Some Seifert surgeries along A'Campo's divide knots and 4-manifolds

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Dedicated to Professor Yukio Matsumoto for his 60th birthday.

We study Dehn surgeries along A'Campo's *divide knots* ([A1, A2] and [H, GHY]:Under "divide" theory, a plane curve represents a knot in S^3). In [Y1], the author pointed out that every knot in the subfamily of J. Berge's lens surgery [Bg]

$$(k(a,b); a^2+ab+b^2) = L(a^2+ab+b^2, -(a/b)^2)$$

is a divide knot and that the corresponding divide (the plane curve) can be obtained by cutting out from the 45° lattice X as $X \cap \mathcal{R}$ such that the area of $\mathcal{R}$ equals to the surgery coefficient a^2+ab+b^2, see the example $(a=2, b=3)$ in Figure 1, where (a,b) is a pair of coprime positive integers and (a/b) is a solution of $bx=a$ in $\mathbf{Z}/(a^2+ab+b^2)\mathbf{Z}$.

In fact, we ([Y1, Y2, Y3]) have studied such a phenomenon

exceptional surgery coefficient $\doteqdot$ area of the region of the curve

for many known examples of *finite surgery*, by which we mean Dehn surgery along a knot yielding a 3-manifold whose π_1 is finite.

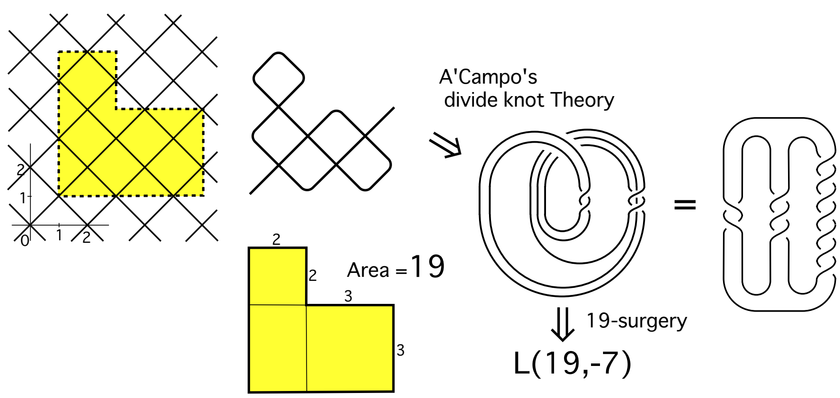

Figure 1: 19-surgery along $k(2,3)$ $(= Pr(-2,3,7))$

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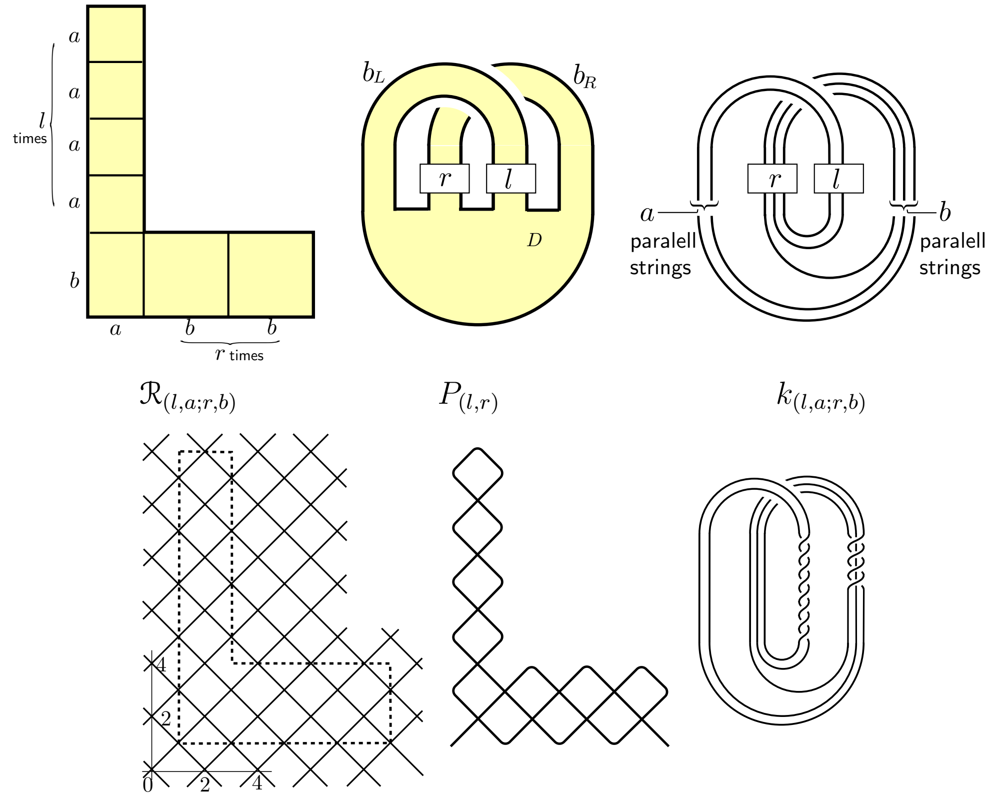

