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RINGS OF INVARIANTS WHICH ARE COMPLETE INTERSECTIONS

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1. Introduction

Let G be a finite subgroup of $GL_n(\mathbb{C})$ acting naturally on an affine space $\mathbb{C}^n$ of dimension n and denote by $\mathbb{C}^n/G$ the quotient variety of $\mathbb{C}^n$ under this action of G. We shall give the complete answer to the next problem.

Problem. When is $\mathbb{C}^n/G$ a complete intersection (abbrev. C.I.)?

Stanley solved this in [20] under the assumption that G is the intersection of a finite reflection group in $GL_n(\mathbb{C})$ and $SL_n(\mathbb{C})$, and conjectured in [22] that if $\mathbb{C}^n/G$ is a C.I., there is a finite reflection group L in $GL_n(\mathbb{C})$ such that G is normal in L and L/G is abelian. But this conjecture was solved negatively ([26]). On the other hand, Watanabe and Rotillon solved the above problem for abelian G (cf. [25]) and for any G in $SL_3(\mathbb{C})$. If G is in $SL_2(\mathbb{C})$, $\mathbb{C}^2/G$'s are hypersurfaces which are rational double points of type (A_m), (D_m), (E_6), (E_7), (E_8) (e.g. [19]).

Recently Goto and Watanabe proved that if $\mathbb{C}^n/G$ is a C.I., then the embedding dimension of $\mathbb{C}^n/G$ is at most 2n-1, which follows from an ideal theoretic result on rational singularities ([4]). Moreover Kac and Watanabe showed that if $\mathbb{C}^n/G$ is a C.I., then G is generated by pseudoreflections and special elements ([8]). Here an element g in $GL_n(\mathbb{C})$ is said to be a pseudo-reflection (resp. a special element) if rank(g-1) = 1 (resp. rank(g-1) = 2). Consequently we can use the classification of some finite linear groups given by Blichfeldt, Huffman and Wales ([1, 6, 23]).

Since C.I.'s are Gorenstein varieties, using the classification of quotient singularities of complex manifolds ([5, 17]), we see that the study in case where G is unimodular is essential ([24]). By [3, 18], the general case follows immediately from the special one.

As an application of our result, in principle, we can classify the representations of simple Lie groups whose algebra of invariants are C.I.'s.

The contents of this note are similar to [15] and the detailed accounts were written in [11, 12, 13, 14].

The following notation will be used except in Sect. 4.

K	the complex number field
V	an n-dimensional vector space over K
G	a finite subgroup of GL(V)
S	the symmetric algebra of V
S(U)	the symmetric algebra of a vector space U
[A,B,...]	the block diagonal matrix (endomorphism) on V, for A in End(U), B in End(W), ..., where $V = U \oplus W \ldots$.
g[[n]]	the permutation matrix associated with g in the symmetric group S_n of degree n
e(m)	a primitive mth root of 1
E_m	the cyclic group <e(m)>
D_m	the binary dihedral group of order 4m
T	the binary tetrahedral group of order 24
O	the binary octahedral group of order 48
I	the binary icosahedral group of order 120
(u,v;H,N)	the group $(E_u \mid E_v;H \mid N)$ defined in [3]
V-det or det	the determinant on V

The notation A(u,v,n), G(u,v,n), W(F) is defined in [3].

