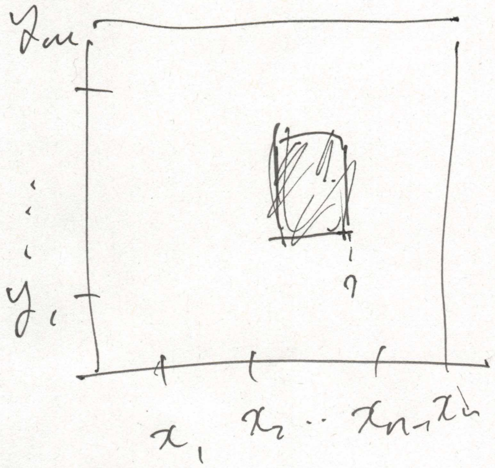
(A1)



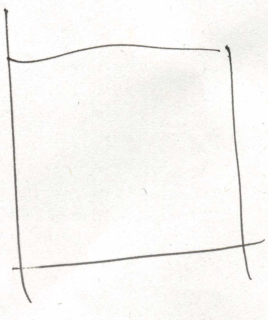
$$U = \sum_{i,j} (x_i + y_j)(x_i - x_{i-1})(y_j - y_{j-1})$$

$$L = \sum_{i,j} (x_{i-1} + y_{j-1})(x_i - x_{i-1})(y_j - y_{j-1})$$

$$U - L = \sum \underline{\{(x_i - x_{i-1}) + (y_j - y_{j-1})\}}\ \underline{(x_i - x_{i-1})(y_j - y_{j-1})}$$

$$\wedge\!| \quad M$$

$$\leq M$$



$$U + L = \sum_{i,j} (x_i + x_{i-1} + y_j + y_{j-1})(x_i - x_{i-1})(y_j - y_{j-1})$$

$$= \sum_{i,j} (x_i^2 - x_{i-1}^2)(y_j - y_{j-1}) + \sum_{i,j} (x_i - x_{i-1})(y_j^2 - y_{j-1}^2)$$

$$= 2$$

$$\int_0^1 dx \int_0^1 dy \; x+y$$

$$\left[xy + \frac{y^2}{2}\right]_0^1$$

$$\int_0^1 dx \left(x + \frac{1}{2}\right)$$

$$\left[\frac{x^2+x}{2}\right]_0^1 = 1$$

$$(y_j - y_{j-1})$$

$$U + L = \sum_j \sum_i$$

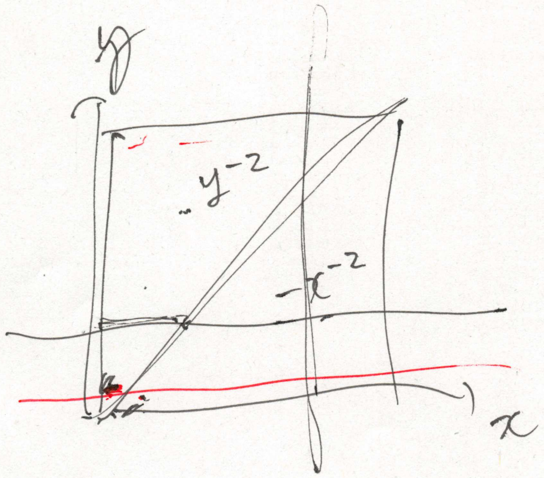
$$\iint_\Omega \left(\partial_x \partial_y f - \partial_{xy} \partial_x f\right)(a) > 0$$

$$> 0$$

$$\boxed{\cdot}$$

$$\iint$$

(A2)



$$\int_0^1 f(x,y)\,dx = \underbrace{\int_0^y y^{-2}\,dx}_{= y^{-1} = \frac{1}{y}} + \underbrace{\int_y^1 -x^{-2}\,dx}_{= \left[\cancel{-}x^{-1}\right]_y^1 = 1-\frac{1}{y}} = 1$$

$$\therefore \iint \cdots \, dx\,dy = 1$$

$$\int_0^1 f(x,y)\,dy = \underbrace{\int_0^x -x^{-2}\,dy}_{= -\frac{1}{x}} + \underbrace{\int_x^1 y^{-2}\,dy}_{= \left[-y^{-1}\right]_x^1 = -1+\frac{1}{x}} = -1$$

$$\therefore \iint \cdots \, dy\,dx = 1$$

(A3)

$$\int_{[a-\varepsilon, a+\varepsilon]\times[b-\delta, b+\delta]} (\partial_x \partial_y f - \partial_y \partial_x f)(x,y)\, dx\, dy$$

$$\| \text{ Fubini}$$

$$\int_{[b-\delta, b+\delta]} \left(\int_{[a-\varepsilon, a+\varepsilon]} \partial_x \partial_y f(x,y)\, dx \right) dy$$

$$- \int_{[a-\varepsilon, a+\varepsilon]} \left(\int_{[b-\delta, b+\delta]} \partial_y \partial_x f(x,y)\, dy \right) dx$$

$$\|$$

$$\int_{[b-\delta, b+\delta]} \left(\partial_y f(a+\varepsilon, y) - \partial_y f(a-\varepsilon, y) \right) dy$$

$$- \int_{[a-\varepsilon, a+\varepsilon]} \left(\partial_x f(x, b+\delta) - \partial_x f(x, b-\delta) \right) dx$$

