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Parametrices of effectively hyperbolic operators

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Introduction

Following Hörmander[3], we call effective hyperbolic the class of weakly hyperbolic operators which have non-vanishing real eigenvalues in their Hamilton maps (fundamental matrices) defined at the critical sets of the principal symbols (See Appendix I; For the terminology, we refer to Ivrii-Petkov[5]. Also [3]).

We list some examples:

$$D_t^2 - t^2 D_x^2 + \cdots, \tag{1}$$

$$D_t^2 - (t^2+x^2) D_x^2 + \cdots, \tag{2}$$

$$D_t^2 - t^2 D_x^2 - D_y^2 + \cdots, \tag{3}$$

$$D_t^2 - (t^2+x^2+y^2) D_x^2 - D_y^2 + \cdots. \tag{4}$$

Here $\cdots$ stands for lower order terms, and

$$D_t = -\sqrt{-1}\ \partial/\partial t, \quad D_x = -\sqrt{-1}\ \partial/\partial x \quad \text{etc.}$$

The Hamilton maps of the above operators are given in Appendix I.

The operator (1) has a smoothly factorizable principal symbol.

A class containing this operator is now extensively studied (References omitted). It is worth noting the following observation of Chi[2]: for a non-negative integer N, the solution u(t,x) of

$$D_t^2 u - t^2 D_x^2 u + \sqrt{-1}\,(4N+1) D_x u = 0 \qquad (1')$$

$$u|_{t=0} = f(x), \quad D_t u|_{t=0} = 0,$$

is given by

$$u(t,x) = \sum_{k=0}^{N} \frac{\sqrt{\pi}\, t^{2k}}{k!(n-k)!\Gamma(k+1/2)} [(\sqrt{-1}\, D_x)^k f](x+t^2/2)$$

(See also Oleinik[6]).

This explicit formula in fact led me to a study of the above mentioned class of effective hyperbolic operators, with a motivation to know how the quantity N expressing the loss of derivatives of the solution with respect to the initial data is related to the lower order terms of the operator (here $\sqrt{-1}\,(4N+1)D_x$). Now I know that this quantity is expressed in terms of the subprincipal symbol at the double characteristic points combined with the real eigenvalues of the Hamilton map. This is in a sense common to effective hyperbolicity.

Returning to the operators listed in the above, a parametrix of the operator (3) was constructed by Alinhac[1]. On the other hand, between the operators (2) and (3) there is a difference in the symplectic geometries of the characteristic sets. Owing to this fact, Alinhac's construction does not work for the operators of the form (2) (See Appendix II for related discussions).

The purpose of the present talk is to give parametrices to the Cauchy problem associated to a class of operators containing the operator (2). We also indicate some properties of these parametrices, such as estimates and wave-front sets. The details are expounded in Yoshikawa[7].

As to the operator (4), purely imaginary eigenvalues appear in the Hamilton map (See Appendix I). For such operators, by the method of separation of variables, we can apply the results about the operators of the form (2) or (3). However, as for detailed properties of solutions of the operators of the form (4), including propagations of singularities, there still remains much to investigate (cf. Alinhac[1]).

It should also be mentioned that the operators close to the above types were discussed by Ivrii[4] in a way analogous to Oleinik[6]. Our explicit construction makes some of Ivrii's observations precise.

Main Results.

As our main results, we give representation formulae of parametrices to a class of effectively hyperbolic operators containing the operators (2) and (3). Properties of parametrices are more or less derived from these formulae.

Hypothesis. We begin by stating our hypothesis. Let U be a bounded open neighborhood of the origin in the Euclidean n-space R^n. Consider the operator:

$$P = D_t^2 - A(t) + B(t)D_t, \tag{5}$$

where A(t), B(t) are classical pseudo-differential operators in U, respectively of order 2 and of order 0, both properly supported, and depending on a C^∞ parameter t running in a neighborhood of t = 0.

