different examples of realizability toposes, we do not have an intrinsic definition of what 'is' a realizability topos, a situation that stands in contrast to Grothendieck toposes where such an intrinsic definition is given by the Giraud theorem.

The presented works is intended to be a step towards such a more abstract – or systematic – theory of realizability toposes. It is well known that given a partial combinatory algebra (pca) A, we can construct a topos $RT(A)$ and a finite limit preserving functor $D : Set \to RT(A)$. We characterize the pairs $(E, D : Set \to E)$ of topos and functor that arise in this way, using Pieter Hofstra's work on 'Basic combinatory objects'. Our approach generalizes to base toposes other than Set.

Compared to the Giraud theorem, which characterizes bare Grothendieck toposes, we have to postulate the functor D explicitely. Since such a functor is equivalent to a fibering of E over Set, one could say that the theory of realizability toposes is a 'theory of fibered toposes'.

8th November (Tuesday)

9:30–11:00 Jean-Yves Girard (IML, Marseille) *Transcendental Syntax (2) Coherent spaces*

11:00–11:15 Coffee Break

11:15–12:00 Claudia Faggian (PPS, Paris) *Geometry of Interaction and Quantum Circuits*

We describe ongoing work about the relationships between quantum computation and linear logic, driven by Geometry of Interaction. In its original formulation, the GoI gives a dynamic interpretation of computation as a flow of information circulating around a net. This flow of information can be formulated both as a token-based interactive machine, or as an algebra of bounded operators on the infinite dimension Hilbert space, which is the canonical state space for quantum computation models. On the other side, the interpretation of a linear logic proof as an operator only makes use of a small fragment of the setting. Our aim is to enrich linear logic in such a way that a larger portion of the GoI semantics is actually exploited, while capturing quantum features.

This lead us to introduce a logical system, QMLL, which extends MLL with a quantum modality, and whose design has been guided by intuitions on the underlying "quantum GoI" interpretation.

QMLL is shown able to capture all unitary quantum circuits. Conversely, any proof is shown to compute, through a concrete GoI interpretation, some quantum circuits.

(Joint work with Ugo Dal Lago)

12:00–12:30 Kazuyuki Asada (National Institute of Informatics) *Bidirectional Transformation and Int Construction*

12:30–14:00 Lunch Break

14:00–15:00 Stefano Guerrini (invited speaker: LIPN, Paris) *Jumps and Boxes*

15:00–15:15 Coffee Break

15:15–16:00 Michele Basaldella (Kyoto Univ.) *Dilators: a Gentle Introduction*

16:00–16:45 Etienne Duchesne (LIPN, Paris) *TBA*

9th November (Wednesday)

9:30–11:00 Jean-Yves Girard (IML, Marseille) *Transcendental Syntax (3) Proof–nets*

11:00–11:15 Coffee Break

11:15–12:00 Nao Hirokawa (JAIST) *Runtime Complexity Analysis and Rewriting Techniques*

Rewrite systems are considered as an abstract model of programs. In this talk we discuss how to analyse (polynomial) time complexity of functions defined by rewrite systems. The talk consists of two parts. The first part is concerned with a basic method and its automation. As easily observed, termination measures on natural numbers give upper-bounds of complexity, and this approach can be effectively implemented by polynomial orders. In the second part we use rewriting techniques to refine the first method.

12:00–12:30 Naohi Eguchi (Tohoku Univ.) *Term-rewriting Approaches to Implicit Computational Complexity*

Implicit Computational Complexity (ICC) aims to give natural machine-independent characterisations of computational complexity classes. Most recursion-theoretic approaches to ICC impose specific constraints on the schema of primitive recursion. Once such a recursion schema together with certain initial functions is given, one will immediately obtain the corresponding term rewrite system by replacing the defining equations by rewriting rules. The term-rewriting characterisation induced by a recursion-theoretic characterisation could be generalised by a reduction ordering compatible with the defining term rewrite system. This talk will give a brief survey of term-rewriting approaches to ICC by means of reduction orderings, including recent results by the speaker.

12:30–14:00 Lunch Break

14:00–15:00 Ugo Dal Lago (invited speaker: Univ. Bologna) *Measuring, Computing and Constraining: Three Ways to Exploit GoI in Implicit Complexity.*

Geometry of Interaction allows to interpret higher-order computation as first-order interaction and, as such, sheds new light on the dynamics of proofs and programs. In this talk, I present some applications of GoI to the field of implicit computational complexity. GoI can be helpful as a computational model, but also as a tool to reason about computation or as a way to isolate programs exhibiting a good behavior among those written in a fully-fledged programming language.