2. Definitions and preliminary results

For any finite group H, a subgroup N of H and a linear representation $\underline{q}$ of H in GL(V), we adopt the following notation and terminology. H is said to be reducible (resp. irreducible, imprimitive, primitive, monomial) in GL(V) if $\underline{q}$ is so. Let $V\langle N\rangle$ be the KN-submodule of V generated by $(g-1)V$ for all g in N and R(V;N) the largest reflection subgroup of $\underline{q}(N)$. A subspace U of codimension one in V is said to be a reflecting hyperplane relative to N if $V^{\langle g\rangle} = U$ for an element g in N. We denote by $\mathbb{P}(V,N)$ the set of reflecting hyperplanes relative to N and, for U in $\mathbb{P}(V,N)$, by $I_U(N)$ the cyclic subgroup of $\underline{q}(N)$ consisting of all elements g in $\underline{q}(N)$ such that U is a subspace of $V^{\langle g\rangle}$. Let $L_U(V,N)$ be a fixed nonzero element in $V\langle I_U(N)\rangle$. For a linear character $\underline{X}$ of H whose kernel contains Ker $\underline{q}$, let $s_U(\underline{X})$ be the smallest natural number u satisfying $\underline{X}(g) = \det(g)^u$ for all g in $I_U(N)$ and $f(V,N,\underline{X})$ be the product of $L_U(V,N)^{s_U(\underline{X})}$ where U runs through $\mathbb{P}(V,N)$. Furthermore $S^{N,\underline{X}}$ denotes the set of all f in S such that $g(f) = \underline{X}(g)f$ for g in N, whose elements are known as $\underline{X}$-invariants (relative invariants) of N.

(2.1) Theorem (Stanley[20]). $S^{N,\underline{X}}$ is a graded free S^N-module of rank one if and only if $f(V,N,\underline{X})$ is a $\underline{X}$-invariant of N. Especially if these equivalent conditions are satisfied, then $S^{N,\underline{X}}$ is generated by $f(V,N,\underline{X})$.

$\mathbb{P}(V,N)/N$ stands for a complete system of representatives of $\mathbb{P}(V,N)$ modulo N under the action of N. The linear characters

$$\det:\langle I_{U'}(N) : U'\in NU\rangle \longrightarrow (K^*)_U$$

induce the natural homomorphism

$$\Phi_{N,V}:R(V;N) \longrightarrow \oplus_{U\in\mathbb{P}(V,N)}(K^*)_U \longrightarrow GL_{|\mathbb{P}(V,N)/N|}(K),$$

where $\oplus_{U\in\mathbb{P}(V,N)}(K^*)_U$ is embedded in $GL_{|\mathbb{P}(V,N)/N|}(K)$ diagonally ([9]). For a linear representation $\underline{q}'$ of a finite group L in GL(V), (R(V;N),L,V) is defined to be a CI-triplet, if R(V;N) contains $\underline{q}'(L)$, $\underline{q}'(L)$ contains the commutator [R(V;N),R(V;N)] of R(V;N) and $\Phi_{N,V}(\underline{q}'(L))$ is conjugate to $G_{\mathbb{D}}$ in $GL_{|\mathbb{P}(V,N)/N|}(K)$ for some datum $\mathbb{D}$ (for definition of the datum $\mathbb{D}$ and $G_{\mathbb{D}}$, see [25]). Furthermore L is said to be extended to a CI-triplet in GL(V), if (M,L,V) is a CI-triplet for a finite reflection group M in GL(V). When N is normal in H and $\underline{q}(N)$ is a reflection group, we denote by V(H#N) a KH/N-submodule of S^N of dimension n which satisfies $S(V(H\#N)) = S^N$ and has a K-basis consisting of graded elements.

(2.2) Theorem (Watanabe[24]). S^N is a Gorenstein ring if and only if f(V,N, V-$\det^{-1}$) is a $\det^{-1}$-invariant of N.

(2.3) Theorem ([9, 10]). Let L be a normal subgroup of a finite reflection group M of GL(V) such that M/L is abelian. Then

(1) S^L is a C.I. if and only if (M,L,V) is a CI-triplet.

(2) If S^L is a C.I., there is a CI-triplet (L*,L,V) such that a regular system of graded parameters of S^{L*} can be extended to a minimal system of graded generators of S^L.

(2.4) Example (The Slice Method). For any v in V, S^{G_v} is etale over S^G at the maximal ideals induced from v, and hence if S^G is a C.I., then S^{G_v} is also a C.I., where G_v is the isotropy group of v in G. (Clearly this can be extended to the case where G is a reductive algebraic group, under the assumption on closedness of the orbit).