Let $A(x,\xi,t)$ be the full symbol of A(t) with the homogeneous expansion:

$$A(x,\xi,t) \sim \sum_{i=0}^{\infty} A^{2-i}(x,\xi,t).$$

We first assume

$$A^2(x,\xi,t) = a(x,\xi) + b(x,\xi,t)^2 t^2, \tag{6}$$

where $a(x,\xi)$ and $b(x,\xi,t)$ are respectively homogeneous of degree 2 and of degree 1 such that

$$a(x,\xi) \geqq 0, \tag{6a}$$

$$b(x,\xi,t) > 0. \tag{6b}$$

We then require a decomposition of coordinates $x = (x',x'')$, $x' = (\cdots,x_{n'})$, $x'' = (x_{n'+1},\cdots,x_n)$, $0 \leqq n' < n$, whence x' may be void, such that

$$a(x,\xi) = 0 \text{ if and only if } x'' = 0, \tag{7}$$

and that the quadratic form in $X'' = (X_{n'+1},\cdots,X_n)$:

$$a_2(x',\xi;X'') = \sum_{|\alpha''|=2} \partial_{x''}^{\alpha''} a(x',0,\xi)\cdot X''^{\alpha''}/\alpha''! \tag{8}$$

is positive definite (See Appendix I). Note that $a_2(x',\xi;X'')$ is essentially the Hessian of $a(x,\xi)$ at its critical set $x'' = 0$.

The operator P is said to be of class $(EH)_0$ if P fulfills the requirements (6) ~ (8) (EH for effective hyperbolicity).

Symbol class. Parametrices of operators of class $(EH)_0$ will be expressed in terms of the symbol class S^d. Let $T > 0$. For real d, we denote by $S^d = S^d(U,T)$ the totality of C^∞ functions $p(x,y,\xi,t,r,s)$, $x, y \in \overline{U}$, $0 \leq s \leq r \leq t \leq T$, vanishing in a neighborhood of $\xi = 0$, for which we have the estimates:

$$|\partial_x^\alpha \partial_y^\beta \partial_\xi^\gamma \partial_t^i \partial_r^j \partial_s^k p(x,y,\xi,t,r,s)| \leq$$

$$\leq C(1+|\xi|)^{d+(|\alpha''|+|\beta''|+i+j+k)/2-|\gamma|}$$

for $x, y \in \overline{U}$, $\xi \neq 0$, $0 \leq s \leq r \leq t \leq T$.

Even if some of the arguments x, y, ξ, t, r, s are absent, we use the notation $p \in S^d$. We often write $p = p(t,r,s)$, $p(t,r)$ or $p(s)$ etc to indicate p really depends on which of the parameters t, r, s. On the other hand, if we write $p = p(x',y'',\xi,t,r)$ etc, this means that p is independent of x'', y', s, etc.

Phase functions. First of the crucial steps in our construction is the following observation concerning the phase functions $\phi^\pm(x',y'',\xi,t,r)$ determined from:

$$\phi_t^\pm = \pm\sqrt{A^2(x',-\phi_{\xi''}^\pm,\phi_{x'}^\pm,\xi'',t)}\ ,$$

$$\phi^\pm|_{t=r} = \langle x',\xi'\rangle - \langle y'',\xi''\rangle$$

when $y'' \neq 0$.

<u>Lemma.</u> When restricted to $(|y''|^2+r^2)|\xi| \geqq \eta_1$, $|\xi| \geqq \eta_0$, η_0, $\eta_1 > 0$,

$$\phi^{\pm},\ \phi^{\pm}_{y''} \in S^1.$$

Furthermore, there is a family of functions $\phi^{\pm}_i(x',y'',\xi,t,r)$ such that

$$\phi^{\pm} \sim \textstyle\sum_{i=0}^{\infty} \phi^{\pm}_i(x',y'',\xi,t,r),$$

$$\phi^{\pm}_0 = \langle x',\xi'\rangle,$$

$$\phi^{\pm}_1 = -\langle y'',\xi''\rangle,$$

$$\phi^{\pm}_2 = \pm b(x',0,\xi,0) \int_r^t \sqrt{\theta^2 + \rho(x',y'',\xi)}\, d\theta,$$

and for $i \geqq 2$

$$\phi^{\pm}_i \sim |\xi|\tau(t)^i \textstyle\sum_{k=0}^{i-1} [\log \frac{\tau(t)}{\tau(r)}]^k \{\textstyle\sum_{j=0}^{\infty} \tau(t)^{-j} \phi^{\pm}_{ikj}(x',y'',\xi,r)\}.$$

Here

$$\rho(x',y'',\xi) = a_2(x',\xi;y'')/b(x',0,\xi,0)^2$$

and

$$\tau(t) = \tau(x',y'',\xi,t) = t + \sqrt{t^2 + \rho(x',y'',\xi)}\ .$$

(For the details, including the meaning of ~, see [7]).

