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Application of the Effective Hamiltonian Method to Relative Diffusion

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The interesting phenomenon that a pair of fluid particles, which are subject to convection in steady, incompressible, statistically isotropic turbulent flow, relatively diffuse toward the perpendicular direction to the initial relative position vector between them faster than toward the parallel direction at the start is found by means of the effective Hamiltonian method. In addition, the fact that the mean square value of the relative distance between them increases exponentially just before Richardson's four-thirds law is satisfied is reported.

§1. Introduction

One of the most interesting features of turbulence is the enhancement of transport processes. Relative diffusion of a pair of fluid particles has been discussed by many authors from various points of view. These points of view are, for instance, the similarity theory,[1-3] the direct interaction approximation,[4] the vortex stretching model,[5] the scaling law[6] and the effective Hamiltonian method.[7-9]

The effective Hamiltonian method, a kind of calculus of variations, is mathematically clear about the applicable limit of its approximation and we can easily develop the degree of its approximation. In addition, this method has the reasonable contents: including the mechanism to suppress the contribution from the energy containing range[7] and the appearance of Richardson's four-thirds law.[10] By using this method, the fact that the mean square value of the relative distance between two fluid particles increases in proportion to the square of the diffusion time, t^2, at the start,[7] faster than t^3 at the next,[9] t^3 during the intermediate time,[8] and t at last[7] has been reported. In those works,[7-9] the trial function, the effective Hamiltonian, has two parameters, and it is implicitly assumed that all of three components of the relative position vector grows equally. However, they will grow differently at the start, so we consider the growth of the perpendicular component and the parallel component to the initial relative position vector with four parameters.

We formulate that in §2 and obtain the set of nonlinear equations to determine the parameters. In §3, we calculate those equations for the concrete cases by Newton method. Finally, we summarize the results in §4.

§2. Formulation

We consider a pair of fluid particles which are subject to convection in steady, incompressible, statistically isotropic turbulent flow. If they are at the position x_0^1 and x_0^2 respectively at the initial time t=0, the probability that we can find them at the positions x^1 and x^2 at the time $t(>0)$ is determined by the probability density function

$$\phi(x^1,x^2,t)=\langle\chi(x^1,x^2,t)\rangle_u, \tag{2.1}$$

where

$$\chi(x^1,x^2,t)=\prod_{i=1}^{2}\delta\,[x^i-X(x_0^i,t)]. \tag{2.2}$$

Here $\langle\ \rangle_u$ denotes the statistical average with respect to the Eulerian velocity field $u(x,t)$, δ is the Dirac delta function in three dimensions, and $X(x_0^i,t)$ denotes the position of the i-th particle at the time t, which is at the position x_0^i at the initial time t=0. The equations of motion are

$$\dot{X}(x_0^i,t)=u[X(x_0^i,t),t] \qquad (i=1,2). \tag{2.3}$$

By using this relation, the time evolution of the probability density function is determined by

$$\Big[\frac{\partial}{\partial t}+\sum_{i=1}^{2} u(x^i,t)\cdot\nabla^i\Big]\chi(x^1,x^2,t)=0, \tag{2.4}$$

where ∇^i is the nabla operator with respect to x^i. Eq.(2.4) can be solved formally as

$$\chi(x^1,x^2,t)=T\Big[e^{-\sum_{i=1}^{2}\int_0^t dt'\,u(x^i,t')\cdot\nabla^i\langle t'\rangle}\Big]\chi(x^1,x^2,0), \tag{2.5}$$

where T is the time ordered operator and $\langle t'\rangle$ denotes the temporal position of the operator. For example, if $t_1<t_2<t_3$,

$$T[\nabla\langle t_2\rangle u(x,t_1)u(x,t_3)]=u(x,t_3)\nabla u(x,t_1). \tag{2.6}$$

