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Zeta functions and invariant hyperfunctions on prehomogeneous vector spaces

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Introduction.

Zeta functions associated with prehomogeneous vector spaces, which are originally introduced by Sato-Shintani [Sm-Sh], are Dirichlet series which are absolutely convergent in a domain of complex numbers with sufficiently large real part and which have meromorphic extensions to the whole complex plane with finite possible poles. These zeta functions satisfy functional equations but may not have Euler products. These are constructed as a Dirichlet series whose coefficients are numbers of equivalence classes of lattice points in a vector space on which a Lie group acts prehomogeneously ,i.e., the union of open orbits is dense in the vector space. The precise definition and fundamental properties of the zeta functions of this type are given in [Sm-Sh].

Our interest on zeta functions associated with prehomogeneous vector spaces is concentrated on the explicit computations of functional equations and residues of the zeta functions. If we read the paper [Sm-Sh] carefully, we can find that relatively invariant hyperfunctions play important roles in calculation of the functional equations and residues of zeta functions. In fact, it is proved that the computation of the functional equation is reduced to the calculation of the Fourier transform of a complex power of a relatively invariant polynomial. Moreover, in some important cases, when we compute the residues, it is essential to compute the Fourier transforms of relatively invariant hyperfunctions whose supports are contained in lower dimensional orbits.

However it is another problem to carry out the explicit computations of the Fourier transforms of invariant hyperfunctions. In [Sm-Sh], they calculated the Fourier transforms for some examples in an elementary way. But it is difficult to apply their method to other examples. On the other hand, the theory of holonomic systems and microlocal analysis provide us a strong tool for calculation. We view relatively invariant hyperfunctions as solutions of holonomic systems and analyze them microlocally. Then we can calculate the Fourier transforms of them explicitly.

In this note we shall explain how relatively invariant hyperfunctions are related to the computations of functional equations and residues of the zeta functions.

§1. Prehomogeneous vector spaces.

The definition of prehomogeneous vector spaces is very simple: we say that a complex vector space $V_{\mathbb{C}}$ is a *prehomogeneous vector space* if a connected complex linear algebraic group $G_{\mathbb{C}}$ in $GL(V_{\mathbb{C}})$ has an open dense orbit in $V_{\mathbb{C}}$. There are many kinds of such vector spaces, which are partly classified by Sato-Kimura [Sm-Ki] and each of them has its own interesting properties. However the condition that $(G_{\mathbb{C}},V_{\mathbb{C}})$ is a prehomogeneous vector space is too rough to give a definition of zeta functions associated to it. We need some additional conditions to define them. In this note we assume rather strong conditions in order to make the explanation simpler. F. Sato [Sf] has given a definition of zeta functions associated to prehomogeneous vector spaces in a more general situation.

Let $V_{\mathbb{C}}$ be a finite dimensional complex vector space and let $G_{\mathbb{C}}$ be a connected complex linear algebraic group. Let ρ be a rational representation of $G_{\mathbb{C}}$ on $V_{\mathbb{C}}$. First we suppose that:

(1.1) $V_{\mathbb{C}}$ decomposes into a finite number of $G_{\mathbb{C}}$-orbits.

Then all $G_{\mathbb{C}}$-orbits except open ones are strictly less dimensional than $V_{\mathbb{C}}$. The connectedness of $G_{\mathbb{C}}$ together with the condition (1.1) implies that there is a unique open dense orbit in $V_{\mathbb{C}}$. This means that the triplet $(G_{\mathbb{C}},\rho,V_{\mathbb{C}})$ is a prehomogeneous vector space.

For a prehomogeneous vector space $(G_{\mathbb{C}},\rho,V_{\mathbb{C}})$, we call the complement of the open orbit the singular set and denote it by $S_{\mathbb{C}}$. Orbits contained in $S_{\mathbb{C}}$ is called *singular orbits*. The second assumption is:

(1.2) The singular set $S_{\mathbb{C}}$ is an irreducible hypersurface.

