4th Kyoto-Nanjing Workshop on Arithmetic and Geometry

Schedule

November 14 (Mon)

11:00 – 12:00 Séverin Philip (Kyoto University)

Title: Fields of definition of endomorphisms and abelian subvarieties

Abstract: I will start by introducing abelian varieties and some results on their endomorphisms algebra. I will then present recent results of Guralnick-Kedlaya and Rémond on the degrees of the fields of definition of those algebras and how to use these results to obtain optimal bounds on the fields of definition of abelian subvarieties.

12:15 – 13:15 Yang Cao (University of Science and Technology of China)

Title: Relative version of weak approximation for abelian schemes

Abstract: Over number field, weak approximation for algebraic varieties is about the density of rational points in adelic points. In this talk, for an abelian scheme over a rational variety, instead of considering the rational points, we consider the relative closed subgroup generated by rational points in adelic points of the total space, and study its relationship with cohomological obstruction.

15:00 – 16:00 Zhiyu Tian (Peking University)

Title: Space of cycles and coniveau filtrations

Abstract: All cycles of a fixed dimension on a variety has more structures than simply being a group. It is could be given a topology or the structure of a sheaf. Invariants coming from this finer structure gives information about the cycle theory on a variety.

In this talk, I will explain a structural result about the space of one cycles on separably rationally connected varieties, either as a topological group or an h sheaf, and discuss some of its applications in understanding the coviveau filtration on cohomology, and certain torsion algebraic classes.

16:15 – 17:15 Tongmu He (IHES)

Title: Sen Operators and Lie Algebras arising from Galois Representations over p-adic Varieties

Abstract: Any finite-dimensional p-adic representation of the absolute Galois group of a p-adic local field with imperfect residue field is characterized by its arithmetic and geometric Sen operators defined by Sen and Brinon. We generalize their construction to the fundamental group of a p-adic affine variety with a semi-stable chart, and prove that the module of Sen operators is canonically defined, independently of the choice of the chart. Our construction relies on a descent theorem in the p-adic Simpson correspondence developed by Tsuji. When the representation comes from a $\mathbb{Q}_p$ representation of a p-adic analytic group quotient of the fundamental group, we relate the infinitesimal action of inertia subgroups with Sen operators, which is a generalization of a result of Sen and Ohkubo. These Sen operators can be extended continuously to certain infinite-dimensional representations. As an application, we prove that the geometric Sen operators annihilate locally analytic vectors, generalizing a result of Pan.

November 15 (Tue)

11:00 – 12:00 Ziquan Zhuang (Johns Hopkins University)

Title: Equivariant K-stability

Abstract: Suppose X is a Fano manifold defined over a number field K, and suppose it has a Kähler–Einstein metric. Then is the metric also "defined over K"? This question is motivated by a classical theorem of Matsushima that says the Kähler–Einstein metric is invariant under some maximal compact subgroup of the automorphism group of X (as a complex manifold); what we ask about is the "Galois invariance" of the metric. I will talk about an answer to this question in terms of equivariant K-stability, an algebraic stability condition that characterizes the existence of Kähler–Einstein metrics.

12:15 – 13:15 Haoyang Guo (MPIM)

Title: Prismatic approach to Fontaine's C_{crys} conjecture

Abstract: Given a smooth proper scheme over a p-adic ring of integers, Fontaine's C_{crys} conjecture says that the étale cohomology of its generic fiber is isomorphic to the crystalline cohomology of its special fiber, after base changing them to the crystalline period ring. In this talk, we give a prismatic proof of the conjecture, for general coefficients, in the relative setting, and allowing ramified base rings. This is a joint work with Emanuel Reinecke.

15:00 – 16:00 Keita Goto (Kyoto Univeristy)

Title: On affine structures coming from Berkovich Geometry

Abstract: In the SYZ mirror symmetry context, SYZ fibrations are often studied. In particular, it is well known that they induce affine structures on their base spaces. On the other

hand, Kontsevich and Soibelman introduce a non-Archimedean analog of SYZ fibrations, which is called non-Archimedean SYZ fibrations later. Non-Archimedean SYZ fibrations induce affine structures as well as SYZ fibrations. Moreover, they predict a certain equivalence between SYZ fibrations and non-Archimedean SYZ fibrations for maximally degenerating family of polarized Calabi-Yau varieties. In particular, two affine structures coming from these different origins are expected to coincide. We proved the equivalence for K-trivial finite quotients of polarized abelian varieties by introducing what we call "hybrid SYZ fibrations". In this talk, we introduce the above our theorem. This talk is based on the joint work with Yuji Odaka.

16:15 – 17:15 Béranger Seguin (Université de Lille)

Title: Geometric methods for inverse Galois theory

Abstract: Inverse Galois theory belongs to a family of more general counting problems for field extensions over some base field K. These questions are related with the count of covers defined over K, and incidentally with the count of K-points on a moduli space of covers, called a Hurwitz space. The ring of components is a tool introduced by Ellenberg, Venkatesh and Westerland to study the homology of these moduli spaces. We will discuss variations on their idea. We first look at the ring of components of covers of the complex projective line, which is commutative, and relate the geometry of its spectrum to group-theoretic and combinatorial quantities. We also define a version of their ring which is aware of arithmetic phenomena.

November 16 (Wed)

11:00 – 12:00 Mingshuo Zhou (Tianjin University)

Title: Moduli space of parabolic bundles over a curve

Abstract: We firstly recall the degeneration argument of moduli space for Verlinde formula, and then, use above technique to prove F-splitting of moduli space of parabolic bundles in positive characteristic (for generic curve and generic parabolic points). This is a joint work with Professor Xiaotao Sun.