15:00–15:15 Coffee Break

15:15–16:00 Michele Pagani (LIPN, Paris) *The Computational Meaning of Probabilistic Coherent Spaces*

We study the probabilistic coherence spaces – a denotational semantics interpreting programs by power series with non negative real coefficients. We prove that this semantics is adequate for a probabilistic extension of the untyped λ-calculus: the probability that a term reduces to a head normal form is equal to its denotation computed on a suitable set of values. The result gives, in a probabilistic setting, a quantitative refinement to the adequacy of Scott's model for untyped λ-calculus.

This is a joint work with Thomas Ehrhard and Christine Tasson.

16:00-16:45 Rene Vestergaard (JAIST) *Towards a Geometry of Regulation*

Can a normal proof exist at a different level of abstraction than the proofs being normalized, and what does this have to do with gene regulation, the central dogma of molecular biology, and the prediction of organism life cycles?

18:30– Workshop Dinner

10th November (Thursday)

9:30–11:00 Jean-Yves Girard (IML, Marseille) *Transcendental Syntax (4) The transcendental status of typing*

11:00–11:15 Coffee Break

11:15–12:15 Marco Gaboardi (invited speaker: Univ. Pennsylvania) *Linear Dependent types for Program Sensitivity and Differential Privacy*

In many applications it is useful to know in advance how much the result of a function can change with respect to input perturbations. One of this application is Differential Privacy. Recently, Pierce and Reed have proposed a linear metric type system for program sensitivity and Differential Privacy. After summarizing Pierce and Reed's approach I will show some limitations of their type system. In order to recover these limitations I suggest a solution that uses linear and dependent types following the ideas of a recent joint work with Ugo Dal Lago in the context of implicit computational complexity.

12:15–13:45 Lunch Break

13:45–14:30 Damiano Mazza (LIPN, Paris) *Metrics on Lambda-terms*

It is well known that the real numbers may be defined as the metric completion of the rational numbers, with the metric induced by the usual absolute value. We seek a computational version of this phenomenon, with the idea that the role of the rationals should be played by the affine lambda-calculus, whose dynamics is finitary; the full lambda-calculus should then appear as a suitable metric completion of the affine lambda-calculus. The aim of this talk is to present a possible technical realization of this idea, and some related results.

14:30–15:00 Marc Bagnol (IML, Marseille) *Cycles and Traces: On the Meaning of Syntaxe Transcendantale's Duality*

15:00–15:15 Coffee Break

15:15–15:45 Guillaume Munch-Maccagnoni (PPS, Paris) *What Can Polarisation Do for the Proofs-as-programs Correspondence?*

In looking for a proofs-as-programs interpretation of real-world mathematics, surely the big scale is interesting (for instance axioms with involved computational content such as dependent choice) but the small scale is equally important (operational details inspired by CPS and linear logic semantics, such as polarities).

One striking polarity-related phenomenon is the symmetry between the call-by-value and the call-by-name evaluation paradigms in the presence of call/cc-like control operators, that corresponds to the symmetry between positive and negative connectives. Curien and Herbelin (2000) captured this phenomenon in an elegant way thanks to a syntax that could express the symmetries of sequent calculus. I will argue that this result is more than a beautiful curiosity — it also provides us with a new tool for investigating the proofs-as-programs correspondence in a unified fashion.

I would like to explain why polarisation gives a better proofs-as-programs interpretation of classical logic

than the usual one with "call/cc" control operators, and then show the importance of phenomena linked to polarisation in Krivine's classical realisability.

The main tool is an extension of classical realisability to Girard's polarised classical logic, based on an extension of Curien and Herbelin's notation (or Krivine machines) to this logic.

(Work-in-progress talk)

15:45–16:15 Naohiko Hoshino (Kyoto Univ.) *Semantics of Higher-Order Quantum Computation via Geometry of Interaction*

Quantum computation attracts growing attention as a new computing paradigm. While much of its current study employs low-level formalisms such as quantum circuits, several high-level languages/calculi have been recently proposed aiming at structured quantum programming. The current work contributes to the semantical study of such high-level quantum languages, by providing interaction-based semantics of a functional quantum programming language—it is based on linear lambda calculus and equipped with features like the ! modality and recursion. We prove soundness and adequacy of our semantics. The construction of our model is by a series of standard techniques taken from the semantics of classical computation as well as from process theory, namely: 1) monads with an order structure as models for branching, used in the coalgebraic study of state-based systems; 2) Girard's Geometry of Interaction, categorically formulated by Abramsky, Haghverdi and Scott, providing interaction-based, game-like semantics for linear logic and computation; and 3) the realizability technique that turns an (untyped) combinatorial algebra into a categorical model of a typed calculus. The mathematical genericity of these techniques—mostly due to their categorical formulation—is exploited for our move from classical to quantum.

