$\boxed{7A}$ (1) $\sqrt{2}$ (2) $-\frac{\pi}{4}+2\pi n \ (n\in\mathbb{Z})$ (3) $1+i$

(4) $5+2i$ (5) $-3-4i$ (6) $7-i$

(7) $\dfrac{z}{w}=\dfrac{(1-i)(4-3i)}{25}=\dfrac{1-7i}{25}$

(8) $\dfrac{w}{z}=\dfrac{(4+3i)(1+i)}{2}=\dfrac{1+7i}{2}$

$\boxed{7B}$ (1) $1=1(\cos\theta+i\sin\theta)$ $\theta=2\pi n \ (n\in\mathbb{Z})$

(2) $i=1(\cos\theta+i\sin\theta)$ $\theta=\frac{\pi}{2}+2\pi n \ (n\in\mathbb{Z})$

(3) $1-i=\sqrt{2}(\cos\theta+i\sin\theta)$ $\theta=-\frac{\pi}{4}+2\pi n \ (n\in\mathbb{Z})$

(4) $\sqrt{3}+i=2(\cos\theta+i\sin\theta)$ $\theta=\frac{\pi}{6}+2\pi n \ (n\in\mathbb{Z})$

(5) $-1-\sqrt{3}i=2(\cos\theta+i\sin\theta)$ $\theta=-\frac{2\pi}{3}+2\pi n \ (n\in\mathbb{Z})$

(6) $-\sqrt{2}+i\sqrt{2}=2(\cos\theta+i\sin\theta)$ $\theta=\frac{3\pi}{4}+2\pi n \ (n\in\mathbb{Z})$

$\boxed{7C}$ (1) $Q=0,\ R=0$

(2) $Q=0,\ R=1$

(3) $Q=0,\ R=1$

(4) $Q=1,\ R=-(1+i)$

(5) $x^3=(x^2+1+i)x-(1+i)x$ より $Q=x,\ R=-(1+i)x$

(6) $x^2=D\cdot 1-(1+i)$ より

$x^4=D^2-2D(1+i)-2i$ だから $Q=D-2(1+i)=x^2-(1+i)$

$R=-2i$

(7) $x^4 = DQ_4 + R_4$ $\left(\deg R_4 < \deg D \ (\text{or } R_4 = 0)\right)$ のとき

$x^5 = D(xQ_4) + xR_4$ で、実際 $\deg R_4 = 0$ だったから

$Q = xQ_4 = x^3 - (1+i)x$, $R = xR_4 = -2ix$

(8) $x^2 = DQ_2 + R_2$ (R_2, R_4 は D でわった余り) のとき

$x^4 = DQ_4 + R_4$

$x^6 = D(Q_2Q_4D + R_2Q_4 + Q_2R_4) + R_2R_4$

で $\deg(R_2R_4) < \deg D$ となっているから $R = R_2R_4 = 2i(1+i)x \left(= (2i-2)x\right)$

$Q = Q_2Q_4D + R_2Q_4 + Q_2R_4$

$= Q_4D + R_2Q_4 + R_4$

$= Q_4x^2 - 2i$

$= x^4 - (1+i)x^2 - 2i$

$\boxed{7D}$ (1) $\omega^2 = \dfrac{-1-\sqrt{3}i}{2}$ (2) $\omega + \omega^2 = -1$ (3) $\omega^3 = 1$

(4) $\omega^{100} = (\omega^3)^{33} \cdot \omega = \omega$

(5) $S = \omega + \cdots + \omega^{100}$ について $(\omega-1)S = \omega^{101} - \omega = \omega^2 - \omega = \omega(\omega-1)$

$\therefore S = \omega$

$\boxed{7E}$ (1) $x^3 - x^2 - x + 1 = (x-1)(x^2-1) = (x-1)^2(x+1)$

よって 根は 1 (重複度 2) と -1 (重複度 1)

(2) $x^4+2x^2+1=(x^2+1)^2=(x+i)^2(x-i)^2$ より

根は $\pm i$ (重複度2)

(3) $x^3-3ix^2-4i=(x+i)(x^2-4ix-4)$

$$=(x+i)\left(x-\frac{2i+\sqrt{-2+4}}{2}\right)\left(x-\frac{2i-\sqrt{-2+4}}{2}\right)$$

より 根は $-i$, $i\pm\frac{\sqrt{2}}{2}$

$\boxed{7F}$ (1) $x^2-1=(x-1)(x+1)$ in $\mathbb{R}[x]$ or $\mathbb{C}[x]$

(2) $x^2+1=x^2+1$ in $\mathbb{R}[x]$

$=(x+i)(x-i)$ in $\mathbb{C}[x]$

(3) $x^3+1=(x+1)(x^2-x+1)$ in $\mathbb{R}[x]$

$=(x+1)\left(x-\frac{1+\sqrt{3}i}{2}\right)\left(x-\frac{1-\sqrt{3}i}{2}\right)$ in $\mathbb{C}[x]$

(4) $x^4-1=(x-1)(x+1)(x^2+1)$ in $\mathbb{R}[x]$

$=(x-1)(x+1)(x+i)(x-i)$ in $\mathbb{C}[x]$

(5) $x^4-4x^3+6x^2-4x=x(x^3-4x^2+6x-4)$

$=x(x-2)(x^2-2x+2)$ in $\mathbb{R}(x)$

$=x(x-2)(x-(1+i))(x-(1-i))$ in $\mathbb{C}(x)$