12:15 – 13:15 Nobuo Sato (National Taiwan University)

Title: On relations among block-indexed multiple zeta values

Abstract: The multiple zeta values, or MZV in short, is a generalization of the Rieman zeta functions evaluated at positive integers. In my talk, I would like to address recent results on linear relations among block-indexed MZVs obtained in a serial collaboration with Minoru Hirose. To motivate our work, I will start with reviewing a conjecture by Borwein-Bradley-Broadhurst-Lisonek (abbreviated as BBBL in the sequel) and Charlton's generalization of BBBL's conjecture (and some other conjectures) introducing the block-indexed MZVs. The main result of our work is a family of relations among block-indexed MZVs, which we call block shuffle relations. Via combinatorial identities, all the conjectures above (BBBL,

Charlton) follows from the block shuffle relation, so our work simultaneously resolve and generalizes those conjectures. In terms of the dimension, the block shuffle relation seems to give a "big" portion of the linear relations of block-indexed MZVs, yet not all of them. I will end my talk by remarking on observations on certain exotic relations not contained in the block shuffle relations.

15:00 – 16:00 Quentin Gazda (MPIM)

Title: Anderson A-Motives and their regulators

Abstract: Interesting zeta values in function field arithmetic arise through L-series of Anderson A-motives. My dream would be to have a Beilinson type conjecture for those objects, relating these L-values to some regulator. I will report on some work towards this program: after general recalls on the theory of A-motives, I will explain how to define an "A-motivic cohomology". I will also discuss how to construct a regulator given an A-motive. If time allows, I will conclude on some recent speculations and computations joint with A. Maurischat.

16:15 – 17:15 Gregorio Baldi (IHES)

Title: The Hodge locus

Abstract: I will report on a joint work with Klingler and Ullmo. Given a polarizable variation of Hodge structures on a smooth complex quasi-projective variety S (e.g. the one associated to a family of pure motives over S), Cattani, Deligne and Kaplan proved that its Hodge locus (the locus of closed points of S where exceptional Hodge tensors appear) is a *countable* union of closed algebraic subvarieties of S. In this talk I will discuss when this Hodge locus is actually algebraic.

If time permits I will explain how such an algebraicity result complements the Lawrence-Venkatesh method.

November 17 (Thu)

11:00 – 12:00 Sean Howe (The University of Utah)

Title: Special subvarieties of basic local Shimura varieties and their non-miniscule analogs

Abstract: Complex Shimura varieties are constructed as moduli of Hodge structures, and the special subvarieties are cut out by imposing conditions on the Mumford-Tate groups, or Tannakian structure groups, of these Hodge structures (equivalently, by enforcing that certain tensors be Hodge tensors). In this talk, reporting on joint work with Christian Klevdal, we will explain how basic local Shimura varieties and their non-miniscule analogs (as constructed by Scholze) can be viewed as moduli of p-adic Hodge structures, with special subvarieties cut out by conditions on the Mumford-Tate group of a Tannakian category of p-adic Hodge structures (or basic admissible pairs). We will discuss an Ax-Lindemann type theorem in this context, which is the main source of our interest, but the focus of the talk

will instead be on explaining why non-reductive Mumford-Tate groups are essential to the theory and explaining some basic facts and questions about their geometry.

12:15 – 13:15 Zhiyuan Ding (ShanghaiTech University)

Title: The Manin-Drinfeld problem for shtukas

Abstract: In this talk, we will discuss linear equivalences between cuspidal divisors on the moduli stack of Drinfeld shtukas. We exhibit explicit linear equivalences with the help of auxiliary moduli spaces of toy shtukas. Then we use compactification to determine the space of principal cuspidal divisors. Unlike the Manin-Drinfeld theorem for modular curves, in our case, principal cuspidal divisors form a nontrivial subspace of zero degree cuspidal divisors.

15:00 – 16:00 Tony Feng (University of California, Berkeley)

Title: Arithmetic Fourier analysis and modularity

Abstract: Classical theta series are generating functions for counting vectors in a lattice. They turn out to have a miraculous symmetry property called modularity, which is proved by some simple (by modern standards) Fourier analysis. Kudla discovered analogs of theta series in arithmetic geometry, called arithmetic theta series, which are generating functions composed of algebraic cycles in moduli spaces. These are also expected to enjoy modularity, but this is unknown in most cases and has been very difficult in the known cases. In joint work with Zhiwei Yun and Wei Zhang, we give a proof of a modularity property for arithmetic theta series in the function field context, which works in total generality. The argument is built on a new variant of Fourier theory that we call arithmetic Fourier analysis.

16:15 – 17:15 Alexander Petrov (MPIM)

Title: Decomposability of the de Rham complex of varieties in positive characteristic

Abstract: Deligne and Illusie proved that if a smooth variety X over $\mathbb{F}_p$ lifts to $\mathbb{Z}/p^2$ then the truncation in degrees $< p$ of its de Rham complex is quasi-isomorphic to the direct sum of its cohomology sheaves. As a consequence, they proved that the Hodge-to de Rham spectral sequence of a liftable variety of dimension $\leq p$ degenerates.

I will describe the obstruction to decomposing the truncation in degrees $< p + 1$ of the de Rham complex of a liftabe variety, and will use this description to give an example of a smooth projective variety over $\mathbb{F}_p$ that lifts to $\mathbb{Z}_p$ but whose Hodge-to de Rham spectral sequence does not degenerates at the first page. The proof relies on the existence of Hodge-Tate cohomology, and along the way gives a way to describe the action of the Sen operator on a truncation of the de Rham complex. The key computation needed for the proof will be reminiscent of the algebraic construction of Steenrod operations by May.