Figure 2: $k_{(l,a;r,b)}$ ex. $k_{(4,2;2,3)}$

Now, we extend such a study, see Figure 2: The area of the region $\mathcal{R}_{(l,a;r,b)}$ is $p_{(l,a;r,b)} := la^2 + ab + rb^2$, where (a, b) is a coprime pair of integers with $1 < a < b$ and l, r are non-negative integers. The plane curve $X \cap \mathcal{R}_{(l,a;r,b)}$ represents the divide knot $k_{(l,a;r,b)}$, which is in the embedded once-punctured torus $P_{(l,r)}$ in S^3. Our main theorem is the following:

Theorem 1 *For each $(l, a; r, b)$ above, the resulting manifold $(k_{(l,a;r,b)}; p_{(l,a;r,b)})$ of $p_{(l,a;r,b)}$-surgery along the knot $k_{(l,a;r,b)}$ is "at most" a graph manifold obtained by splicing two Seifert manifolds over S^2.*

In fact, $(k_{(l,a;r,b)}; p_{(l,a;r,b)})$ bounds a plumbing manifold ([O, p.22]) described by the framed link in Figure 3. The integers n_L, n_R and the framings, $\{a_j\}$ ($a_{-(n_R+1)}$ is always -1) are decided from (a, b), independent from l, r. An algorithm to decide them is given in the author's talk ([Y4]). It is related to the resolution of the singularity of the complex curve of $z^a - w^b = 0$, i.e., Euclidean algorithm.

Theorem 1 includes the following Dehn-surgeries, which has been discovered one by one in the history:

(1) ([Mo]) ab-surgery along $T(a, b)$ is $-(L(a, b) \sharp L(b, a))$ as the cases $(l, a; r, b) = (0, a; 0, b)$.

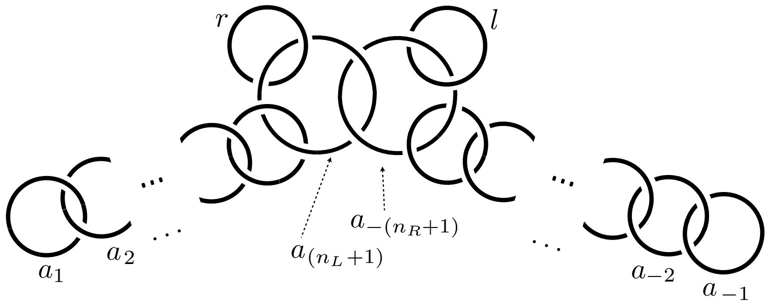

Figure 3: $(k_{(l,a;r,b)}, p_{(l,a;r,b)})$

(2) A subfamily of Berge's lens surgery [Bg] (denoted by $k^+(a,b)$ in [Y1]) as the cases $(l,a;r,b) = (1,a;1,b)$, including 19-surgery along the pretzel $(-2,3,7)$ discovered by R. Fintushel and R. Stern [FS].

(3) $(4l+15)$-surgery on the pretzel knot $Pr(-2,3,2l+5)$ is a Seifert manifold ([BH, Proposition 16]) as the case $(l,a;r,b) = (l,2;1,3)$ with $l \geq 3$.

If $l = 1$ (or $r = 1$) the resulting manifold is a Seifert manifold and bounds a negative definite manifold by Kirby calculus ([K, GS]) in Figure 4.

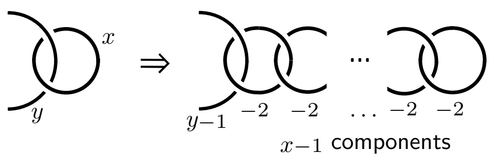

Figure 4: A Kirby calculus

We also study the following question in the 4-dimensional topology, which was the motivation of this study.

Question. (by Masaaki Ue [U1, Remark 1]) Suppose that a (Seifert) 3-manifold S
∘ bounds a *negative* smooth 1-connected 4-manifold N^4 and
∘ also bounds a *positive* smooth 1-connected 4-manifold P^4.
Then the union $N \cup_\partial -P$ along the boundary S is homeomorphic to the connected sum $\sharp_{\beta_2}\overline{\mathbf{C}P^2}$ of some copies of $\overline{\mathbf{C}P^2}$ by the great works by S. Donaldson and M. Freedman in ′80. Is $N \cup_\partial -P$ diffeomorphic to $\sharp_{\beta_2}\overline{\mathbf{C}P^2}$?

How about the case that N is the plumbing manifold and P is the one from an Seifert surgery ($B^4 \cup$ a 2-handle)?

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