If we assume that the distribution for the Eulerian velocity field is statistically stationary, statistically isotropic, and joint Gaussian whose mean and variance are given by

$$\langle u(x,t)\rangle_u=0, \tag{2.7}$$

$$S_{kl}(x^i\langle t'\rangle-x^j\langle t''\rangle,t'-t'')\equiv\langle u_k(x^i,t')u_l(x^j,t'')\rangle_u$$
$$=\int dl\ S(l,t'-t'')\Delta_{kl}(l)e^{il\cdot(x^i\langle t'\rangle-x^j\langle t''\rangle)}, \tag{2.8}$$

where

$$\Delta_{kl}(l)=\delta_{kl}-\frac{l_k l_l}{l^2} \qquad (i,j=1,2;\ k,l=1,2,3), \tag{2.9}$$

then eq.(2.1) becomes

$$\phi(\boldsymbol{x}^1,\boldsymbol{x}^2,t)=\int \frac{\mathrm{d}\boldsymbol{k}^1\mathrm{d}\boldsymbol{k}^2}{(2\pi)^6}\, T\left[e^{\frac{1}{2}\sum_{i,j=1}^{2}\sum_{k,l=1}^{3}\int_0^t dt'dt''\, S_{kl}(\boldsymbol{x}^i\langle t'\rangle-\boldsymbol{x}^j\langle t''\rangle,t'-t'')\nabla_k^i\langle t'\rangle\nabla_l^j\langle t''\rangle}\right] \times e^{i\sum_{i=1}^{2}\boldsymbol{k}^i\cdot(\boldsymbol{x}^i-\boldsymbol{x}_0^i)}. \tag{2.10}$$

This is the formal expression of the probability density function which gives the probability that a pair of fluid particles are at the position $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$ respectively at the time $t(>0)$.

In order to calculate furthermore, we employ the effective Hamiltonian method, in which the probability density function is approximated to

$$\phi_{eff}(\boldsymbol{x}^1,\boldsymbol{x}^2,t)=\int \frac{\mathrm{d}\boldsymbol{k}^1\mathrm{d}\boldsymbol{k}^2}{(2\pi)^6}\, e^{-I(\boldsymbol{k}^1,\boldsymbol{k}^2,t)}e^{i\sum_{i=1}^{2}\boldsymbol{k}^i\cdot(\boldsymbol{x}^i-\boldsymbol{x}_0^i)}, \tag{2.11}$$

where $I(\boldsymbol{k}^1,\boldsymbol{k}^2,t)$ is determined such that the error

$$E(A)=\langle A\rangle-\langle A\rangle_{eff} \tag{2.12}$$

becomes minimum. Here $\langle A\rangle$ and $\langle A\rangle_{eff}$ are the statistical average of a certain physical quantity $A(\boldsymbol{x}^1,\boldsymbol{x}^2,t)$ with respect to $\phi(\boldsymbol{x}^1,\boldsymbol{x}^2,t)$ and $\phi_{eff}(\boldsymbol{x}^1,\boldsymbol{x}^2,t)$ respectively:

$$\langle A\rangle=\int \mathrm{d}\boldsymbol{x}^1\mathrm{d}\boldsymbol{x}^2\, A(\boldsymbol{x}^1,\boldsymbol{x}^2,t)\phi(\boldsymbol{x}^1,\boldsymbol{x}^2,t), \tag{2.13}$$

$$\langle A\rangle_{eff}=\int \mathrm{d}\boldsymbol{x}^1\mathrm{d}\boldsymbol{x}^2\, A(\boldsymbol{x}^1,\boldsymbol{x}^2,t)\phi_{eff}(\boldsymbol{x}^1,\boldsymbol{x}^2,t). \tag{2.14}$$

The effective probability density function $\phi_{eff}(\boldsymbol{x}^1,\boldsymbol{x}^2,t)$ satisfies

$$\left[\frac{\partial}{\partial t}+H(-i\nabla^1,-i\nabla^2,t)\right]\phi_{eff}(\boldsymbol{x}^1,\boldsymbol{x}^2,t)=0, \tag{2.15}$$

$$\phi_{eff}(\boldsymbol{x}^1,\boldsymbol{x}^2,0)=\prod_{i=1}^{2}\delta\,[\boldsymbol{x}^i-\boldsymbol{x}_0^i], \tag{2.16}$$

where $H(-i\nabla^1,-i\nabla^2,t)$ is the effective Hamiltonian, a trial function, which is associated with $I(\boldsymbol{k}^1,\boldsymbol{k}^2,t)$ by

$$I(\boldsymbol{k}^1,\boldsymbol{k}^2,t)=\int_0^t \mathrm{d}t'\, H(\boldsymbol{k}^1,\boldsymbol{k}^2,t'). \tag{2.17}$$