Let

$$\Phi^{\pm}(t,r) = \phi^{\pm}(x',y'',\xi,t,r) - \langle y',\xi'\rangle + \langle x'',\xi''\rangle.$$

In view of Lemma, we can introduce, as an oscillatory integral, the following variant of Fourier integral operators:

$$I(p(t,r,s);\Phi^{\pm}(t,r))u(x) =$$

$$= (2\pi)^{-n} \iint e^{\sqrt{-1}\Phi^{\pm}(t,r)} p(t,r,s)u(y)\, dyd\xi \qquad (9)$$

for $p(t,r,s) = p(x,y,\xi,t,r,s) \in S^d$ supported in $|\xi| \geqq \eta_0$, $(|y''|^2+r^2)|\xi| \geqq \eta_1$; $u \in \mathcal{E}'(U)$. A similar notation to (9) is used when $\Phi^{\pm}$ is replaced by

$$\Psi = \langle x-y,\xi\rangle.$$

Thus, $I(p;\Psi)u$ is a pseudo-differential operator with the amplitude function $p = p(x,y,\xi,t,r,s) \in S^d$.

Parametrices. Let $s \geqq 0$. Consider the problem:

$$Pu = 0, \quad t \geqq s,$$
$$D_t^i u|_{t=s} = f_i, \quad i = 0, 1, \qquad (10)$$

with prescribed f_i, $i = 0, 1$. For sufficiently small U and T, we are now ready to give a pair of parametrices $E_i(t,s)$, $i = 0, 1$, to the problem (10) such that $PE_i(t,s)$ are of C^∞ kernels in $x, y \in \overline{U}$, $0 \leqq s \leqq t \leqq T$, and that $D_t^i E_k(t,s)|_{t=s} - \delta_k^i I$ are of C^∞ kernels in $x, y \in \overline{U}$, $0 \leqq s \leqq T$.

Let

$$m^{\pm} = \frac{1}{2}\sup[\pm\frac{\operatorname{Im} A^1(x',0,\xi,0)}{b(x',0,\xi,0)}],$$

the suprema being taken over $(x',0) \in \overline{U}$, $\xi \in R^n\backslash\{0\}$. Note that $-A^1(x',0,\xi,0)$ is the value of the subprincipal symbol of the operator P at the double characteristic points and $\pm b(x',0,\xi,0)$ are the real eigenvalues of the Hamilton map.

Let $S^{d+0} = \bigcap_{\eta>0} S^{d+\eta}$. Our basic results now read as follows.:

Theorem 1. Let P be an effectively hyperbolic operator of class $(EH)_0$. Assume U and T are sufficiently small. Then we have pairs of amplitude functions:

$$p^{i\pm}(t,s) \in S^{1/4-3i/4+|m^{\pm}|/2+0},$$

$$q^{i\pm}(t,r,s) \in S^{1-i/2+|m^{\pm}|/2+0},$$

$$q^{i}(t,s) \in S^{-i/2},$$

$i = 0, 1$, such that the oscillatory integrals:

$$E_i(t,s)u =$$

$$= \sum_{\pm} I(p^{i\pm}(t,s);\Phi^{\pm}(t,s))u + I(q^{i}(t,s);\Psi)u +$$

$$+ \sum_{\pm} \int_s^t I(q^{i\pm}(t,r,s);\Phi^{\pm}(t,r))u\, dr,$$

$u \in \mathcal{E}'(U)$, give a pair of local parametrices to the problem (10). Here $p^{i\pm}(t,s)$ are chosen independent of x" in U and supported in $|\xi| \geqq \eta_0$, $(|y''|^2+s^2)|\xi| \geqq \eta_1$, $q^{i\pm}(t,r,s)$ are supported in $|\xi| \geqq \eta_0$, $(|y''|^2+s^2)|\xi| \leqq \eta_2$, $(|y''|^2+r^2)|\xi| \geqq \eta_3$, and $q^{i}(t,s)$ in $|\xi| \geqq \eta_0$, $(|y''|^2+s^2)|\xi| \leqq \eta_2$, $(|x''|^2+t^2)|\xi| \leqq \eta_4$. Here $\eta_i > 0$, $i = 0,1,2,3,4$.

