3RD KYOTO-HEFEI WORKSHOP ON ARITHMETIC GEOMETRY

Speaker: Taylor Dupuy (The University of Vermont)
Title: Angle Ranks of Abelian Varieties over Finite Fields
Abstract: Abstract: Using the formalism of Newton hyperplane arrangements, we resolve the open questions regarding angle rank left over from the Dupuy-Kedlaya-Roe-Vincent paper on the LMFDB database of isogeny classes of abelian varieties over finite fields:

https://arxiv.org/abs/2003.05380.

The database can be found here (check it out!):

https://www.lmfdb.org/Variety/Abelian/Fq/

As a consequence we end up generalizing theorems of Lenstra-Zarhin and Tankeev proving several new cases of the Tate conjecture for abelian varieties over finite fields. The is joint work with Kiran Kedlaya and David Zureick-Brown.

Speaker: Ziyang Gao (Leibniz Universität Hannover)
Title: A proof of the Uniform Mordell-Lang Conjecture
Abstract: Let A be an abelian variety and let X be a subvariety, both defined over $\overline{\mathbb{Q}}$. For any finite rank subgroup Γ of $A(\overline{\mathbb{Q}})$, the famous Mordell-Lang Conjecture predicts that each component of $X \cap \Gamma$ is a coset of A. This conjecture is proved by Faltings and one also needs a result of Hindry to handle division points.

The Uniform Mordell-Lang Conjecture predicts that the number of irreducible components concerned above is bounded solely in terms of $\dim A$, $\deg X$ and the rank of Γ. The question was posed by Mazur and David-Philippon. Recently this conjecture is proved in a series of work (Dimitrov-Gao-Habegger, Kühne; Gao-Ge-Kühne). In this talk I will report this proof. I will focus on the case of rational points on curves and then explain how to generalize this method to the general case. This is a joint project with Vesselin Dimitrov, Philipp Habegger; Tangli Ge, Lars Kühne.

Speaker: Wanlin Li (Université de Montréal)
Title: A generalization to Elkies's theorem
Abstract: Elkies proved that for a fixed elliptic curve over $\mathbb{Q}$, there exist infinitely many primes at which its reductions are supersingular. In this talk, we give the first generalization to Elkies's theorem for some curves of genus > 2. We consider families of

cyclic covers of the projective line ramified at 4 points whose moduli space is embedded in a Shimura curve. This is joint work in progress with Elena Mantovan, Rachel Pries, and Yunqing Tang.

Speaker: Dongwen Liu (Zhejiang University)
Title: Period relations for Rankin-Selberg convolutions
Abstract: In this talk we explain the period relations for critical values of Rankin-Selberg L-functions, for the case of $GL(n) \times GL(n-1)$. The main novelty is a proof of archimedes period relations, which is based on an explicit comparison between Rankin-Selberg integrals and the integrals over the open orbit of certain spherical variety. It is based on a recent joint work with Jian-Shu Li and Binyong Sun.

Speaker: Arata Minamide (RIMS, Kyoto University)
Title: Anabelian group-theoretic properties of the absolute Galois groups of Henselian discrete valuation fields
Abstract: Let K be a Henselian discrete valuation field of characteristic zero whose residue field k is a field of characteristic $p > 0$. Write G_K for the absolute Galois group of K. In the case where k is finite, it is known that G_K, as well as any almost pro-p-maximal quotient of G_K, satisfies an "anabelian" group-theoretic property called slimness (i.e., the property that the center of every open subgroup is trivial). In this talk, we introduce the notion of strong internal indecomposability, which is a stronger property than slimness, and show that G_K, as well as any almost pro-p-maximal quotient of G_K, satisfies strong internal indecomposability (without imposing the finiteness assumption on k). As an application, we give a group-theoretic algorithm for reconstructing the prime number p from G_K. This is joint work with Shota Tsujimura.

Speaker: Ippei Nagamachi (RIMS, Kyoto University)
Title: On behavior of conductors, Picard schemes, and Jacobian numbers of varieties over imperfect fields
Abstract: Let X be a regular geometrically integral variety over a field K of characteristic p. If K is imperfect, X may have singular points. Various invariants for such singular points and X have been studied. For example, Tate proved that the difference between the genus of the normalization of $X \otimes_K K^{1/p}$ and the genus of X is divisible by $(p-1)/2$ in the case where X is a curve. Patakfalvi and Waldron proved that the conductor divisors of such singular points are divisible by $p - 1$. In this talk, we introduce a new invariant q and calculate arithmetic genera and conductors by using q. Moreover, by using q, we explain the relation of Jacobian numbers and arithmetic genera. This is a joint work with Teppei Takamatsu.