Then there is an irreducible homogeneous polynomial $P(x)$ on $V_{\mathbb{C}}$ whose zero describes the singular set: $S_{\mathbb{C}}=\{x\in V_{\mathbb{C}};P(x)=0\}$, and it is determined up to a constant multiple. The last condition we suppose for complex prehomogeneous vector space is that:

(1.3) Hessian of $P(x) := \det\left(\frac{\partial P(x)}{\partial x_i \partial x_j}\right) \not\equiv 0$.

The triplets $(G_{\mathbb{C}}, \rho, V_{\mathbb{C}})$ satisfying the above three conditions (1.1),(1.2) and (1.3) forms a class of regular prehomogeneous vector spaces. For the definition of regular prehomogeneous vector space, see [Sm-Sh]. The most important and fundamental class of prehomogeneous vector spaces would be the set of regular prehomogeneous vector spaces whose group representation ρ is irreducible. Such prehomogeneous vector spaces are completely classified in [Sm-Ki]. Since many of them satisfies the conditions (1.1),(1.2) and (1.3), the above three conditions are not too strong to develop the theory of zeta functions associated with prehomogeneous vector spaces. We can construct many new zeta functions under the above conditions.

Consider the dual triplet $(G_{\mathbb{C}}, \rho^*, V_{\mathbb{C}}^*)$ with $V_{\mathbb{C}}^*$ the dual space and ρ^* the contragredient representation. Then the dual triplet $(G_{\mathbb{C}}, \rho^*, V_{\mathbb{C}}^*)$ also satisfies the conditions (1.1),(1.2) and (1.3). We denote by $S_{\mathbb{C}}^*$ the singular set in $V_{\mathbb{C}}^*$. Let $Q(y)$ be an irreducible polynomial defining the singular set $S_{\mathbb{C}}^*$. Then the degree of $Q(y)$ coincides with that of $P(x)$. Since $P(x)$ varies up to constant multiple by the action of $G_{\mathbb{C}}$, there exists a rational character χ of $G_{\mathbb{C}}$ such that $P(\rho(g)\cdot x)=\chi(g)P(x)$ for all $g\in G_{\mathbb{C}}$. Similarly we see that $Q(\rho^*(g)\cdot y)=\chi(g)^{-1}Q(y)$ for all $g\in G_{\mathbb{C}}$. That is to say, $P(x)$ and $Q(y)$ are relatively invariant polynomials corresponding to the characters χ and χ^{-1}, respectively.

Next we consider a real forms of a prehomogeneous vector space.

Let $G_{\mathbb{R}}$ be a real form of $G_{\mathbb{C}}$ (as a linear algebraic group) and let $V_{\mathbb{R}}$ be a real form of $V_{\mathbb{C}}$ (as a vector space). We say that $(G_{\mathbb{C}},\rho,V_{\mathbb{C}})$ is a real form of $(G_{\mathbb{C}},\rho,V_{\mathbb{C}})$ (as a prehomogeneous vector space) if $\rho(G_{\mathbb{R}}) \subset GL(V_{\mathbb{R}})$. Then $(G_{\mathbb{R}},\rho^*,V_{\mathbb{R}}^*)$ becomes automatically a real form of $(G_{\mathbb{C}},\rho^*,V_{\mathbb{C}}^*)$. For a real form $(G_{\mathbb{R}},\rho,V_{\mathbb{R}})$, we suppose that:

(1.4) we can take a relatively invariant polynomial $P(x)$ as a polynomial with real coefficients on $V_{\mathbb{R}}$.

Hen we naturally see that $Q(y)$ can be taken as a polynomial with real coefficients on $V_{\mathbb{R}}^*$.