According to the normalization condition of the effective probability density function

$$\int \mathrm{d}\boldsymbol{x}^1\mathrm{d}\boldsymbol{x}^2\, \phi_{eff}(\boldsymbol{x}^1,\boldsymbol{x}^2,t)=1, \tag{2.18}$$

$I(\boldsymbol{k}^1,\boldsymbol{k}^2,t)$ must satisfy

$$I(0,0,t)=0. \tag{2.19}$$

Substituting equations (2.13) and (2.14) into eq.(2.12), using equations (2.10) and (2.11), and neglecting the terms of order $O(S^2)$, we can minimize the $E(A)$ at the first approximation,

$$E(A)=\int \frac{\mathrm{d}\boldsymbol{k}^1\mathrm{d}\boldsymbol{k}^2}{(2\pi)^6}\mathrm{d}\boldsymbol{x}^1\mathrm{d}\boldsymbol{x}^2\, A(\boldsymbol{x}^1,\boldsymbol{x}^2,t)$$

$$\times\left[I(\boldsymbol{k}^1,\boldsymbol{k}^2,t)+\frac{1}{2}\sum_{i,j=1}^{2}\sum_{k,l=1}^{3}\int_0^t \mathrm{d}t'\mathrm{d}t''\int \mathrm{d}\boldsymbol{l}\, S(\boldsymbol{l},t'-t'')\Delta_{kl}(\boldsymbol{l})e^{i\boldsymbol{l}\cdot(\boldsymbol{x}^i-\boldsymbol{x}^j)}\nabla_k^i\nabla_l^j\right] \tag{2.20}$$

$$\times e^{-I(\boldsymbol{k}^1,\boldsymbol{k}^2,t)+i\sum_{i=1}^{2}\boldsymbol{k}^i\cdot(\boldsymbol{x}^i-\boldsymbol{x}_0^i)}=0,$$

where eq.(2.8) and the incompressibility are used. It should be mentioned that this is not a simple perturbation expansion with respect to S but rather is a kind of Bethe-Salpeter treatment[11] in the sense that repeated part of higher order terms are included.

There is no obstacle to calculate the statistical averages of any physical quantities that are associated with the relative diffusion. We are interested in the relative distance between two fluid particles, so we exchange the variables $\boldsymbol{x}^1$, $\boldsymbol{x}^2$, $\boldsymbol{k}^1$, $\boldsymbol{k}^2$ into

$$\boldsymbol{r}=\boldsymbol{x}^1-\boldsymbol{x}^2,\ \boldsymbol{r}_0\equiv\boldsymbol{x}_0^1-\boldsymbol{x}_0^2, \tag{2.21}$$

$$\boldsymbol{R}=\frac{\boldsymbol{x}^1+\boldsymbol{x}^2}{2},\ \boldsymbol{R}_0\equiv\frac{\boldsymbol{x}_0^1+\boldsymbol{x}_0^2}{2}, \tag{2.22}$$

$$k=\frac{k^1-k^2}{2}, \tag{2.23}$$
$$K=k^1+k^2. \tag{2.24}$$

Considering the perpendicular component of r to r_0, $r_\perp$, and the parallel one, $r_{//}$, we assume the form of the $I(k,K,t)$ as

$$I(k,K,t)=\frac{1}{2}\alpha_\perp(t)K_\perp^2+\frac{1}{2}\alpha_{//}(t)K_{//}^2+\frac{1}{2}\beta_\perp(t)k_\perp^2+\frac{1}{2}\beta_{//}(t)k_{//}^2, \tag{2.25}$$

where $K_\perp$ is the projection of K toward the perpendicular direction to r_0, $K_{//}$ the projection of K toward the parallel direction, and so on. These $\alpha_\perp(t)$, $\alpha_{//}(t)$, $\beta_\perp(t)$, $\beta_{//}(t)$ are the unknown functions which are determined by solving the equations,

$$E(R_\perp^2)=0,\ E(R_{//}^2)=0, \tag{2.26}$$
$$E(r_\perp^2)=0,\ E(r_{//}^2)=0. \tag{2.27}$$

