Fundamental groups: Geometry and Arithmetic

Organizers: Yuichiro Hoshi (RIMS), Zhi Hu (USTC), Yu Yang (RIMS)
Date: August 18, 2019-August 22, 2019
Place: USTC, Hefei, Anhui, China

Speaker: Kazumi Higashiyama (RIMS, Kyoto University)
Title: The mono-anabelian geometry of geometrically pro-p arithmetic fundamental groups of second configuration spaces
Abstract: The n-th configuration space of a hyperbolic curve is the scheme which parametrizes n-tuples of pairwise distinct points in the hyperbolic curve. Mochizuki proved the Grothendieck conjecture for hyperbolic curves. We discuss a certain pro-p version of the Grothendieck conjecture for hyperbolic curves. In this talk, we reconstruct group-theoretically the function field of a hyperbolic curve of type $(0, 3)$ from the pro-p fundamental group of the associated second configuration space equipped with the collection of decomposition groups.

Speaker: Toshiro Hiranouchi (Kyushu Institute of Technology)
Title: Finiteness theorems on fundamental groups with restricted ramification
Abstract: We introduce the fundamental group with restricted ramification for a variety over a finite field and show a finiteness property: such a fundamental group has only finitely many open subgroups with given index. In the case of a curve, the finiteness gives a function field analogue of the classical Hermite-Minkowski theorem.

Speaker: Yuichiro Hoshi (RIMS, Kyoto University)
Title: An anabelian open basis for a smooth variety
Abstract: Grothendieck predicted, in his letter to Faltings, that every smooth variety over a field finitely generated over the field of rational numbers has an open basis consisting of "anabelian" varieties. Moreover, Schmidt and Stix proved this prediction. In this talk, we discuss the generalization of this result to a smooth variety over a generalized sub-p-adic field.

Speaker: Yu Iijima (Hiroshima University)
Title: On the centralizer of the image of the universal outer monodromy representation of the moduli stack of pointed smooth curves
Abstract: By considering the etale homotopy exact sequence induced by the tautological curve over the moduli stack of pointed smooth curves, the etale fundamental group

of the moduli stack of pointed smooth curves has a natural outer action on the profinite surface group. This outer action is called the universal outer monodromy representation. It is known that the universal outer monodromy representation satisfies a geometric analogue of the Grothendieck Conjecture for hyperbolic curves. In this talk, I explain my recent study concerning the centralizer of the image of the universal outer monodromy representation.

Speaker: Takahiro Murotani (RIMS, Kyoto University)

Title: A p-adic analytic approach to the absolute Grothendieck conjecture

Abstract: Let K be a finite extension of $\mathbb{Q}_p$, X a hyperbolic curve over K and $\pi_1(X)$ the étale fundamental group of X. The p-adic absolute Grothendieck conjecture asks: Is it possible to recover X group-theoretically, solely from $\pi_1(X)$? To consider this problem, we introduce a certain p-adic analytic invariant defined by Serre (which we call i-invariant). Then, the absolute p-adic Grothendieck conjecture can be reduced to the following problems: (A) determining whether a proper hyperbolic curve admits a rational point from the data of i-invariants of the sets of rational points of the curve and its coverings; (B) recovering the i-invariant of the set of rational points of a proper hyperbolic curve group-theoretically. Our main results give a complete affirmative answer to (A) and a partial affirmative answer to (B). In this talk, we present these results and sketches of their proofs.

Speaker: Ippei Nagamachi (The University of Tokyo)

Title: On the Hom version of the Grothendieck Conjecture for hyperbolic polycurves of dimension 2

Abstract: Mochizuki proved the Hom version of the Grothendieck Conjecture for hyperbolic curves, that is, a conjecture that the map

$$\mathrm{Mor}_K^{\mathrm{dom}}(Y, X) \to \mathrm{Hom}_{G_K}^{\mathrm{open}}(\pi_1(Y, *), \pi_1(X, *))/\mathrm{Inn}$$

is bijective for any sub-p-adic field K, any hyperbolic curve X over K, and any smooth variety Y over K. In the case where X is a hyperbolic polycurve of dimension 2, Hoshi proved the injectivity of the map and that any element $\phi \in \mathrm{Hom}_{G_K}^{\mathrm{open}}(\pi_1(Y, *), \pi_1(X, *))/\mathrm{Inn}$ with topologically finitely generated kernel is in the image of the map. In this talk, we give a "group theoretic" condition for ϕ such that ϕ satisfies this condition if and only if ϕ is in the image of the map. We also show that any open outer homomorphism is induced by a morphism of the varieties under the assumption that the Grothendieck Section Conjecture holds.