Let $G_{\mathbb{R}}^+$ be the connected component of $G_{\mathbb{R}}$ containing the identity. The real form of $G_{\mathbb{C}}$-open orbit is given by $V_{\mathbb{R}}-S_{\mathbb{R}}$ with $S_{\mathbb{R}}:=S_{\mathbb{C}}\cap V_{\mathbb{R}}$. Each connected component of $V_{\mathbb{R}}-S_{\mathbb{R}}$ is a $G_{\mathbb{R}}^+$-orbit. Similarly $V_{\mathbb{R}}^*-S_{\mathbb{R}}^*$ with $S_{\mathbb{R}}^*:=S_{\mathbb{C}}^*\cap V_{\mathbb{R}}^*$ is the real form of the $G_{\mathbb{C}}$-open orbit in $V_{\mathbb{C}}^*$ and each connected component of $V_{\mathbb{R}}^*-S_{\mathbb{R}}^*$ is a $G_{\mathbb{R}}^+$-orbit. We denote by

$$V_1\cup V_2\cup\cdots\cup V_\ell=V_{\mathbb{R}}-S_{\mathbb{R}} \qquad \text{and} \qquad V_1^*\cup V_2^*\cup\cdots\cup V_\ell^*=V_{\mathbb{R}}^*-S_{\mathbb{R}}^*$$

the connected component decompositions of $V_{\mathbb{R}}-S_{\mathbb{R}}$ and $V_{\mathbb{R}}^*-S_{\mathbb{R}}^*$, respectively.

There is a relatively invariant hyperfunction supported on each closure $\bar{V}_i$ (resp. $\bar{V}_j^*$). Namely, we put:

$$(1.5) \qquad |P(x)|_i^s := \begin{cases} |P(x)|^s, & \text{if } x\in V_i \\ 0, & \text{if } x\notin V_i \end{cases} \qquad (i=1,\cdots,\ell).$$

$$\text{(resp.}\quad |Q(y)|_j^s := \begin{cases} |Q(y)|^s, & \text{if } y \in V_i^* \\ 0, & \text{if } y \notin V_i^* \end{cases} \quad (j=1,\cdots,\ell)\ )$$

for complex number s with sufficiently large real part. Then it is a continuous function on $V_{\mathbb{R}}$ (resp. $V_{\mathbb{R}}^*$) and s is a holomorphic parameter. We can easily check the identity:

$$Q(\frac{\partial}{\partial x})\cdot|P(x)|_i^{s+1} = b(s)\cdot|P(x)|_i^s$$

$$\text{(resp.}\quad P(\frac{\partial}{\partial y})\cdot|Q(x)|_i^{s+1} = b(s)\cdot|Q(x)|_i^s\ ),$$

where b(s) is a polynomial in s called the b-function. By using this identity repeatedly, we can prove that $|P(x)|_i^s$ (resp. $|Q(y)|_j^s$) is extended meromorphically with respect to s to the whole complex plane. In other words, let $\mathcal{S}(V_{\mathbb{R}})$ be the space of rapidly decreasing functions on $V_{\mathbb{R}}$. Then for $f\in\mathcal{S}(V_{\mathbb{R}})$ (resp. $f\in\mathcal{S}(V_{\mathbb{R}}^*)$), the linear functional:

$$f \longmapsto \int f(x)\cdot|P(x)|_i^s\,dx\ ,$$

$$\text{(resp.}\quad f \longmapsto \int f(y)\cdot|Q(y)|_j^s\,dy\ )$$

defines a tempered distribution with a meromorphic parameter $s\in\mathbb{C}$. From the definition, $|P(x)|_i^s$ (resp. $|Q(y)|_j^s$) is a relatively invariant hyperfunction corresponding to the character χ^s (resp. χ^{-s}), i.e., $|P(\rho(g)\cdot x)|_i^s=\chi(g)^s\cdot|P(x)|_i^s$ (resp. $|Q(\rho^*(g)\cdot y)|_i^s=\chi(g)^{-s}\cdot|Q(y)|_i^s$) for all $g\in G_{\mathbb{R}}^+$. The complex power $|P(x)|_i^s$ and $|Q(y)|_j^s$ are called *local zeta functions* of prehomogeneous vector spaces $(G_{\mathbb{R}},\rho,V_{\mathbb{R}})$ and $(G_{\mathbb{R}},\rho^*,V_{\mathbb{R}}^*)$, respectively. They will naturally appear in the definition of global zeta functions in the next section.