Substituting eq.(2.25) into eq.(2.11) and integrating it with respect to k and K, we obtain the effective probability density function,

$$\phi_{eff}(r,R,t)=\frac{1}{(2\pi)^3\alpha_\perp(t)\beta_\perp(t)\sqrt{\alpha_{//}(t)\beta_{//}(t)}} \times e^{-(R_\perp-R_{0\perp})^2/2\alpha_\perp(t)-(R_{//}-R_{0//})^2/2\alpha_{//}(t)-r_\perp^2/2\beta_\perp(t)-(r_{//}-r_0)^2/2\beta_{//}(t)}. \tag{2.28}$$

Therefore, we obtain

$$\langle r_\perp^2\rangle\simeq\langle r_\perp^2\rangle_{eff}=2\beta_\perp(t), \tag{2.29}$$
$$\langle r_{//}^2\rangle\simeq\langle r_{//}^2\rangle_{eff}=r_0^2+\beta_{//}(t), \tag{2.30}$$
$$\langle R_\perp^2\rangle\simeq\langle R_\perp^2\rangle_{eff}=R_{0\perp}^2+2\alpha_\perp(t), \tag{2.31}$$
$$\langle R_{//}^2\rangle\simeq\langle R_{//}^2\rangle_{eff}=R_{0//}^2+\alpha_{//}(t) \tag{2.32}$$

by means of eq.(2.14). We make use of eq.(2.20) and eq.(2.25) so as to calculate eq.(2.27). Then the first of eq.(2.27) becomes

$$E(r_\perp^2)=\int\frac{dk\,dK}{(2\pi)^6}dr\,dR\; r_\perp^2\Bigg[I(k,K,t)-\int_0^t d\tau\,(t-\tau)\int dl\, S(l,\tau) \times\Big\{\frac{1}{3}K^2+\frac{4}{3}k^2+\Big[\frac{1}{2}\Big(K^2-\frac{(K\cdot l)^2}{l^2}\Big)-2\Big(k^2-\frac{(k\cdot l)^2}{l^2}\Big)\Big]e^{il\cdot r}\Big\}\Bigg] \times e^{-I(k,K,t)+ik\cdot(r-r_0)+iK\cdot(R-R_0)}=0. \tag{2.33}$$

By integrating with respect to R, K, and r, eq.(2.33) becomes

$$E(r_\perp^2)=\int dk\left[-\left(\frac{1}{2}\beta_\perp k_\perp^2+\frac{1}{2}\beta_{//}k_{//}^2\right)\Delta_{k_\perp}\delta(k)-\int_0^t d\tau\,(t-\tau)\int dl\, S(l,\tau)\right.$$
$$\left.\times\left\{-\frac{4}{3}k^2\Delta_{k_\perp}\delta(k)+2\left(k^2-\frac{(k\cdot l)^2}{l^2}\right)\Delta_{k_\perp}\delta(k+l)\right\}\right]e^{-\beta_\perp k_\perp^2/2-\beta_{//}k_{//}^2/2-ik_{//}r_0}=0, \tag{2.34}$$

where $\Delta_{k_\perp}$ denotes the Laplacian with respect to $k_\perp$. Finally, integrating with respect to $\boldsymbol{k}$, we obtain

$$\beta_\perp=\int_0^t d\tau\,(t-\tau)\int dl\, S(l,\tau)\left[\frac{8}{3}-\left(4-\frac{2l_\perp^2}{l^2}\right)e^{-\beta_\perp l_\perp^2/2-\beta_{//}l_{//}^2/2+il_{//}r_0}\right]. \tag{2.35}$$

Similarly, the second of eq.(2.27) becomes

$$\beta_{//}=\int_0^t d\tau\,(t-\tau)\int dl\, S(l,\tau)\left[\frac{8}{3}-\left(4-\frac{4l_{//}^2}{l^2}\right)e^{-\beta_\perp l_\perp^2/2-\beta_{//}l_{//}^2/2+il_{//}r_0}\right]. \tag{2.36}$$

Equations (2.35) and (2.36) are the set of nonlinear equations with respect to $\beta_\perp$ and $\beta_{//}$. Same procedure can be used for calculating eq.(2.26), but we are only interested in the relative distance between two fluid particles.