(2.5) Example. Let G be a 6-dimensional representation of nonsplitting central extension of Z/3Z by A_6 of order 1080 such that G is generated by special elements of order 2. The Taylor series expansion of the Poincare series of S^G is

$$1+2T^3+7T^6+16T^9+38T^{12}+\ldots$$

([7]). From this we can easily see that $\mathrm{emb}(S^G) > 11$, and S^G is not a C.I..

(2.6) Example. Suppose that $n = 4$ and G is monomial in $SL(V)$ such that the permutation group which is induced from the action of G on a complete system of imprimitivities for the KG-module V is $\langle(12)(34),(13)(24)\rangle$ in S_4. Then S^G is a C.I. if and only if G is conjugate to one of the following groups: 1) $\langle g_1,g_2,h_1,h_2\rangle$ $(4a|c)$, 2) $\langle g_3,g_4,g_7,h_1,h_2\rangle$ $(a < c/2,\ a|c/2,\ 2|c)$, 3) $\langle g_3^2,g_5^2,g_7,h_1,h_2\rangle$ $(4|c)$, 4) $\langle g_3^2,g_5^2,g_7,h_1',h_2\rangle$ $(a = c/4)$, 5) $\langle g_2,g_5^2,g_6,h_1,h_2\rangle$ $(4a|c,\ b-a = c/2,\ a < c/4,\ b/a \equiv 3 \bmod 4$; assume this condition for the groups below), 6) $\langle g_2,g_5^2,g_6,h_1',h_2\rangle$, 7) $\langle g_2,g_5,g_6,h_1,h_2\rangle$, 8) $\langle g_2,g_5,g_6,h_1',h_2\rangle$. Here $g_1 = [e(c),1,1,e(c)^{-1}]$, $g_2 = [1,1,e(a),e(a)^{-1}]$, $g_3 = [e(c/2),e(c/2)^{-1},1,1]$, $g_4 = [1,e(a),e(a)^{-1},1]$, $g_5 = [1,e(c/2),e(c/2)^{-1},1]$, $g_6 = [e(c)^{-b/a},e(c)^{-1},e(c)^{b/a},e(c)]$, $g_7 = [e(c),e(c)^{-1},e(c)^{-1},e(c)]$, $h_1 = (12)(34)[[4]]$, $h_2 = (13)(24)[[4]]$, $h_1' = [1,1,e(2a),e(2a)^{-1}](12)(34)[[4]]$ and a, b, c are natural numbers.

In general we have

(2.7) Lemma. Suppose that $n = 4$ and G is a finite imprimitive irreducible subgroup of $SL(V)$ generated by special elements such that the KG-module V has a system of imprimitivities with 2-dimensional subspaces. Then S^G is a C.I. if and only if G is conjugate to one of the groups in (2.6) or there is a system W_* of imprimitivities consisting of 2-dimensional subspaces W_i $(i = 1,2)$ for the KG-module V which satisfies the following conditions.

(1) $S^{L(W_*)}$ is a C.I..

(2) $L(W_*)$ is the intersection of kernels of the restrictions of some linear characters of $L^*(W_*)G$ to $L^*(W_*)$.

(3) In $GL(V(L^*(W_*)\#R(V;L^*(W_*)G)))$, $L^*(W_*)G$ is extended to a CI-triplet or conjugate to one of the groups in (2.6).

Here $L(W_*)$ is the subgroup of G generated by all elements g in G preserving W_* such that rank of $g-1$'s on W_1 or W_2 is smaller than 2, and $L^*(W_*)$ is the direct product of the images of $L(W_*)$ in $GL(W_i)$ $(i = 1, 2)$ ($GL(W_i)$'s are naturally regarded as subgroups of $GL(V)$).

(2.8) Remark. We note the next remarks on (2.7).

(2.8.1) If S^G is a C.I. and $L^*(W_*)G$ is not extended to a CI-triplet in $GL(V(L^*(W_*)\#R(V;L^*(W_*)G)))$, $R(V;L^*(W_*)G) = E_4D_2 \oplus E_4D_2$.