Speaker: Akira Sarashina (RIMS, Kyoto University)

Title: Reconstruction of curves in positive characteristic by their geometric fundamental groups

Abstract: Our main question is whether the isomorphism class as a scheme of a curve X over $\overline{\mathbb{F}}_p$ can be reconstructed by the étale fundamental group of X. First we discuss the reconstruction of various invariants. Then we introduce the reconstruction of the isomorphism class as a scheme of a curve in special cases.

Speaker: Koichiro Sawada (RIMS, Kyoto University)

Title: Reconstruction of invariants of configuration spaces of hyperbolic curves from associated Lie algebras

Abstract: Configuration spaces of hyperbolic curves are typical examples of higher dimensional anabelian varieties. Hoshi, Minamide, and Mochizuki gave certain explicit group-theoretic algorithm for reconstructing some objects from the étale fundamental group of the configuration space of a hyperbolic curve over an algebraically closed field of characteristic zero. In this talk, we consider "Lie algebra analogues" of these reconstruction algorithms. More precisely, we discuss algorithms for reconstructing some geometric invariants from (the Lie algebra obtained by forgetting the grading of) the graded Lie algebra associated to the configuration space of a hyperbolic curve. If time permits, we also discuss various applications to anabelian geometry.

Speaker: Ruiran Sun (Mainz University)
Title: TBA
Abstract: TBA

Speaker: Naotake Takao (RIMS, Kyoto University)
Title: On anabelian geometry of moduli spaces of curves

Abstract: After reviewing several results on the Grothendieck conjecture on moduli spaces of curves of genus less than two, I will discuss a proof of the Grothendieck conjecture for a certain type of finite étale covers of the moduli space of curves of genus two with ordered r marked points over a field of characteristic zero. I will also give a proof of the Grothendieck conjecture for a certain type of hyperbolic polycurves (i.e., algebraic varieties in the form of successive fiberations by hyperbolic curves).

Speaker: Shota Tsujimura (RIMS, Kyoto University)
Title: Combinatorial anabelian geometry and resolution of nonsingularities

Abstract: In this talk, we first introduce the theory of combinatorial anabelian geometry [developed by Y. Hoshi, S. Mochizuki] and the theory of resolution of nonsingularities [developed by A. Tamagawa, E. Lepage]. After introducing these theories, we also discuss two applications of these theories. Let us write GT_p for the p-adic Grothendieck-Teichmüller group defined by Y. André. The first application is a construction of a natural surjection from GT_p to the absolute Galois group of $\mathbb{Q}_p$. The second application is ongoing joint work with Y. Yang on the finiteness of the set of isomorphism classes of genus 0 curves whose geometric tempered fundamental groups are isomorphic.

Speaker: Jinbang Yang (Mainz University)
Title: TBA
Abstract: TBA

Speaker: Yu Yang (RIMS, Kyoto University)
Title: On the topological and combinatorial structures of pointed stable curves in positive characteristic

Abstract: Some developments of F. Pop-M. Saïdi, M. Raynaud, and A. Tamagawa from the 1990's showed evidence for very strong anabelian phenomena for curves over algebraically closed fields of characteristic $p > 0$. In this situation, the Galois group of the

base field is trivial, and the arithmetic fundamental group coincides with the geometric fundamental group, thus in a total absence of a Galois action of the base field. This kind of anabelian phenomena go beyond Grothendieck's anabelian geometry, and shows that a lot of geometric information of pointed stable curves can be carried out by their geometric fundamental groups. In this talk, I will discuss some results concerning the anabelian phenomena for the topological and combinatorial structures of pointed stable curves which were proved by Tamagawa and me. In particular, I will explain an anabelian formula for topological types (i.e., (g, n)) of arbitrary pointed stable curves.

Speaker: Kang Zuo (Mainz University)

Title: An algebraic characterization of modular curves

Abstract: Given a smooth projective curve C over a number ring together with a divisor D of finitely many points in C. We show $C \setminus D$ is a modular curve if and only if the logarithmic uniformizing Higgs bundle on C is 1-periodic at a p-adic place and the traces of Frobenius on the F-crystal corresponding to the periodic Higgs bundle are rational at all closed points in C/k_p. The proof relies on the recent work by T. Abe on Deligne's conjecture on p to l companies and Drinfeld's work on Langlands program over function field. This is a joint project with Raju Krishnamoorthy and Jinbang Yang.