§3. Numerical Calculation

In order to calculate $\beta_\perp$ and $\beta_{//}$ concretely, we assume the form of the two-time correlation function as

$$S(k,t)=A(k)e^{-k^{4/3}t^2/2}, \tag{3.1}$$

where

$$A(k)=\begin{cases}\dfrac{C}{k_0^{11/3}+k^{11/3}} & (0\le k\le k_d)\\ 0 & (k_d<k)\end{cases} \tag{3.2}$$

or

$$A(k)=\begin{cases} \dfrac{C}{(k_0^{11/3}+k^{11/3})(k_d'^4+k^4)} & (0\leq k\leq k_d) \\ 0 & (k_d<k) \end{cases} \qquad (3.3)$$

Here C is determined such that $\langle u^2\rangle_u=1$ from eq.(2.8) and k_0 is the inverse of the typical length of turbulence. Both equations (3.2) and (3.3) satisfy Kolmogorov's five-thirds law because of the energy spectrum $E(k)=4\pi k^2A(k)$.

Substituting eq.(3.1) into equations (2.35) and (2.36), and integrating them with respect to τ, we obtain

$$\beta_\perp=8\pi\int_0^{k_d} dk\, kA(k)T(k,t)\left[\frac{4}{3}k-\int_0^k dk_{//}\left(1+\frac{k_{//}^2}{k^2}\right)e^{-\beta_\perp(k^2-k_{//}^2)/2-\beta_{//}k_{//}^2/2}\cos(k_{//}r_0)\right], \qquad (3.4)$$

$$\beta_{//}=16\pi\int_0^{k_d} dk\, kA(k)T(k,t)\left[\frac{2}{3}k-\int_0^k dk_{//}\left(1-\frac{k_{//}^2}{k^2}\right)e^{-\beta_\perp(k^2-k_{//}^2)/2-\beta_{//}k_{//}^2/2}\cos(k_{//}r_0)\right], \qquad (3.5)$$

where

$$T(k,t)=\frac{t^2}{2z}\left[\sqrt{\pi z}\,\mathrm{erf}(\sqrt{z})+e^{-z}-1\right], \qquad (3.6)$$

$$z=\frac{k^{4/3}t^2}{2}. \qquad (3.7)$$

We calculate the set of nonlinear equations (3.4) and (3.5) by the Newton method for the cases;

(¡) $k_0=10^{-4}$, $k_d=10^2$, $r_0=0.1$, $\langle u^2\rangle_u=1$, where $A(k)$ is given by eq.(3.2),

(¡¡) $k_0=10^{-1}$, $k_d'=10^2$, $k_d=10^5$, $r_0=0.1$, $\langle u^2\rangle_u=1$, where $A(k)$ is given by eq.(3.3),

(¡¡¡) $k_0=10^{-1}$, $k_d=10^2$, $r_0=0.1$, $\langle u^2\rangle_u=1$, where $A(k)$ is given by eq.(3.2).

Cases (¡) are shown in Fig.1, (¡¡) in Fig.2, (¡¡¡) in Fig.3. In Fig.n.(d), where n=1,2,3, we can see that $\beta_\perp$ increases faster than $\beta_{//}$ does at the start. This means that the relative distance between two fluid particles grows toward the perpendicular direction to the initial relative position vector faster than toward the parallel direction at the start. We can also see that $\langle r^2 \rangle$ is proportional to t^2 at first in Fig.n.(b), increases exponentially at the second place in Fig.n.(c), grows up to almost k_0^{-2}, the square of the typical length of turbulence, in proportion to t^3 at the third place in Fig.n.(a), and is proportional to t at last.