(2.8.2) The conditions in (3.1) can be replaced by a concrete classification of some subgroups of $GL(V)$, but it is rather complicated.

3. The classification

We now state the classification of G whose invariant subring is a C.I. under the following circumstances. Let V_i's be irreducible KG-submodules of V with $\dim V_i = n_i$ which satisfy $V = \oplus_{i=1}^{m} V_i$ and p_i the representation of G in $GL(V_i)$ afforded by the KG-module V_i. Let G^* be the direct product of all $p_i(G)$'s where $p_i(G)$'s are naturally regarded as subgroups of $GL(V)$, and, for simplicity, put $R = R(V;G^*)$, $G(i) = p_i(G)$, $G[i] = \mathrm{Ker}(\oplus_{j \neq i} p_j)$, $G\langle i\rangle = p_i(G[i])$ and $R(i) = p_i(R)$ respectively.

(3.1) Proposition ([12]). Suppose that G is a subgroup of $SL(V)$. Then S^G is a C.I. if and only if G is generated by special elements, $(R, R \cap G, V)$ is a CI-triplet and, for each i, the following conditions are satisfied.

(1) For any linear character $\underline{X}$ of G^* which is trivial on G, $f(V_i, G(i), \underline{X})$ is an invariant of $G\langle i\rangle$.

(2) $S(V_i)$ is a C.I..

This is a formal solution to our problem, if the irreducible case is solved.

(3.2) Theorem ([12, 13]). S^G is a C.I. if and only if G is generated by special elements, $(R, R \cap G, V)$ is a CI-triplet and, for each i, the following conditions are satisfied.

<u>CASE</u> I "$p_i(R(V;G)) = 1$":

<u>Case</u> A "R <u>is</u> <u>irreducible</u> <u>in</u> $GL(V_i)$". If $R(i) \neq G(i)$, up to conjugacy, the groups $G(i)$, $R(i)$, $G\langle i\rangle$ respectively agree with one of the following triplets; 1) $G(i) = \langle[e(2^b),-1/e(2^b)],R(i)\rangle$, $R(i) = G(u,u,2)$, $G\langle i\rangle = G(i)\cap SL(V_i)$; 2) E_4T, E_4D_2, $G(i)\cap SL(V_i)$; 3) E_6O, E_6T, $G(i)\cap SL(V_i)$; 4) $\langle -1,R(i)\rangle$, $G(u,u,3)$, $G(i)\cap SL(V_i)$ (u odd) ; 5) $\langle[e(3u)^{-2},e(3u),e(3u)],R(i)\rangle$, $G(u,v,3)$, $\langle[e(3u)^{-2},e(3u),e(3u)],G(u,u,3)\cap SL(V_i)\rangle$ $(u > 1)$; 6) $\langle W(L_3)\cap SL(V_i),R(i)\rangle$, $G(3,3,3)$, $G(i)\cap SL(V_i)$; 7) $E_9R(i)$, $W(L_3)$, $G(i)\cap SL(V_i)$; 8) $E_9R(i)$, $W(M_3)$, $G(i)\cap SL(V_i)$; 9) $E_{18}R(i)$, $W(M_3)$,$G(i)\cap SL(V_i)$; 10) $E_3R(i)$, $W(J_3(4))$, $G(i)\cap SL(V_i)$; 11) $\langle[e(2^b),e(2^b),1/e(2^b),1/e(2^b)],R(i)\rangle$, $G(u,v,4)$, $\langle[e(2^b),e(2^b),1/e(2^b),1/e(2^b)],G(u,u,4)\cap SL(V_i)\rangle$; 12) $E_4R(i)$, $W(F_4)$, $G(i)\cap SL(V_i)$; 13) $E_2R(i)$, $W(A_4)$, $G(i)\cap SL(V_i)$; 14) $E_{12}R(i)$, $W(L_4)$, $G(i)\cap SL(V_i)$; 15) $E_8R(i)$, $EW(N_4)$, $G(i)\cap SL(V_i)$; 16) $E_2R(i)$, $W(A_5)$, $G(i)\cap SL(V_i)$; 17) $E_6R(i)$, $W(K_5)$, $G(i)\cap SL(V_i)$; 18) $E_2R(i)$, $W(E_6)$, $G(i)\cap SL(V_i)$.

