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# Moduli spaces of holomorphic mappings into hyperbolically imbedded complex spaces and hyperbolic fibre spaces (abridged and revised version)

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## 1. Introduction.

Classical de Franchis theorem asserts that there are only a finite number of non-constant holomorphic mappings of a fixed compact Riemann surface $X$ into another fixed compact Riemann surface $Y$ of genus greater than one. Since the nonconstant mappings are regarded as the nontrivial section of the trivial fibre space $(Y \times X,\ P_X,\ X)$, the following is regarded as a generalization of the above theorem;

**Finiteness theorem for sections.** *Let $R$ be a Riemann surface of finite type. If a holomorphic family $W$ of compact Riemann surfaces with fixed genus greater than one over $R$ is non-trivial, then it has only finitely many non-constant holomorphic sections.*

This has been proved by Manin, Grauert and Miwa independently, and has an impotant implication in Diophantine problem (it was so-called Mordell's conjecture over function fields). In this article, we report the results (cf. [21]) obtained recently about the structure of the moduli spaces of holomorphic mappings considering a higher dimensional analogue in non-compact case of the above theorems. Noguchi [14,13] and Imayoshi and Shiga [6] gave another proof of the above finiteness theorem in function theoretic method independently. In the case of non-compact fibres, Imayoshi and Shiga [6] and Zaidenberg [24] obtained the finiteness results. Our method is heavily depend on the theory developed by Noguchi [14,15,17] so that we work within the category of hyperbolic geometry. About de Franchis theorem, many generalizations to higher dimensional cases have already been obtained. In the case where the target spaces are general type, the finiteness theorem was proved by Kobayashi and Ochiai [10] and Tsushima [22] (for noncompact case). In the case where the target spaces are the quotient spaces of the bounded symmetric domains in the complex vector space under some conditions (see §2), Sunada [20], Noguchi and Sunada [19], Imayoshi [4,5] and Noguchi [15] obtained the results which contained the one about the detailed structure of the moduli spaces of holomorphic mappings to such spaces. Under the assumption that the tangent spaces of the target spaces are negative in some sense, Kalka, Shiffman and Wong [7] and Urata [23] proved the finiteness theorem for surjective mappings. In the case of compact hyperbolic Kähler manifolds, Horst [2,3] obtained the finiteness results. On the other hand, about twenty years ago Lang [11] conjectured the higher dimensional analogue (in compact hyperbolic case) of the above finiteness theorems in a standpoint of Diophantine problem, which was recently solved by Noguchi [17](see §2 and §3 below). Noguchi [17] also conjectured a non-compact version of his theorems, which is our main theorem.

Let $Y$ be a complete hyperbolic complex space. We assume that $Y$ is hyperbolically imbedded into an irreducible compact complex space $\overline{Y}$ as it's Zariski open subset. Let $X$ be a Zariski open subset of an irreducible compact complex space. We denote by $Hol(X, Y)$ (resp. $Mer_{dom}(X, Y)$) the set of all holomorphic (resp. dominant meromorphic) mappings of $X$ into $Y$, where a mapping is said to be dominant if it's image contains a nonempty open subset.

**Finiteness Theorem for mappings in non-compact case.** *Let $X$ and $Y$ be as above. Then $Mel_{dom}(X, Y)$ is a finite set.*

We can also obtain a finiteness theorem of non-constant holomorphic sections and of splitting fibre subspaces in the case of non-compact trivial fibre spaces. Let $Y$ be as in the above Finiteness Theorem and $X$ be a nonsingular Zariski open subset of an irreducible compact complex space $\overline{X}$. Let $(Y \times X, P, X)$ be the trivial hyperbolic fibre space with the natural projection $P : Y \times X \longrightarrow X$. We obtain

**Finiteness Theorem for splitting fibre subspaces.** *Let $X$ and $Y$ be as above. Then $(Y \times X, P, X)$ contains only finitely many meromorphically trivial fibre subspaces, and carries only finitely many holomorphic sections except for constant ones in those bimeromorphic trivializations.*

By making use of the above results, we can see the detailed structure of the moduli space $Hol(X, Y)$.

Throughout this article, we assume that all complex spaces are paracompact and reduced and that all cmplex manifolds are connected. The term "hyperbolic" is always used in the sense of Kobayashi.