§4. Summary

We consider the time evolution of the mean square value of the relative position vector between two fluid particles, in which the relative position vector is decomposed into the perpendicular component and the parallel component to the initial relative position vector. As a result, we understand that the relative distance between two fluid particles grows toward the perpendicular direction to the initial relative position vector faster than toward the parallel direction at the start. Moreover, we find that the mean square value of the relative distance increases exponentially just before Richardson's four-thirds law is satisfied, which is consistent with Y.Inaba and M.Suzuki (1985).[6]

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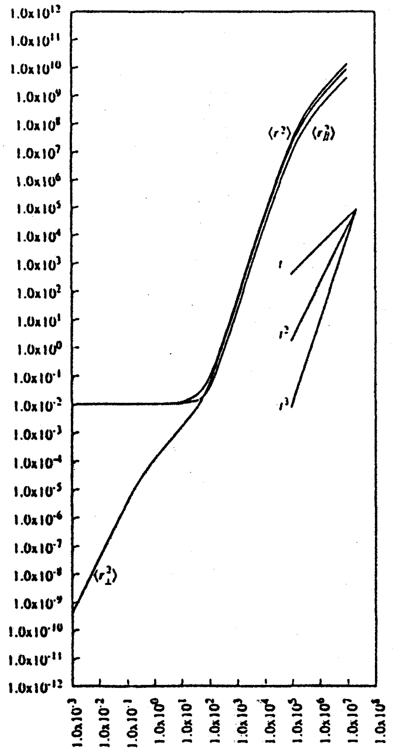

Fig.1.(a) The time evolution of $\langle r^2\rangle$, $\langle r_\perp^2\rangle$, $\langle r_{/\!/}^2\rangle$ for the case $k_0=10^{-4}$, $k_d=10^2$, $r_0=0.1$, $\langle u^2\rangle_e=1$, where $A(k)$ is given by eq.(3.2).

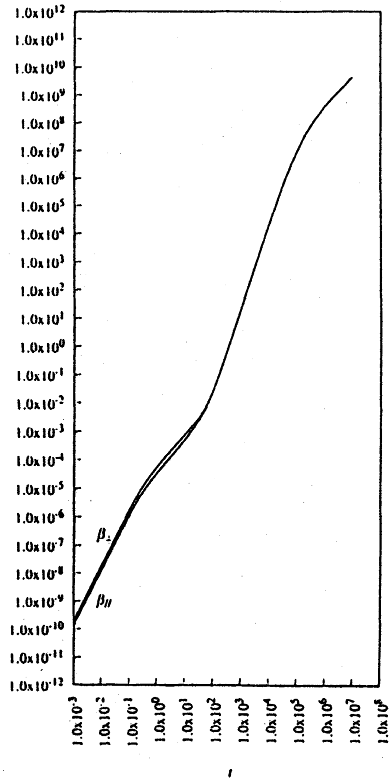

Fig.1.(d) The time evolution of $\beta_\perp$, $\beta_{/\!/}$ for the case $k_0=10^{-4}$, $k_d=10^2$, $r_0=0.1$, $\langle u^2\rangle_e=1$, where $A(k)$ is given by eq.(3.2).

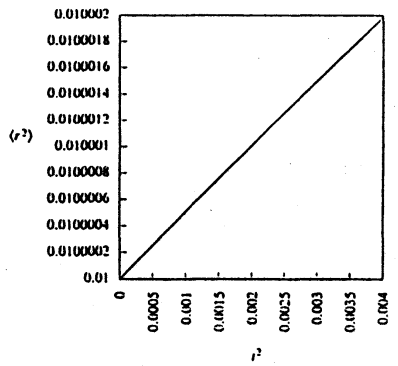

Fig.1.(b) t^2 dependence of $\langle r^2\rangle$ for the case $k_0=10^{-4}$, $k_d=10^2$, $r_0=0.1$, $\langle u^2\rangle_e=1$, where $A(k)$ is given by eq.(3.2).

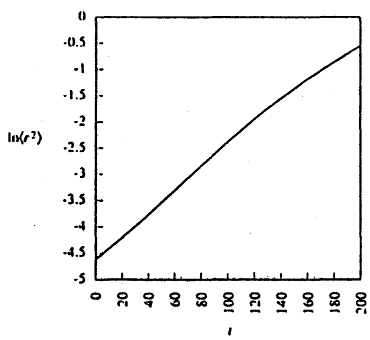

Fig.1.(c) Exponential dependence of $\langle r^2\rangle$ for the case $k_0=10^{-4}$, $k_d=10^2$, $r_0=0.1$, $\langle u^2\rangle_e=1$, where $A(k)$ is given by eq.(3.2).