§2. Zeta functions.

The definition of the a zeta function associated with a prehomogeneous vector space is a little complicated. We need some preliminaries. Let $(G_{\mathbb{C}},\rho,V_{\mathbb{C}})$ be a prehomogeneous vector space satisfying the conditions (1.1),(1.2),(1.3). Let $(G_{\mathbb{R}},\rho,V_{\mathbb{R}})$ be a fixed real form of $(G_{\mathbb{C}},\rho,V_{\mathbb{C}})$ satisfying the condition (1.4). Consider a $\mathbb{Q}$-form of $(G_{\mathbb{R}},\rho,V_{\mathbb{R}})$ where $\mathbb{Q}$ is the rational number field. Namely, let $G_{\mathbb{Q}}$ and $V_{\mathbb{Q}}$ be $\mathbb{Q}$-forms of $G_{\mathbb{R}}$ and $V_{\mathbb{R}}$, respectively, satisfying $\rho(G_{\mathbb{Q}})\subset GL(V_{\mathbb{Q}})$. We put $G_{\mathbb{R}}^1:=\{g\in G_{\mathbb{R}}^+;\chi(g)=1\}$ and suppose that $G_{\mathbb{R}}^1$ is a unimodular group. Let L be a $\mathbb{Z}$-lattice in $V_{\mathbb{Q}}$ and let Γ be a discrete subgroup in $G_{\mathbb{R}}^1\cap G_{\mathbb{Q}}$. We suppose that L is Γ-invariant ,i.e., $\rho(\Gamma)\cdot L=L$. For a rapidly decreasing function f(x) on $V_{\mathbb{R}}$, we consider the integral:

$$Z_i(f,L,s):=\int_{g\in G_{\mathbb{R}}^+/\Gamma}\sum_{x\in L\cap V_i} f(\rho(g)\cdot x)\cdot\chi(g)^s\,dg,$$

where dg is a Haar measure on $G_{\mathbb{R}}^+$ and $s\in\mathbb{C}$. We suppose that the integral $Z_i(f,L,s)$ is absolutely convergent for any $f\in\mathcal{S}(V_{\mathbb{R}})$ if the real part of s is sufficiently large. When f is a positive function and the real part of s is sufficiently large, the integral $Z_i(f,L,s)$ is rewritten as:

$$Z_i(f,L,s) = \xi_i(s,L)\cdot\int_{V_i} f(x)\cdot|P(x)|^{s-(n/d)}\,dx\ ,\quad (i=1,\cdots,\ell)$$

with $\xi_i(s,L):=\sum_{x\in L\cap V_i/\sim} \mathrm{Vol}(G^+_{\mathbb{R}x}/\Gamma_x)\cdot|P(x)|^{-s}$. Here, d is the homogeneous degree of P(x); n is the dimension of $V_{\mathbb{R}}$; $\sim$ is the equivalence relation by Γ; $G^+_{\mathbb{R}x}$ and Γ_x are isotropy subgroups at x of $G^+_{\mathbb{R}}$ and Γ, respectively; $\mathrm{Vol}(G^+_{\mathbb{R}x}/\Gamma_x)$ means the volume of the fundamental domain $G^+_{\mathbb{R}x}/\Gamma_x$. The assumption of the convergence of $Z_i(f,L,s)$ implies the convergence of the Dirichlet series $\xi_i(s,L)$ for any $s\in\mathbb{C}$ with sufficiently large real part. In the dual prehomogeneous vector space $(G_{\mathbb{R}},\rho^*,V_{\mathbb{R}}^*)$, we can define the same integral:

$$Z^*_i(f,L^*,s):=\int_{g\in G^+_{\mathbb{R}}/\Gamma} \sum_{y\in L^*\cap V^*_i} f(\rho^*(g)\cdot y)\cdot\chi(g)^{-s}\,dg,$$
$$= \xi_j{}^*(s,L^*)\cdot\int_{V^*_j} f(y)\cdot|Q(y)|^{s-(n/d)}\,dy\ ,\quad (j=1,\cdots,\ell)$$