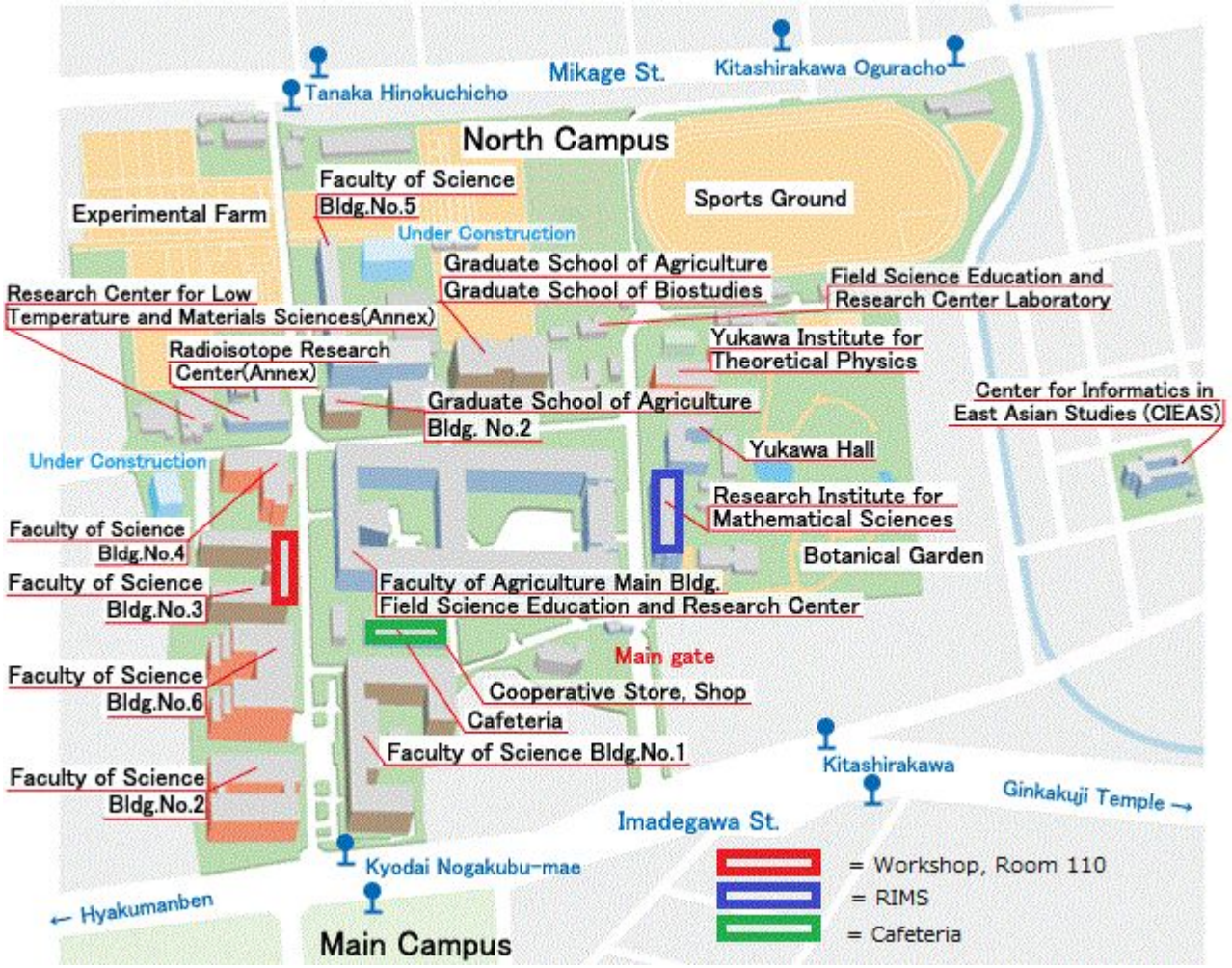
(Joint work with Ichiro Hasuo)

11th November (Friday)

9:30– Open Discussion

Information

North campus map



Conference room

Room 110, Faculty of Science Building No.3, Kyoto University.

Internet

Wireless LAN is available in the conference room. To access the web, it is necessary to fill a form and set the laptop appropriately. Ask to the local organizers.

Coffee Breaks

Tea, coffee and snacks are available in Room 108 (close to Conference room). Symbolic offerings/donations from the participants are greatly appreciated.

Lunch Breaks

A map of some restaurants near the campus is available in Conference room.