Speaker: Masao Oi (Kyoto University)
Title: A recovering result of Deligne-Lusztig representations from their characters
Abstract: Let us suppose that a representation of a group is given. When we want to determine what this representation is, it is often convenient to look at the (trace) characters of the representation. In this talk, we consider such a problem for Deligne-Lusztig representations, which are representations of a finite reductive group constructed by Deligne and Lusztig in 1976. Especially, I will explain that Deligne-Lusztig representations can be

determined only by their characters at regular semisimple elements of a finite reductive group. This is a joint work with Charlotte Chan (Univ. of Michigan).

Speaker: Kenji Sakugawa (Shinshu University)
Title: On the $\mathbb{R}$-mixed Hodge structure on the relative pro-unipotent fundamental group of $\mathcal{M}_{1,1}$
Abstract: Let $\mathcal{M}_{1,1}$ be the moduli stack of elliptic curves. The relative pro-unipotent fundamental group of $M_{1,1}$ is a Tannakian fundamental group classifying local systems over $\mathcal{M}_{1,1}$ whose simple factors are isomorphic to relative middle cohomology groups of open Kuga-Sato varieties over $\mathcal{M}_{1,1}$. The mixed Hodge structure on it was first defined in a more general context by Hain, and more detailed studies have recently been started by Hain and Brown. In this talk, we will discuss real mixed Hodge structure on the relative pro-unipotent fundamental group in length two.

Speaker: Junyi Xie (BICMR, Beijing)
Title: Geometric Bogomolov conjecture
Abstract: With Xinyi Yuan, we prove the full geometric Bogomolov conjecture. We first reduce it to the case that the extension of the base fields has transcendence degree 1, and then we prove the later case by intersection theory. The proof uses Yamaki's reduction theorem on the geometric Bogomolov conjecture and the Manin-Mumford conjecture proved by Raynaud and Hrushovski.

Speaker: Daxin Xu (MCM, Beijing)
Title: Parallel transport for Higgs bundles on p-adic curves
Abstract: Faltings conjectured that under the p-adic Simpson correspondence, finite dimensional p-adic representations of the geometric étale fundamental group of a smooth proper p-adic curve X are equivalent to semi-stable Higgs bundles of degree zero over X. We will talk about an equivalence between these representations and Higgs bundles, which potentially admit a strongly semi-stable of degree zero reduction for their underlying vector bundle. These Higgs bundles are semi-stable of degree zero and we will investigate some evidences for Faltings' conjecture.

Speaker: Bogdan Zavyalov (MPI, Bonn)
Title: Mod-p Poincaré Duality in p-adic Analytic Geometry
Abstract: Étale cohomology of $\mathbb{F}_p$-local systems does not behave nicely on general smooth p-adic rigid-analytic spaces; e.g., the $\mathbb{F}_p$-cohomology of the 1-dimensional closed unit ball is infinite. However, it turns out that the situation is much better if one considers only proper rigid-analytic spaces. These spaces have finite $\mathbb{F}_p$-cohomology groups and these groups satisfy Poincaré Duality if X is smooth and proper.

I will explain how one can prove such results using the concept of almost coherent sheaves that allows to "localize" such questions in an appropriate sense and actually reduce to some local computations. I will also mention some generalizations of these results in the relative setting (in progress).

Speaker: Hongjie Yu (IST Austria)

Title: Deligne's conjectures on the number of l-adic local systems over a punctured curve

Abstract: Deligne conjectured that the number of l-adic local systems over a curve in positive characteristics with fixed local monondromies and fixed by the Frobenius endomorphism should look like the number of $\mathbb{F}_q$ points of a variety, although the naive way to define a moduli space of l-adic local systems does not work. In this talk, I will introduce his conjectures over a punctured curve. I will also introduce a residue morphism on the moduli space of Higgs bundles and prove his conjectures for tame regular cases. If time permits, I may also discuss automorphic forms for split reductive groups.