$$\|$$

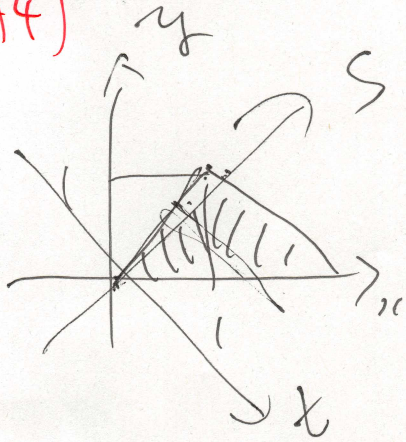
$$\left(f(a+\varepsilon, b+\delta) - f(a+\varepsilon, b-\delta)\right) - \left(f(a-\varepsilon, b+\delta) - f(a-\varepsilon, b-\delta)\right)$$

$$- \left\{ \left(f(a+\varepsilon, b+\delta) - f(a-\varepsilon, b+\delta)\right) - \left(f(a+\varepsilon, b-\delta) - f(a-\varepsilon, b-\delta)\right) \right\}$$

$$\|$$

$$0$$

(A4)



$x - y = t$

$x + y = s$

$$\iint_D \underbrace{x^2 - y^2}_{\substack{= (x+y)(x-y) \\ = st}} dx\,dy$$

$$= \frac{1}{2}\int_0^2 ds \int_0^s dt\, st$$

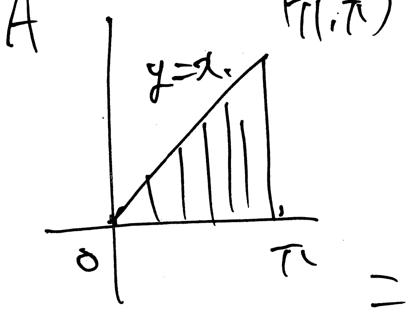
$$= \frac{1}{2}\int_0^2 ds\ \frac{s^3}{2}$$

$$= \frac{1}{4}\int_0^2 s^3 ds$$

$$= \frac{1}{4}\ \frac{2^4}{4} = 1$$

A5.

$$\iint f\,dxdy = \int_0^{\pi} dx \int_0^{x} dy\ 2xy \cos\left(\frac{y^2}{x}\right)$$



$$= \int_0^{\pi} dx \left[x^2 \sin\left(\frac{y^2}{x}\right) \right]_0^{x}$$

$$= \int_0^{\pi} dx\ (x^2 \sin x) \qquad x^2(-\cos x)'$$

$$= \left[-x^2 \cos x \right]_0^{\pi} + \int_0^{\pi} 2x(+\cos x)\,dx$$

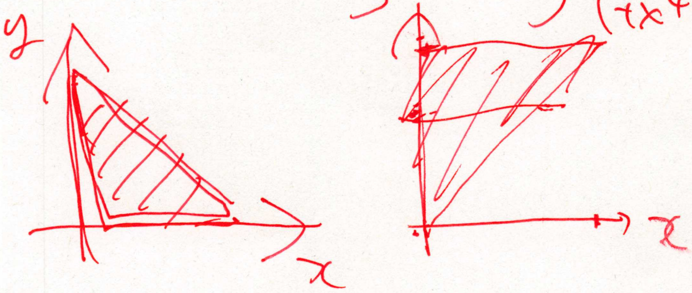
$$= \pi^2 + \int_0^{\pi} 2x \cos x\,dx$$

$$= \pi^2 + \left[2x \sin x \right]_0^{\pi} - \int_0^{\pi} 2 \sin x\,dx$$

$$= \pi^2 - 4.$$

(A6)

$$\int \frac{dx}{1+x^4}$$



$$\iint \frac{dx\,dy}{1+(x+y)^4} = \pi/8$$

$x+y=s$

$$\iint \frac{dx\,ds}{1+s^4}$$

$$=$$

$$\int_0^1 ds \int_0^s dx \, \frac{1}{1+s^4}$$

$$=$$

$$\int_0^1 ds \, \frac{s}{1+s^4}$$

$u = s^2$

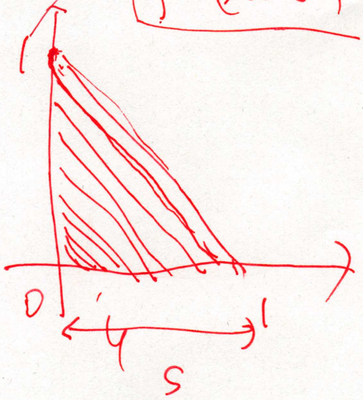
$\frac{du}{ds} = 2s$

$$=$$

$$\frac{1}{2}\int_0^1 \frac{1}{1+u^2}\, du = \pi/8$$

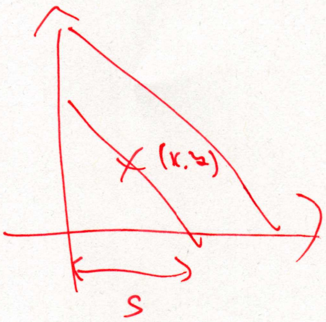
$\boxed{\iint f(x+y)}$

$x+y=s$



$$\sum s \cdot f(s)\,\Delta s$$

$$= \int_0^1 s f(s)\, ds$$



$$\boxed{\int_0^1 \int_0^{1-y} \frac{dx}{1+(x+y)^4}\, dy}$$