The proof of Theorem 1 and related discussions are found in [7]. In particular, corresponding to the operator (3) we have a microlocalized subclass of $(EH)_0$, for which the above representation formulae take a much simpler form (microlocally).

Wave-front sets. To describe the wave-front sets of $E_i(t,s)$, we first extend the bicharacteristic relations associated to $\xi_0 \pm \sqrt{A^2(x,\xi,t)}$. Let $(X^{\pm}(y,\zeta,t,r),\Xi^{\pm}(y,\zeta,t,r))$ be the solutions of

$$\dot{X}^{\pm} = \mp A^2_{\xi}(X^{\pm},\Xi^{\pm},t)/2\sqrt{A^2(X^{\pm},\Xi^{\pm},t)},$$

$$\dot{\Xi}^{\pm} = \pm A^2_{x}(X^{\pm},\Xi^{\pm},t)/2\sqrt{A^2(X^{\pm},\Xi^{\pm},t)},$$

with the initial data:

$$(X^{\pm},\Xi^{\pm})|_{t=r} = (y,\zeta), \qquad y'' \neq 0,\ \zeta \neq 0.$$

For $U_1 \subset\subset U$, we set

$$\mathcal{B}^{\pm}(t,r;U_1) =$$

$$= \{(x,\xi,y,\zeta);\ x=X^{\pm}(y,\zeta,t,r), \xi=\Xi^{\pm}(y,\zeta,t,r), y\in U_1, y''\neq 0\}.$$

Now let us set

$$\mathcal{B}_0^{\pm}(t,r;y',\zeta) =$$

$$= \bigcap_{\eta>0} \overline{\{(X^{\pm}(y,\zeta,t,r),\Xi^{\pm}(y,\zeta,t,r),y,\zeta);\ 0<|y''|\leqq\eta\}}$$

and

$$\mathcal{B}_0^{\pm}(t,r;U_1) = \bigcup_{(y',0)\in U_1,\zeta\neq 0} \mathcal{B}_0^{\pm}(t,r;y',\zeta).$$

Finally we put

$$\tilde{\mathcal{B}}^{\pm}(t,r;U_1) = \mathcal{B}^{\pm}(t,r;U_1) \cup \mathcal{B}_0^{\pm}(t,r;U_1).$$

It is not difficult to see that $\tilde{\mathcal{B}}^{\pm}(t,r;U_1)$ are closed subsets of $(T^*U_1\backslash\{0\})\times(T^*U_1\backslash\{0\})$. $\tilde{\mathcal{B}}^{\pm}(t,r;U_1)$ are thus the extended bicharacteristic relations (See [7] for detail).

Now we have the following (probably rather rough) estimates of the wave-front sets of $E_i(t,s)$, $i = 0, 1$.

Theorem 2. Let P be of class $(EH)_0$. For sufficiently small $U_1 \subset\subset U$ and $T > 0$, we have

$$WF(E_i(t,s))|_{U_1} \subset \bigcup_{\pm} \tilde{\mathcal{B}}^{\pm}(t,s;U_1) \cup \bigcup_{\pm} \overline{\bigcup_{s \leqq r \leqq t} \mathcal{B}_0^{\pm}(t,r;U_1)}$$

when $0 \leqq s \leqq t \leqq T$. If $s > 0$, the second union in the right hand side is unnecessary. Here $WF(E_i(t,s))|_{U_1}$ is the restriction of $WF(E_i(t,s))$ to $(T^*U\setminus\{0\})\times(T^*U_1\setminus\{0\})$, and for the distribution kernels $K_i(t,s)$ of $E_i(t,s)$,

$$WF(E_i(t,s)) = WF'(K_i(t,s)) =$$
$$= \{(x,\xi,y,\zeta);\ (x,\xi,y,-\zeta) \in WF(K_i(t,s))\}.$$

For the proof, see [7]. We remark that $\tilde{\mathcal{B}}^{\pm}(t,r;U_1)$ (in their micro-localized version) as well as Theorem 2 are much simplified for the operators corresponding to (3) (Appendix II. cf. Alinhac [1]. Also [7]).