with the Dirichlet series: $\xi_j{}^*(s,L^*):=\sum_{y\in L^*\cap V^*_i/\sim} \mathrm{Vol}(G^+_{\mathbb{R}y}/\Gamma_y)\cdot|Q(y)|^{-s}$, where L^* is the dual lattice of L. We also suppose that $Z^*_j(f,L^*,s)$ is absolutely convergent for all $f\in\mathcal{S}(V^*_{\mathbb{R}})$ if the real part of s is sufficiently large. Then the Dirichlet series $\xi^*_j(s,L^*)$ is also convergent when the real part of s is sufficiently large.

Theorem 2.1. (Sato-Shintani [Sm-Sh])

(1) The Dirichlet series $\xi_i(s,L)$ $(i=1,\cdots,\ell)$ *and* $\xi^*_j(s,L^*)$ $(j=1,\cdots,\ell)$ *can be extended to the whole complex plane as meromorphic functions and satisfy the functional equation of the form:* $\xi_i(s,L) = \sum_{j=1}^{n} c_{ij}(s)\cdot\xi^*_j(\frac{n}{d}-s,L^*)$. *Here* $c_{ij}(s)$ $(i,j=1,\cdots,\ell)$ *are meromorphic*

functions in s.

(2) The locations of poles of $\xi_i(s,L)$ *and* $\xi_j^*(s,L^*)$ *are contained in the set* $\{k;b(-k)=0\}$. *Here* $b(s)$ *is the b-function of* $P(x)$.

It has been proved by Sato-Shintani [Sm-Sh] that the explicit determinations of $c_{ij}(s)$ $(i,j=1,\cdots,\ell)$ are reduced to the computation of the Fourier transforms of $|P(x)|_i^s$ $(i=1,\cdots,\ell)$. The Fourier transform of $|P(x)|_i^s$ is written as a linear combination of $|Q(y)|_j^{-s-(d/n)}$ $(j=1,\cdots,\ell)$ with meromorphic coefficients in s. The problem is to compute the coefficients. Sato-Shintani [Sm-Sh] computed them in a typical example in an elementary way which is not applicable to other examples. At the present time, the most powerful way of the computation would be the method using holonomic systems and micro-local analysis. Kashiwara-Miwa [Ka-Mi] explained the method of microlocal analysis precisely and [Mr1],[Mr3] gave some examples of explicit computations by means of microlocal analysis.

On the other hand, the explicit computation of the residues (or the principal parts) of the poles of zeta functions is far more difficult. We may say that there is no routine way to compute them yet. However if we read the paper [Sm-Sh] carefully, we realize that invariant hyperfunctions supported in the singular set would be important for the calculations. In some important examples, we observe that some orbits in the singular set admit $G_{\mathbb{R}}^1$-invariant measure on them. They are extended uniquely as a relatively invariant tempered distributions. Indeed, [Sm-Sh] has computed the residues of example by reducing the problem to the calculation of the Fourier transforms of the relatively invariant measures on the orbits in the singular set. Consequently we may say that the calculus of relatively invariant hyperfunctions is essential also in the calculation of the residues.

In [Mr1] and [Mr3], we have proved for four examples that any orbit in the singular set has a $G_{\mathbb{R}}^1$-invariant measure on it and compute the Fourier transforms of it by microlocal analysis. Moreover we can prove remarkable properties of relatively invariant distributions supported on the singular set by means of the theory of holonomic systems. In the next section we shall state some theorems on invariant distributions on prehomogeneous vector spaces.

§3. Singular invariant hyperfunctions.

Let $(G_{\mathbb{C}},\rho,V_{\mathbb{C}})$ be a prehomogeneous vector space satisfying the conditions (1.1),(1.2) and (1.3), and let $(G_{\mathbb{R}},\rho,V_{\mathbb{R}})$ be a fixed real form of $(G_{\mathbb{C}},\rho,V_{\mathbb{C}})$ satisfying the condition (1.4). We shall use the same notations in the preceding sections. In this section, we suppose that:

(3.1) Any $G_{\mathbb{C}}$-orbit in $S_{\mathbb{C}}$ is a $G_{\mathbb{C}}^1$-orbit where $G_{\mathbb{C}}^1:=\{g\in G_{\mathbb{C}};\chi(g)=1\}$.