## 2. Finiteness of mappings in noncompact case.

Let $X$ be a Zariski open subset of an irreducible compact complex space $\overline{X}$. Let $Y$ be an irreducible complex space. A mapping $f$ of $X$ into $Y$ is said to be *dominant* if the image of $X$ by $f$ contains a non-empty open subset of $Y$. We denote by $Mer_{dom}(X, Y)$ (resp. $Hol_{dom}(X, Y)$) the set of all dominant meromorphic (resp. holomorphic) mappings from $X$ into $Y$. Recently Noguchi [17] proved the following finiteness theorem, which was conjectured by Lang [11];

**Noguchi's finiteness theorem.** *If $Y$ is compact hyperbolic, then $Mer_{dom}(X, Y)$ is* *a finite set.*

First, We prepare for a noncompact version of the above finiteness theorem. Let $X$ be a Zariski open subset of a compact complex manifold $\tilde{X}$ such that the boundary $\partial X := \tilde{X} - X$ of $X$ is a hypersurface with only normal crossings. Let $Y$ be a connected Zariski open subset of a compact complex space $\overline{Y}$. Assume that $Y$ is complete hyperbolic and hyperbolically imbedded into $\overline{Y}$. The spaces $Hol(X,\ Y)$ and $Hol(\tilde{X},\ \overline{Y})$ are equipped with compact-open topology. The extension and convergence theorem of Noguch [15] implies that the extension mapping $Hol(X,\ Y) \longrightarrow Hol(\tilde{X},\ \overline{Y})$ is homeomorphic onto the image of the mapping and by the natural identification $Hol(X,\ Y)$ is regarded as the topological subspace of $Hol(\tilde{X},\ \overline{Y})$. In fact the following structure theorem due to Noguchi [15] holds:

**Noguchi's structure theorem.** **i)** *The space $Hol(X,\ Y)$ is a Zariski open subset of the compact analytic subspace $\overline{Hol(X,\ Y)}$ of $Hol(\tilde{X},\ \overline{Y})$ where $\overline{Hol(X,\ Y)}$ is the closure of $Hol(X,\ Y)$ in $Hol(\tilde{X},\ \overline{Y})$ and the evaluation mapping*

$$\Phi : Hol(X,\ Y) \times X \ni (f,\ x) \longmapsto f(x) \in Y$$

*is holomorhpic and extends to a holomorphic mapping*

$$\overline{\Phi} : \overline{Hol(X,\ Y)} \times \tilde{X} \longrightarrow \overline{Y}.$$

**ii)** (universality) *For a complex space $T$ and a holomorphic mapping $\psi : T \times X \longrightarrow Y$, the natural mapping*

$$T \ni t \longmapsto \psi(t,\ \bullet) \in Hol(X,\ Y)$$

*is holomorphic.*

We set

$$Hol(k; X,\ Y) := \{f \in Hol(X,\ Y);\ \operatorname{rank} f = k\},$$

where $k$ is a nonnegative integer. Then we know the following

**Proposition 2.1** (cf. Noguchi [15]). *$Hol(k; X,\ Y)$ is open and closed in $Hol(X,\ Y)$, hence it carries a structure of complex space. In particular $Hol(n; X,\ Y)$ is a compact complex space where $n = \dim Y$.*

For any element $g \in Hol(X,\ Y)$, we denote the extension of $g$ to $\tilde{X}$ by the same letter $g$. Put $\partial Y := \overline{Y} - Y$. The next assertion essentially follows from the proof of Noguchi's structure theorem, i) (cf. the proof of Theorem 2.8, i) in Noguchi [15], pp. 23-24).

**Lemma 2.2** (cf. [21]). *Let $Z$ be a connected component of $Hol(X,\ Y)$. Take an element $g_0 \in Z$ and put $\partial X_0 := g_0^{-1}(\partial Y)$ and $Z_0 := \{g \in Z;\ g^{-1}(\partial Y) = \partial X_0\}$. Then $Z = Z_0$.*

Now our finiteness theorem for mappings of non-compact version is the following. The use of Lemma 2.2 was pointed out by Professor J. Noguchi and makes the proof of Theorem 2.3 simpler than the original one.