<u>Case</u> B "R(i) <u>is</u> <u>reducible</u> <u>and</u> <u>not</u> <u>abelian</u>". (1) $n_i = 4$. (2) $G(i)/R(i)$ is conjugate in $GL(V_i(G*\#R))$ to one of the groups in (2.6) or is extended to a CI-triplet in $GL(V_i(G*\#R))$. (3) For any element x in V_i with $\dim V_i\langle G[i]_x\rangle = 3$, in $GL(V_i\langle G[i]_x\rangle)$, $G[i]_x$ is extended to a CI-triplet or conjugate to one of the groups in [26, Sect. 3]. (4) If, for an irreducible KR-submodule U of V_i, $(G[i])_U$ is not contained in R, up to conjugacy, the groups $(G(i))_U$, $(R(i))_U$ and $(G\langle i\rangle)_U$ agree, in $GL(V_i\langle R_U\rangle)$, respectively with $G(i)$, $R(i)$ and $G\langle i\rangle$ of $n_i = 2$ listed in Case A, where L_U denotes the intersection of the isotropy groups L_x of all x's in U in a group L.

<u>Case</u> C "R(i) <u>is</u> <u>reducible</u> <u>in</u> $GL(V_i)$ <u>and</u> <u>nontrivial</u> <u>abelian</u>". For each g in $G\langle i\rangle$, the product of nonzero entries of the matrix $[g_{ij}]$ of g is equal to one, where $[g_{ij}]$ of g is afforded by a K-basis on which R(i) is represented as a diagonal group, and $G\langle i\rangle$ is conjugate in $GL(V_i)$ to one of the groups;

$G(u,u,n_i)\cap SL(V_i)$ $((u,u,n_i)\neq(2,2,2))$, $\langle G(u,u,4)\cap SL(V_i),[e(2^b),e(2^b),1/e(2^b),1/e(2^b)]\rangle$, the groups in (2.6), $\langle G(u,u,3)\cap SL(V_i),[e(3u)^{-2},e(3u),e(3u)]\rangle$ $(u > 1)$, $\langle G(u,u,3)\cap SL(V_i),[e(7u),e(7u)^2,e(7u)^{-3}]\rangle$.

Case D "$R(i) = 1$". $G(i)$ can be extended to a CI-triplet in $GL(V_i)$ or is conjugate in $GL(V_i)$ to one of the following groups; 1) $G\langle i\rangle$'s in Case A; 2) the groups in (2.7) $(n_i = 4)$; 3) $\langle A(u,u,4),g,(123)[[4]],(234)[[4]]\rangle$, $\langle G(u,u/2,4)\cap SL(V),g\rangle$ $(n = 4)$ where $g = [e(2^b),e(2^b),1/e(2^b),1/e(2^b)]$; 4) $E_{2m}T^{\otimes 2}$, $E_{2m}O^{\otimes 2}$, $E_{2m}I^{\otimes 2}$ $(m = 1, 2)$, $(4,2;O,T)^{\otimes 2}$ $(n = 4)$; 5) the groups in [26].

CASE II "$p_i(R(V;G)) \neq 1$ and $p_i(R(V;G)) \neq R(i)$":

Case E "$R(V;G)$ is irreducible in $GL(V_i)$". If $G(i) \neq R(i)$, the groups $p_i(R(V;G))$, $R(i)$ and $G\langle i\rangle$ are respectively listed in the following triplets; 1) $p_i(R(V;G)) = G(u,v,3)$, $R(i) = G(u,v',3)$, $G\langle i\rangle = \langle G(p,q,3),[e(3u)^{-2},e(3u),e(3u)]\rangle$; 2) $W(L_3)$, $W(M_3)$, $E_9W(L_3)$; 3) $G(u,v,4)$, $G(u,v',4)$, $\langle G(u,v,4),[e(2^b),e(2^b),1/e(2^b),1/e(2^b)]\rangle$.