Ideas ?

Our discussions are based on the R_+-action of the form:

$$g_\lambda : (x',x'',y',y'',\xi,t,r,s) \to$$
$$\to (x',\lambda^{-1/2}x'',y',\lambda^{-1/2}y'',\lambda\xi,\lambda^{-1/2}t,\lambda^{-1/2}r,\lambda^{-1/2}s),$$

$\lambda > 0$, and their natural restrictions, also denoted by g_λ, to smaller sets of arguments. This R_+-action is closely connected with the critical set of $A^2(x,\xi,t)$. The homogeneity associated to the R_+-action g_λ is called (strong) semi-homogeneity. Thus, $p(x,y,\xi,t,r,s)$ is (strongly) semi-homogeneous of degree d if

$$(g_\lambda^* p)(x,y,\xi,t,r,s) =$$

$$= p(x',\lambda^{-1/2}x'',y',\lambda^{-1/2}y'',\lambda\xi,\lambda^{-1/2}t,\lambda^{-1/2}r,\lambda^{-1/2}s) =$$

$$= \lambda^d \; p(x,y,\xi,t,r,s).$$

Actually building of the phase functions of Lemma is based on such considerations. In particular, each $\phi_i^\pm$ is (strongly) semi-homogeneous of degree 1-i/2.

In this way, we expand everything into (strongly) semi-homogeneous parts, and apply, say, the method of indeterminate coefficients. That such a procedure makes sense is assured by the asymptotic properties of the solutions $u^\pm = u^\pm(x',y'',\xi,t)$ of the ordinary differential equations:

$$\{D_t^2 \pm \frac{2b(x',0,\xi,0)}{\sqrt{-1}}[\sqrt{t^2+\rho}\;\frac{\partial}{\partial t}+\frac{1}{2}\;\frac{1}{\sqrt{t^2+\rho}} \mp \frac{\sqrt{-1}A^1(x',0,\xi,0)}{2b(x',0,\xi,0)}]\}u^\pm = 0,$$

$\rho = \rho(x',y'',\xi)$, enjoying the asymptotic expansions:

$$u^\pm \sim (t^2+\rho)^{-1/2}\tau(t)^{1/2}\{|\xi|^{1/2}\tau(t)\}^{\mu^\pm(x',\xi)}\textstyle\sum_{j=0}^\infty u_j^\pm(x',y'',\xi,t),$$

when $y'' \neq 0$. Here

$$\mu^\pm(x',\xi) = -\frac{1}{2} \pm \frac{\sqrt{-1}}{2}\,\frac{A^1(x',0,\xi,0)}{b(x',0,\xi,0)},$$

$$u_0^\pm = 1,$$

and $u_j^\pm$ are determined from:

$$\left\{\sqrt{t^2+\rho}\ \frac{\partial}{\partial t}+\frac{1}{2}\ \frac{1}{\sqrt{t^2+\rho}} \mp \frac{\sqrt{-1}}{2}\ \frac{A^1(x',0,\xi,0)}{b(x',0,\xi,0)}\right\} u_j^{\pm} =$$

$$= \mp \frac{\sqrt{-1}}{2b(x',0,\xi,0)}\ D_t^2 u_{j-1}^{\pm}.$$

The details are again transferred to [7].

The above asymptotic results are valid when handling the amplitude functions in the domain $(|y''|^2+r^2)|\xi| \geqq \eta > 0$. To complete our construction, we need a further modification of the R_+-action g_λ and related symbol classes. Again we can say that for the operators of type (3) this remaining part becomes simpler.

Appendix I

Let P represent one of the operators (1) ~ (5), $\sigma(P)$ its principal symbol, and Σ_P the critical set of $\sigma(P)$, i.e., the double characteristic points. Let H_P be the Hessian matrix of $\sigma(P)$ at Σ_P and F_P the Hamilton map (fundamental matrix). Then if J is the matrix representation of the symplectic structure of $T^*(U\times R)$,

$$F_P = -J\, H_P.$$

For the operator (1): $\Sigma_P = \{t = \xi_0 = 0,\ \xi \neq 0\}$.

$$F_P = \left(\begin{array}{c:cc} 0 & \multicolumn{2}{c}{0} \\ \hdashline 0 & 0 & -1 \\ & -|\xi|^2 & 0 \end{array}\right)$$

eigenvalues: $0,\ \pm|\xi|$.