**Theorem 2.3** (cf. [21]). *Let $Y$ be a complete hyperbolic complex space which is hyperbolically imbedded into an irreducible compact complex space $\overline{Y}$ and is a Zariski open subset of $\overline{Y}$. Let $X$ be a Zariski open subset of an irreducible compact complex space $\overline{X}$. Then $Mer_{dom}(X,\ Y)$ is finite.*

**Proof.** Assume that $Mer_{dom}(X,\ Y)$ is not a finite set. Let $\overline{X}^* \xrightarrow{\alpha} \overline{X}$ be a resolution of singularities due to Hironaka and put $X^* := \alpha^{-1}(X)$. Then $f\circ(\alpha\ |_{X^*}) \in Mer(X^*,\ Y)$ for $f \in Mer(X,\ Y)$. Since $X^*$ is nonsingular and $Y$ is hyperbolic, $f\circ(\alpha\ |_{X^*}) \in Hol(X^*,\ Y)$. Then replacing $X^*$ by $X$ and putting $\tilde{X} := \overline{X}^*$, $\partial X := \tilde{X} - X$, we may assume that $\tilde{X}$ is a compact complex manifold, $X$ a Zariski open subset of $\tilde{X}$ and $\partial X$ a hypersurface with only normal crossings. Assume that $Hol_{dom}(X,\ Y)$ is not finite. It follows from Proposition 2.1 that $Hol(n; X,\ Y)$ is a compact complex space with positive dimension where $n = dim\, Y$. Take an irreducible component $Z$ of $Hol(n; X,\ Y)$ with $\dim Z > 0$ and an element $g_0 \in Z$. If we put $\partial X_0 := g_0^{-1}(\partial Y)$, we see from Lemma 2.2 that $g^{-1}(\partial Y) = \partial X_0$ for any $g \in Z$. Put $X_0 := \tilde{X} - \partial X_0$ and take a point $x_0 \in X_0$. Then the subset $Z(x_0) := \{\Phi(z, x_0) \in \overline{Y}; z \in Z\}$ of $\overline{Y}$ is a compact hyperbolic complex subspace of $Y$, where $\Phi$ is the evaluation mapping. Let $Y_0$ be an irreducible compact hyperbolic complex subspace of $Y$ containing $Z(x_0)$ with the maximum dimension among those subspaces. Take an element $z_0 \in Z$ at which $Z$ is nonsingular. Since the mapping $z_0\ |_{X_0}\colon X_0 \longrightarrow Y$ is proper holomorphic, $(z_0\ |_{X_0})^{-1}(Y_0)$ is a compact subvariety in $X_0$. Let $X_0'$ be the irreducible component of $(z_0\ |_{X_0})^{-1}(Y_0)$ containing $x_0$. Then the mapping $z_0\ |_{X_0'} : X_0' \longrightarrow Y_0$ is surjective. Moreover, the subset $\Phi(Z \times X_0')$ of $Y$ is an irreducible compact hyperbolic complex subspace containing $Y_0$. Thus we see that $\Phi(Z \times X_0') = Y_0$. Because of the finiteness of holomorphic mappings which map a given point to a given point, together with $\dim Z > 0$, it holds that the mapping $Z \longrightarrow Hol_{dom}(X_0',\ Y_0)$ is a non-constant mapping. This contradicts Noguchi's finiteness theorem, and we complete the proof. ∎

The following was proved in the case where $Y$ is nonsingular by Noguchi (cf. [13], Theorem (2.4)).

**Corollary 2.4.** *Let $Y$ be as in Theorem 2.3. Then the holomorphic automorphism group of $Y$ is a finite set.*

Let $D$ be a bounded symmetric domain in the complex vector space and $\Gamma$ a torsion free arithmetic subgroup of the identity component of the holomorphic automorphism group of $D$. Then it is well known that the non-compact quotient $D/\Gamma$ is complete hyperbolic and hyperbolically imbedded into it's Satake compactification (cf. [9]). Thus applying the theorem in this case, we obtain the following, which was first shown by Tsushima [22] within the categoly of general type (see also Noguchi [15]).

