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SPECIFICATION, STABILITY AND INVARIANT MEASURE FOR GROUP AUTOMORPHISMS

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It is well known that Anosov diffeomorphisms have the following properties; that is, the existance of Markov partition, Expansiveness, Specification and psudo-orbit tracing property. These properties are notion to be defined as to homeomorphisms on compact metric space. However, it is unknown yet what kind of homeomorphisms have such properties, except in the case of homeomorphism on manifold or shift space. In this report, I show the results to study in the class of automorphisms on compact metric group, which is not manifold generally. These are the results done by N.Aoki, M.Dateyama and me.

Definition 1. Let σ be a homeomorphism of a compact metric space X with distance function d. The system (X,σ) is said to satisfy specification(SP) if for every $\varepsilon > 0$ there is a positive integer $M(\varepsilon)$ such that for every $k \geq 1$ and k points $x_1,\ldots,x_k \in X$ and for every set of integers $a_1 \leq b_1 < a_2 \leq b_2 < \ldots < a_k \leq b_k$ with $a_i - b_{i-1} \geq M(\varepsilon)$ $(2 \leq i \leq k)$ and for every integer p with $p \geq b_k - a_1 + M(\varepsilon)$, there is a point $x \in X$ such that $d(\sigma^n x, \sigma^n x_i) < \varepsilon$ for $a_i \leq n \leq b_i$ $(1 \leq i \leq k)$ and $\sigma^p x = x$.

The system (X,σ) is said to satisfy weak specification(WSP) if it has the condition of specification except for the periodic condition $\sigma^p x = x$.

Definition 2. Let X be a finit dimensional compact metric abelian group, σ be a group automorphism of X. Let G denote the character group of X (G has the compact open topology). We define the dual automorphism γ of G by $(\gamma g)(x) = g(\sigma x)$, $g \in G$ and $x \in X$. Since X is compact, G is discrete.

G is finitely generated w.r.t. γ if there is a finite subset H of G such that G is generated by $\bigcup_{-\infty}^{\infty} \sigma^n H$.

When X is connected, G is torsion free. Then G can be embedded in $\mathbb{Q}^r$(r = rank G = dim X) as a subgroup, and γ can be extended a linear automorphism $\bar{\gamma}$ of $\mathbb{R}^r$. γ is hyperbolic if $\bar{\gamma}$ has no eigenvalues on the unit circle (i.e. no unitary eigenvalues). γ is central spin if $\bar{\gamma}$ has some unitary eigenvalues, and the Jordan blocks for the unitary eigenvalues have no off-diagonal 1's. γ is aperiodic if $\bar{\gamma}$ has no periodic point except 0.

Definition 3. σ is a Bernoulli automorphism of a compact metric abelian group X if there exists a subgroup H such that $X = \bigoplus_{-\infty}^{\infty} \sigma^n H$.

Theorem 1. Let X be a solenoidal group (i.e. a finite-dimensional compact connected metric abelian group) and σ be an automorphism of X.

(A) The following conditions are equivalent;

1) (X,σ) is expansive. 2) (X,σ) satisfies specification.

3) (X,σ) has a Markov partition.

4) G is finitely generated w.r.t. γ and γ is hyperbolic.

(B) The followings are equivalent;

1) (X,σ) satisfies weak specification but not specification.

2) The dual system (G,γ) satisfies one of the conditions;

(a) γ is hyperbolic and G is not finitely generated w.r.t. γ, (b) γ is aperiodic and central spin.

(C) If (X,σ) is ergodic (w.r.t. the normalized Haar measure), then there is a finite sequence $X = X_0 \supset X_1 \supset \dots \supset X_n = \{0\}$ of σ-invariant subgroups such that for any $i \geqslant 0$, X_i is connected and $(X_i/X_{i+1},\sigma)$ satisfies weak specification.

Remark 1. In the class of solenoidal automorphisms, the following diagram holds; Markov partition $\rightleftarrows$ expansive $\rightleftarrows$ SP $\underset{\not\leftarrow}{\rightarrow}$ WSP $\underset{\not\leftarrow}{\rightarrow}$ ergodic.

Theorem 2. Let X be a zero-dimensional compact metric abelian group and σ be an automorphism of X.

(A) The followings are equivalent;

1) (X,σ) satisfies specification.

2) X contains a sequence $X = F_0 \supset F_1 \supset \dots$ of σ-invariant subgroups such that $\bigcap F_n = \{0\}$ and for every $n \geqslant 0$, $\sigma : F_n/F_{n+1} \longrightarrow F_n/F_{n+1}$ is a Bernoulli automorphism.

(B) If (X,σ) is expansive and ergodic, then (X,σ) satisfies specification.

(C) (X,σ) satisfies weak specification iff (X,σ) is ergodic.

Remark 2. In the class of zero-dimensional compact abelian group automorphisms, the following diagram holds;
Markov partition $\rightleftarrows$ {ergodic & expansive} $\underset{\not\leftarrow}{\rightarrow}$ SP $\underset{\not\leftarrow}{\rightarrow}$ WSP $\rightleftarrows$ ergodic.

About invariant measures for the system with weak specificati
M.Dateyama showed the following results.

Theorem 3. Let X be a compact metric space and σ be a homeomorphism of X. Assume that (X,σ) satisfies weak specification. Then the followings are generic properties for σ-invariant probability measure;

1) non-atomic, 2) positive on all non-empty open set,

3) ergodic, 4) not strongly mixing.

This "generic property" means that in the space of all σ-invariant probability measures on X with weak topology, the set of all element with one of these properties is dense G_δ-set.

In the case of that (X,σ) satisfies specification, K.Sigmund proved same result. Theorem 3 imples Sigmund's result.

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