Case F "$p_i(R(V;G))$ is reducible in $GL(V_i)$ and not abelian". $n_i = 4$. If $R(i)/p_i(R(V;G))$ is abelian, $G(i)$ can be extended to a CI-triplet in $GL(V_i(G\#R(V;G)))$ and $f(V_i(G\#R(V;G)),G/R(V;G),\det)$ is a det-invariant of $G(i)/p_i(R(V;G))$. Otherwise, $R(i)/p_i(R(V;G))$, $G(i)/p_i(R(V;G))$, $G\langle i\rangle/p_i(R(V;G))$ and $V_i(G\#R(V;G))$, respectively satisfy the conditions for $R(i)$, $G(i)$, $G\langle i\rangle$ and V_i in Case B.

Case G "$p_i(R(V;G))$ is reducible in $GL(V_i)$ and abelian". The group $G(i)$ is monomial, and $R(i)/p_i(R(V;G))$, $G(i)/p_i(R(V;G))$, $G\langle i\rangle/p_i(R(V;G))$ and $V_i(G\#R(V;G))$, respectively, satisfy the conditions for $R(i)$, $G(i)$, $G\langle i\rangle$ and V_i, in CASE I.

CASE III "$p_i(R(V;G)) = R(i) \neq 1$": If $R(i) \neq G(i)$ and $G(i)$ is not extended to a CI-triplet in $GL(V_i)$, $G(i)$ is conjugate to $G(i)$ in Case A or $G(i)/R(i)$

satisfies the conditions for $G = G(i)/R(i)$ and $V = V_i(G(i)\#R(i))$ in CASE I. Here the 2-part of u is 2^{b-1}.

(3.3) Remark If necessary, we can replace any condition in (3.2) by some concrete classification of subgroups in $GL(V_i)$. However it is rather complicated.

4. Simple algebraic groups

Let G be a simple algebraic group over the complex number field K and denote by K[p] the symmetric algebra of the dual p* of a finite dimensional linear (rational) complex representation p of G (we confuse representations with their spaces).

By our classification in Sect. 3, we have

(4.1) Theorem ([14]). Fix a simple G. Then, up to outer automorphisms, the set of all representations p's of G such that $K[p]^G$'s are C.I.'s and p's do not have nonzero trivial subrepresentations is finite.

In general, if a system of generators of graded algebras with rational singularities are constructive, then their relation ideals are constructive.

(4.2) Proposition ([12]). Let A be a graded algebra defined over a field k and B an n-dimensional graded polynomial algebra over k. If A is pseudo-rational at A_+ and h is a graded epimorphism from B to A, the intersection of Ker h and $(A_+)^{\dim A+1}$ is contained in A_+Ker h.

The next result follows immediately from [2, 21].

(4.3) Proposition. Let A be a Gorenstein graded K-algebra and suppose that A_{A_+} is a rational singularity. If a system of generators of A as a K-algebra is known, a minimal graded free resolution of A is constructive.

It was proved in [16] that a minimal system of generators of $K[p]^G$ is constructive. Because $K[p]^G$ is a Gorenstein graded K-algebra with rational singularities, by (4.1) and (4.3), (in principle) we can determine the representations of G whose algebras of invariants are C.I.'s.

For example, suppose that $G = SL_2$ and p is the representation of G satisfying $p^G = 0$. Then $K[p]^G$ is a C.I. if and only if p is a subrepresentation of one of q^5, q_6, q^2+q_3, q^2+q^4, $2q+q^2$, $q+2q^2$, $2q^3$, $4q$, $3q^2$, $q+q^3$, $q+q^4$, $2q^4$. Here q is the irreducible representation of SL_2 associated with the fundamental dominant weight and q^i is the ith symmetric power of q.

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