**Corollary 2.5.** *Let $X$ be as in Theorem 2.3. Then $Mer_{dom}(X, D/\Gamma)$ is a finite set.*

## 3. Finiteness of nontrivial sections and of trivial fibre subspaces.

Let $\overline{R}$ and $\overline{W}$ be irreducible compact complex spaces and $\overline{\Pi} : \overline{W} \longrightarrow \overline{R}$ a surjective holomorphic mapping with connected fibres. Let $R$ be a nonsingular Zariski open subset of $\overline{R}$ and $\partial R := \overline{R} - R$. Put

$$W := \overline{W}\,|_R = \overline{\Pi}^{-1}(R),\ \Pi := \overline{\Pi}\,|_W\,.$$

Suppose that each fibre $W_t := \Pi^{-1}(t)$ is irreducible for $t \in R$. We denote by $\Gamma$ the set of all holomorphic sections of the fiber space $(W, \Pi, R)$.

**Definition 3.1** (cf. Noguchi [14], §1). We call a fibre space $(W, \Pi, R)$ a *hyperbolic fibre space* if all the fibres $W_t$ for $t \in R$ are hyperbolic. We say that the fibre space $(W, \Pi, R)$ is *hyperbolically imbedded into* $(\overline{W}, \overline{\Pi}, \overline{R})$ *along* $\partial R$ if for any $t \in \partial R$ there are neighborhoods $U$ and $V$ of $t$ in $\overline{R}$ such that $U$ is relatively compact in $V$ and $W\,|_{U-\partial R}$ is hyperbolically imbedded into $\overline{W}\,|_V$.

Noguchi proved the following global triviality for normal hyperbolic fibre spaces (Noguchi [14], Main Theorem (3.2) and Noguchi [17], Theorem A).

**Noguchi's triviality Theorem for hyperbolic fibre spaces.** *Let $(W, \Pi, R)$ be a hyperbolic fibre space. Suppose that $(W, \Pi, R)$ is hyperbolically imbedde into a compact fibre space $(\overline{W}, \overline{\Pi}, \overline{R})$ along $\partial R$ and that $W$ is normal. If there exists a point $t \in R$ such that $\Gamma(t) := \{s(t) \in W_t : s \in \Gamma\}$ is Zariski dense in $W_t$, then $(W, \Pi, R)$ is holomorphically trivial, i.e., there is a biholomorphic mapping $F : W_t \times R \longrightarrow W$ such that $P = \Pi \circ F$ where $P : W_t \times R \longrightarrow R$ is the natural projection.*

Noguchi considered hyperbolic fibre spaces in a more general setting and obtained the following finiteness theorem for sections and for trivial fibre subspaces of a hyperbolic fibre space, which gave an affirmative answer to the higher dimensional analogue of Mordell's conjecture over function fields posed by Lang [11] (cf. Noguchi [17], Theorem B and it's correction).

**Definition 3.2.** We say that a fibre space $(W, \Pi, R)$ is *meromorphically trivial* if $(W, \Pi, R)$ is bimeromorphically isomorphic to some trivial fibre space over $R$.

**Theorem 3.3** (cf. [21]). *Let $(W, \Pi; R)$ be a hyperbolic fibre space. Assume that $(W, \Pi, R)$ is hyperbolically imbedded into some compact fibre space $(\overline{W}, \overline{\Pi}, \overline{R})$ along $\partial R$.*

*Then $(W,\ \Pi,\ R)$ contains only finitely many meromorphically trivial fibre subspaces with positive dimensional fibres, and carries only finitely many holomorphic sections except for constant ones in those bimeromorphic trivializations.*

In fact, in the proof it is shown that the normalization of each irreducible fibre subspace $W'$ of $W$ whose sections are dense in the total space becomes the trivial fibre subspace and that the normalization of the space of all sections of $W'$ gives the one of each fibre except for a proper subvariety of $R$.

**Corollary 3.4** (cf. [21]). *Let $(W,\ \Pi,\ R)$ be as in Theorem 3.3. If there is a point $t \in R$ such that $\Gamma(t)$ is Zariski dense in $W_t$, then the fibre space $(W_N,\ \Pi_N,\ R)$ obtained by taking the normalization of $W$ is a holomorphically trivial fibre space.*

**Example 3.5.** We give an example of the non-normal hyperbolic fibre spaces with infinitely many sections which are locally nontrivial. The author wishes to thank Professor T. Ueda for his help in constructing this example.