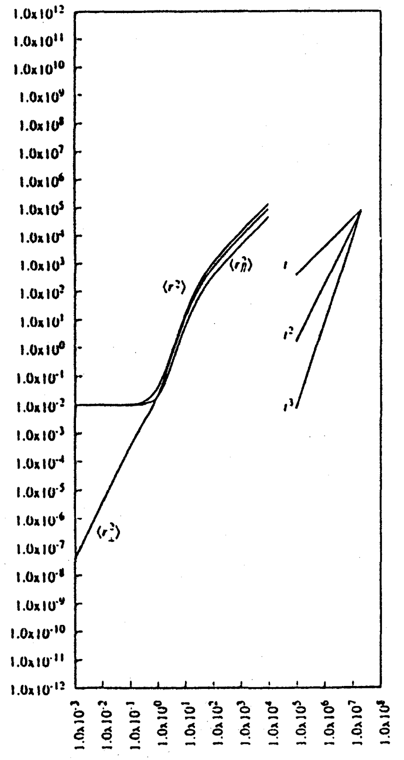

Fig.2.(a) The time evolution of $\langle r^2\rangle$, $\langle r_\perp^2\rangle$, $\langle r_{/\!/}^2\rangle$ for the case $k_0=10^{-1}$, $k_d'=10^2$, $k_d=10^5$, $r_0=0.1$, $\langle u^2\rangle_u=1$, where $A(k)$ is given by eq.(3.3).

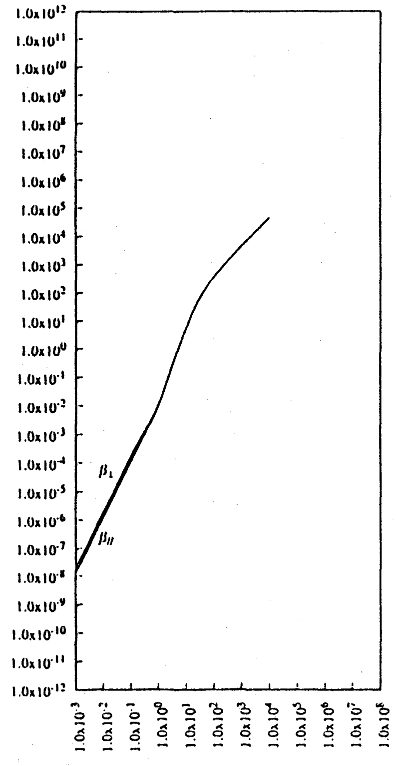

Fig.2.(d) The time evolution of $\beta_\perp$, $\beta_{/\!/}$ for the case $k_0=10^{-1}$, $k_d'=10^2$, $k_d=10^5$, $r_0=0.1$, $\langle u^2\rangle_u=1$, where $A(k)$ is given by eq.(3.3).

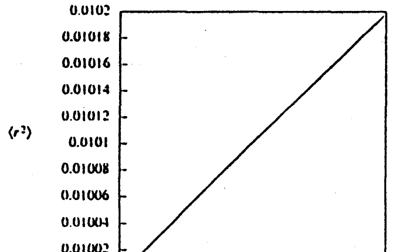

Fig.2.(b) t^2 dependence of $\langle r^2\rangle$ for the case $k_0=10^{-1}$, $k_d'=10^2$, $k_d=10^5$, $r_0=0.1$, $\langle u^2\rangle_u=1$, where $A(k)$ is given by eq.(3.3).

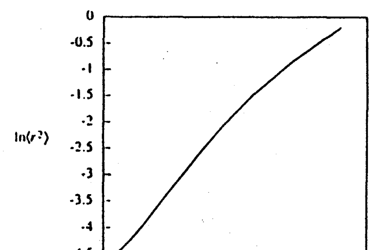

Fig.2.(c) Exponential dependence of $\langle r^2\rangle$ for the case $k_0=10^{-1}$, $k_d'=10^2$, $k_d=10^5$, $r_0=0.1$, $\langle u^2\rangle_u=1$, where $A(k)$ is given by eq.(3.3).