For the operator (2): $\Sigma_P = \{t = \xi_0 = x = 0,\ \xi \neq 0\}$.

$$F_P = \left(\begin{array}{cc|cc} 0 & 0 & & \\ -|\xi|^2 & 0 & \multicolumn{2}{c}{0} \\ \hline & & 0 & -1 \\ \multicolumn{2}{c|}{0} & -|\xi|^2 & 0 \end{array}\right) \qquad \text{eigenvalues: } 0,\ \pm|\xi|.$$

For the operator (3): $\Sigma_P = \{t = \xi_0 = \zeta = 0,\ \xi \neq 0\}$.

$$F_P = \left(\begin{array}{c|cc|cc} 0 & \multicolumn{2}{c|}{0} & \multicolumn{2}{c}{0} \\ \hline & 0 & 1 & & \\ 0 & 0 & 0 & \multicolumn{2}{c}{0} \\ \hline & & & 0 & -1 \\ 0 & \multicolumn{2}{c|}{0} & -|\xi|^2 & 0 \end{array}\right) \qquad \text{eigenvalues: } 0,\ \pm|\xi|.$$

For the operator (4): $\Sigma_P = \{t = \xi_0 = x = y = \zeta = 0,\ \xi \neq 0\}$.

$$F_P = \left(\begin{array}{cc|cc|cc} 0 & 0 & & & & \\ -|\xi|^2 & 0 & \multicolumn{2}{c|}{0} & \multicolumn{2}{c}{0} \\ \hline & & 0 & 1 & & \\ \multicolumn{2}{c|}{0} & -|\xi|^2 & 0 & \multicolumn{2}{c}{0} \\ \hline & & & & 0 & -1 \\ \multicolumn{2}{c|}{0} & \multicolumn{2}{c|}{0} & -|\xi|^2 & 0 \end{array}\right) \qquad \text{eigenvalues: } 0,\ \pm|\xi|,\ \pm\sqrt{-1}\,|\xi|.$$

For the operator (5): $\Sigma_P = \{t = \xi_0 = 0,\ x'' = 0,\ \xi \neq 0\}$.

$$F_P \sim \left(\begin{array}{c|cc|cc} 0 & \multicolumn{2}{c|}{0} & \multicolumn{2}{c}{0} \\ \hline & 0 & 0 & & \\ 0 & -A''(x',\xi) & 0 & \multicolumn{2}{c}{0} \\ \hline & & & 0 & -1 \\ 0 & \multicolumn{2}{c|}{0} & -b(x',0,\xi,0)^2 & 0 \end{array}\right)$$

where

$$\langle X'', A''(x',\xi)X''\rangle = a_2(x',\xi;X'').$$

Eigenvalues: $0, \pm b(x',0,\xi,0)$.

Appendix II

Related to the operator (3) we introduce the following micro-localized subclass $(EH,\Gamma)_{n''}$ of $(EH)_0$.

Let P be of the form (5). Let

$$n' \geqq 1 \tag{7a}$$

and Γ an open cone in $R^n\backslash\{0\}$ in which $\xi' \neq 0$, $\xi = (\xi',\xi'')$. We call the operator P of class $(EH,\Gamma)_{n''}$ if P satisfies (7a) and

$$\{x'', tb(x,\xi,t)\} = 0 \quad \text{when } x'' = 0,\ \xi \in \Gamma \tag{7b}$$

in addition to (6), (6a), (6b), (7), (8) restricted to $\xi \in \Gamma$.

An example of operators of class $(EH,\Gamma)_{n''}$ is:

$$D_t^2 - (t^2+x_2^2)D_{x_1}^2 + \cdots, \tag{3'}$$

$x = (x_1,x_2)$, $(x' = x_1,\ x'' = x_2)$, $n = 2$, $n' = n'' = 1$, $\Gamma = \{\xi' \neq 0\}$. Note the operators (3) and (3') are symplectically related.