Let $R$ be a compact Riemann surface of genus greater than one. Let $\sigma$ be a holomorphic automorphism of $R$ which is not the identity mapping and $\iota$ be the identity mapping of $R$. Put

$$\hat{\sigma}(t) = (\sigma(t),\ t) \in R \times R \text{ for } t \in R$$

and

$$\hat{\iota}(t) = (t,\ t) \in R \times R \text{ for } t \in R.$$

We define an equivalence relation on $R \times R$ as follows: for $y_1,\ y_2 \in R \times R$, $y_1 \sim y_2$ if and only if there exists a point $t \in R$ such that $y_1 = \hat{\sigma}(t)$ and $y_2 = \hat{\iota}(t)$. Put $W := R \times R/\sim$. Then we see that $W$ is a complex space and that the projection $\beta : R \times R \longrightarrow W$ is holomorphic. Let $\Pi$ be the projection such that $\Pi \circ \beta = P_2$ on $R \times R$ where $P_2 : R \times R \longrightarrow R$ is the second projection. Then $(W,\ \Pi,\ R)$ is a hyperbolic space with compact hyperbolic fibres and carries infinitely many sections which come from the trivial fibre space $(R \times R,\ P_2,\ R)$ through the projection $\beta$. The fibre space $(W,\ \Pi,\ R)$ is locally nontrivial. In fact, suppose that there exists a local trivialization $\varphi : W\,|_U \xrightarrow{\cong} W_0 \times U$ where $U$ is an open set in $R$ and $W_0$ is an irreducible curve. We take the normalizations of the domain and the image of the localization and consider the lifting $\tilde{\varphi}$ of the mapping $\varphi$ to the normalizations. Then we see that $\tilde{\varphi}$ generates infinitely many holomorphic automorphisms of $R$. This is absurd since $R$ is compact hyperbolic.

Next we consider about finiteness of trivial fibre subspaces in the case where all the fibres are noncompact. We treat only trivial fibre spaces. Let $\overline{X}$ be an irreducible compact complex space and $X$ a nonsingular Zariski open subset of $\overline{X}$. Making use of the idea of the proof of Theorem 3.3 and of Theorem 3.3 itself, we get the follwong

**Theorem 3.6** (cf. [21], Theorem 4.1). *Let $X$ be as above. Let $Y$ be an irreducible* *complete hyperbolic complex space. Suppose that $Y$ is hyperbolically imbedded into some compact complex space $\overline{Y}$ and $Y$ is Zariski open in $\overline{Y}$. Then the trivial fibre space $(Y \times X,\ P,\ X)$ contains only finitely many meromorphically trivial fibre subspaces where $P$ is the natural projection (i.e., any meromorphically trivial fibre subspace of $(\overline{Y} \times \overline{X},\ P,\ \overline{X})$ is a trivial fibre subspace of one of them ) and carries only finitely many holomorphic sections except for constant ones in those bimeromorphic trivialization.*

Also in the case of non-trivial fibre spaces, under some condition on the imbeddedness of total space we can prove the finiteness theorem of above type.

## 4. Structure of the moduli space $Hol(X,\ Y)$.

We can obtain some information about the moduli spaces of holomorphic mappings in our situation. Let $X$ be a Zariski open subset of a compact complex manifold $\tilde{X}$. We assume that $\partial X := \tilde{X} - X$ is a hypersurface with only normal crossings. Let $Y$ be a Zariski open subset of an irreducible compact complex $\overline{Y}$. Assume that $Y$ is complete hyperbolic and hyperbolically imbedded into $\overline{Y}$. Let $Z$ be a connected component of $Hol(X,\ Y)$. Then the closure $\overline{Z}$ of $Z$ in $Hol(\tilde{X},\ \overline{Y})$ is a compact complex subspace of $Hol(\tilde{X},\ \overline{Y})$ and $Z$ is a Zariski open subset of $\overline{Z}$ by Noguchi's structure theorem.

**Proposition 4.1** (Noguchi [15], Miyano and Noguchi [13]).
**i)** *$Z$ is complete hyperbolic and hyperbolically imbedded into $\overline{Z}$.*
**ii)** *If $Y$ is quasi-projective algebraic and carries a projective compactification $\overline{Y}$ such that $Y$ is hyperbolically imbedded into $\overline{Y}$, then $Z$ is quasi-projective.*

**Proposition 4.2.** *The space $Hol(k; X,\ Y)$ is compact for $k > \dim \partial Y$ where $\partial Y := \overline{Y} - Y$.*

The proof is same as in Noguchi [15], Theorem (3.3), i). We obtain an estimate of the dimension of moduli.