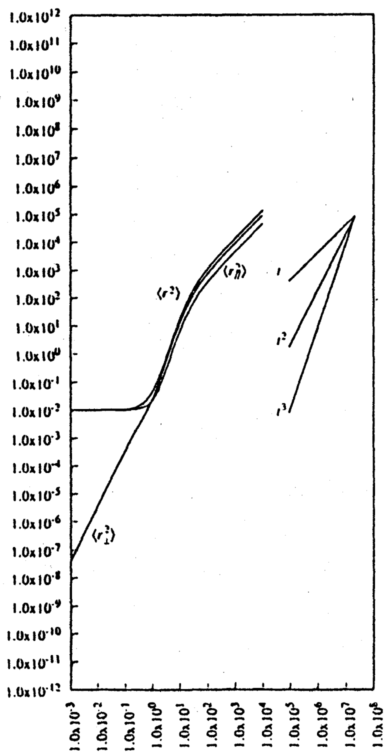

Fig.3.(a) The time evolution of $\langle r^2\rangle$, $\langle r_\perp^2\rangle$, $\langle r_\parallel^2\rangle$ for the case $k_0=10^{-1}$, $k_c=10^2$, $r_0=0.1$, $\langle u^2\rangle_u=1$, where $A(k)$ is given by eq.(3.2).

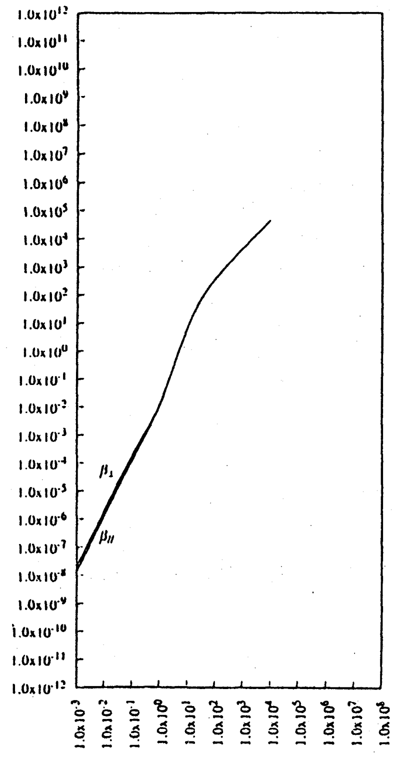

Fig.3.(d) The time evolution of $\beta_\perp$, $\beta_\parallel$ for the case $k_0=10^{-1}$, $k_c=10^2$, $r_0=0.1$, $\langle u^2\rangle_u=1$, where $A(k)$ is given by eq.(3.2).

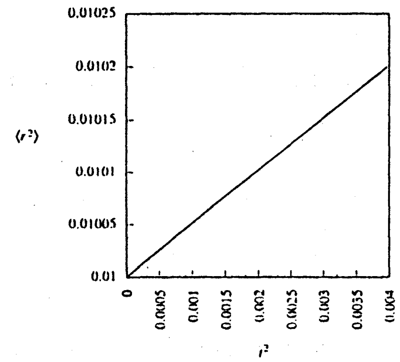

Fig.3.(b) t^2 dependence of $\langle r^2\rangle$ for the case $k_0=10^{-1}$, $k_c=10^2$, $r_0=0.1$, $\langle u^2\rangle_u=1$, where $A(k)$ is given by eq.(3.2).

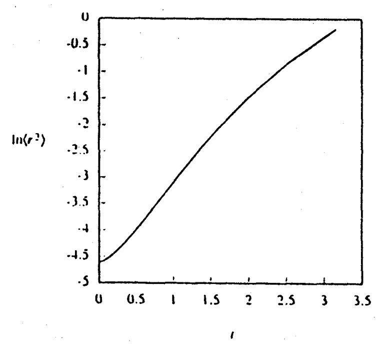

Fig.3.(c) Exponential dependence of $\langle r^2\rangle$ for the case $k_0=10^{-1}$, $k_c=10^2$, $r_0=0.1$, $\langle u^2\rangle_u=1$, where $A(k)$ is given by eq.(3.2).