Since $G_{\mathbb{R}}^1:=G_{\mathbb{R}}^+\cap G_{\mathbb{C}}^1$, the condition (3.1) implies that any $G_{\mathbb{R}}^+$-orbit in $S_{\mathbb{R}}$ is a $G_{\mathbb{R}}^1$-orbit. We call a $G_{\mathbb{R}}^+$-orbit in $S_{\mathbb{R}}$ a real singular orbit. The real singular set $S_{\mathbb{R}}$ decomposes into a finite number of $G_{\mathbb{R}}^1$-orbits from the assumption (1.1).

The main theme of this section is $G_{\mathbb{R}}^1$-invariant hyperfunctions whose support is contained in $S_{\mathbb{R}}$. We call such functions *singular hyperfunctions*. But explicit construction of singular hyperfunction is difficult in general. We shall try to construct them from the complex powers of relatively invariant polynomials P(x).

We have introduced $G_{\mathbb{R}}^1$-invariant hyperfunctions $|P(x)|_i^s$. The hyperfunctions $|P(x)|_i^s$, which are tempered distributions, are relatively invariant with respect to the action of $G_{\mathbb{R}}^+$ corresponding to the character χ^s. They depend on $s\in\mathbb{C}$ meromorphically. Let s_0 be a point in $\mathbb{C}$ at which $|P(x)|_i^s$ has a pole of order m_0. Then we have the Laurent expansion:

$$(3.2)\quad |P(x)|_i^s = \sum_{k=-m_0}^{\infty} A_i{}^k{}_{s_0}(x)\cdot(s-s_0)^k ,$$

where $A_i{}^k{}_{s_0}(x)$ $(k=-m_0,-m_0+1,\cdots)$ are tempered distribution on $V_{\mathbb{R}}$. They are not relatively invariant any longer except for $A_i{}^k{}_{s_0}(x)$. If k is a negative integer, then the support of $A_i{}^k{}_{s_0}(x)$ is contained in the singular set $S_{\mathbb{R}}$. It is natural that we expect that any singular $G_{\mathbb{R}}^1$-invariant tempered distribution would be written as a linear combination of $A_i{}^k{}_{s_0}(x)$. In fact, we have the following theorem.

Theorem 3.1

We suppose the conditions (1.1)-(1.4) and (3.1). Then any $G_{\mathbb{R}}^1$*-invariant singular hyperfunctions on* $V_{\mathbb{R}}$ *is written as a linear combinations of the coefficients of the Laurent expansions of* $|P(x)|_i^s$ $(i=1,\cdots,\ell)$ *of negative order if any relatively invariant hyperfunction is written as a linear combination of* $|P(x)|_i^s$

$(i=1,\cdots,\ell)$.

The proof is given in [Mr2]. The condition that any relatively invariant hyperfunction is written as a linear combination of $|P(x)|_i^s$ $(i=1,\cdots,\ell)$ can be proved in some cases by microlocal analysis. If a singular invariant tempered distribution is written as a linear combination of Laurent coefficients of $|P(x)|_i^s$, then the Fourier transform of the tempered distribution is computed from the data of the Fourier transform of $|P(x)|_i^s$ $(i=1,\cdots,\ell)$. For example, the Fourier transform of $A_{i}{}^{k}{}_{s_0}(x)$ defined in (3.2) is obtained by computing the Laurent expansion of the Fourier transform of $|P(x)|_i^s$ at $s=s_0$. Therefore the problem that we have to solve is to compute the explicit presentation of a singular hyperfunction by Laurent expansions of $|P(x)|_i^s$. In order to carry out this computation, we need more precise information about the microlocal structure of $|P(x)|_i^s$. It would be possible after establishing the theory of real principal symbols of regular holonomic microfunction the author is now studying.

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