**Theorem 4.3.** *Let $Z$ be an irreducible component of $Hol(X,\ Y)$. If $Z$ contains a non-constant holomorphic mapping, then $\dim Z \leq \dim Y - 1$.*

**Proof.** Since $\overline{Z}$ is compact, for any $x \in X$ the extension of the evaluation mapping

$$\overline{\Phi}(\bullet,\ x) : \ \overline{Z} \ni z \longmapsto z(x) \in \overline{Y}$$

is finite. Thus $\dim \overline{Z} \leq \dim \overline{Y}$. The case where $\dim Z = \dim Y$ contradicts Theorem 2.3 from the assumption of $Z$. ■

In the case where $Y$ is a noncompact quotient of a bounded symmetric domain in the complex vector space by a torsion free arithmetic subgroup of the identity component of it's holomorphic automorphism group, more effective estimates were obtained in Sunada [20], Theorem B, and Noguchi [15], Theorem (4.7), (4.10) (see for the compact quotient case Noguchi and Sunada [19], and Imayoshi [4,5]).

**Proposition 4.4.** *Suppose that* $\operatorname{codim} \partial Y \geq 2$. *Let* $z_0$ *be an element in* $Hol(n-1; X,\ Y)$ *such that* $z_0(X)$ *is relatively compact in* $Y$ $(n = \dim Y)$. *Then* $\dim_{z_0} Hol(n-1; X,\ Y) = 0$.

This follows from Proposition 4.2 and Noguchi's finiteness theorem.

**Theorem 4.5** (cf. [21], Corollary 4.2). *Take* $f \in Hol(X,\ Y)$. *If* $f(X)$ *is not relatively compact in* $Y$, *then the dimension of the irreducible component of* $Hol(X,\ Y)$ *which contains* $f$ *is not greater than the dimension of* $\partial Y$.

In the case where $Y$ is the quotient space of a bounded symmetric domain by a torsion free arithmetic discrete subgroup of the identity component of the holomorphic automorphism group, Theorem 4.5 was obtained in Noguch [15], Theorem 4.7 (iv).

Making use of the theory of harmonic mappings, Miyano and Noguchi proved the following sharper version of the finiteness theorem for mappings under Kähler condition.

**Theorem 4.6** (cf. [13], Theorem 2.15). *Let* $X$ *be a Zariski open subset of a compact Kähler manifold* $\bar{X}$ *and* $Y$ *be a quasiprojective algebraic manifold which carries a projective compactification* $\bar{Y}$ *such that* $Y$ *is hyperbolically imbedded into* $\bar{Y}$. *Suppose that* $Y$ *carries a complete Kähler metric whose Riemannian sectional curvatures are non-positive and holomorphic sectional curvatures are negatively bounded away from zero. Then*

$$\dim Hol(k; X,\ Y) \leq \dim Y + k.$$

Note that in the above theorem $Y$ becomes complete hyperbolic and then $\mathrm{Hol}(k; X,\ Y)$ has the complex structure with universal properties. In the case that $X$ and $Y$ are compact, algebraic manifolds, Goloff [1] obtained the same result under some different negativity conditions of $Y$. Recentry, Imayoshi [5] proved the same result in the case that $Y$ is a Carathéodory hyperbolic manifold and that $X$ is a projective algebraic manifold. A complex manifold $Y$ is said to be Carathéodory hyperbolic if $Y$ has a covering whose Carathéodory pseudo-distance is actually a distance (cf. Kobayashi [8], p. 129).

**Conjecture 4.7.** *Let* $Y$ *be a complete hyperbolic complex space which is hyperbolically imbedded into an irreducible compact complex space* $\overline{Y}$ *and is a Zariski open subset of* $\overline{Y}$. *Let* $X$ *be a nonsingular Zariski open subset of an irreducible compact complex* $\bar{X}$. *Then*

$$\dim Hol(k; X,\ Y) \leq \dim Y + k.$$

Relating to this conjecture, the following seems to be true (cf. [13]).

**Conjecture 4.8** *Under the same assumptions on $X$ and $Y$ as in Conjecture 4.7, the complex space $Hol(k;\ X, Y)$ is isometrically immersed into $Y$ with respect to Kobayashi metric.*

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