We indicate how Theorems 1 and 2 are simplified (although micro-locally) for operators of class $(EH,\Gamma)_{n''}$ (See [7]. Compare with [1]).

Let

$$\Theta^{\pm}(t,s) = \Theta_0^{\pm}(t,s) + \langle x'',\xi''\rangle$$

where

$$\partial_t \Theta_0^{\pm} = \pm tb(x',-\partial_{\xi''}\Theta_0^{\pm},\partial_{x'}\Theta_0^{\pm},\xi'',t),$$

$$\Theta_0^{\pm}|_{t=s} = \langle x'-y',\xi'\rangle - \langle y'',\xi''\rangle.$$

We use an analogous notation $I(p;\Theta^{\pm})u$ to (9), replacing $\Phi^{\pm}$ by $\Theta^{\pm}$.

<u>Theorem 1bis.</u> Let P be of class $(EH,\Gamma)_{n''}$. If U and T are sufficiently small, there exist pairs of amplitude functions:

$$p^{i\pm}(t,s) \in S^{1/4-3i/4+|m^{\pm}|/2+0},$$

$$q^{i\pm}(t,s) \in S^{1/4-3i/4+|m^{\pm}|/2+0},$$

$i = 0, 1$, such that the oscillatory integrals:

$$E_i(t,s)u =$$
$$= \textstyle\sum_{\pm} I(p^{i\pm}(t,s);\Phi^{\pm}(t,s))u + \sum_{\pm} I(q^{i\pm}(t,s);\Theta^{\pm}(t,s))u,$$

$u \in \mathcal{E}'(U)$, give a pair of micro-local parametrices in $U\times\Gamma$, $0 \leqq s \leqq t \leqq T$. Here $p^{i\pm}(t,s)$ and $q^{i\pm}(t,s)$ are chosen independent of x'' in U. $p^{i\pm}(t,s)$ are supported in $|\xi| \geqq \eta_0$, $(|y''|^2+s^2)|\xi| \geqq \eta_1$, and $q^{i\pm}(t,s)$ in $|\xi| \geqq \eta_0$, $(|y''|^2+s^2)|\xi| \leqq \eta_2$, $\eta_j > 0$, $j = 0,1,2$.

To describe the wave-front sets in these cases, we observe the following. Let $(X_0^{\pm}(y,\zeta,t,r),\Xi_0^{\pm}(y,\zeta,t,r))$ be the solutions of the Hamilton-Jacobi systems:

$$\dot{X}_0^{\pm} = \mp tb_{\xi}(X_0^{\pm},\Xi_0^{\pm},t),$$

$$\dot{\Xi}_0^{\pm} = \pm tb_x(X_0^{\pm},\Xi_0^{\pm},t)$$

with the initial condition:

$$(X_0^{\pm},\Xi_0^{\pm})|_{t=r} = (y,\zeta), \qquad \zeta \in \Gamma.$$

Then (7b) implies

$$B_0^{\pm}(t,r;y',\zeta) =$$

$$= \{(X_0^{\pm}(y',0,\zeta,t,r),\Xi_0^{\pm}(y',0,\zeta,t,r),y',0,\zeta)\}.$$

Thus, if $\tilde{B}^{\pm}(t,r;U_1,\Gamma_1)$ denotes $\tilde{B}^{\pm}(t,r;U_1)$ micro-localized to those $\zeta \in \Gamma_1 \subset\subset \Gamma$, we have a simplified version of Theorem 2.

<u>Theorem 2bis.</u> Let P be of class $(EH,\Gamma)_{n''}$. For sufficiently small $U_1 \subset\subset U$, $\Gamma_1 \subset\subset \Gamma$, and $T > 0$, we have for $0 \leqq s \leqq t \leqq T$

$$WF(E_i(t,s))|_{U_1\times\Gamma_1} \subset \bigcup_{\pm} \tilde{B}^{\pm}(t,s;U_1,\Gamma_1).$$

Here $WF(E_i(t,s))|_{U_1\times\Gamma_1}$ is the restriction of $WF(E_i(t,s))$ to $(T^*U\backslash\{0\})\times U_1\times\Gamma_1$.

<u>References.</u>

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