26.

On the Weights of Simple K3 Singularities

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Let X be a normal 2-dimensional analytic space. Then the singular points of X are discrete. There are rational singularities, quotient singularities, elliptic singularities, cusp singularities and so on. In the theory of two-dimensional singularities, simple elliptic singularities and cusp singularities are regarded as the next most reasonable class of singularities after rational singularities. What are natural generalizations in three-dimensional case of those singularities. They are purely elliptic singularities. When X is a two-dimensional analytic space, purely elliptic singularities are quasi-Gorenstein singularities. In higher dimensions, however, purely elliptic singularities are not always quasi-Gorenstein. Simple elliptic singularities and cusp singularities are characterized as two-dimensional purely elliptic singularities of (0,1)-type and of (0,0)-type, respectively. The notion of a simple K3 singularity is defined as a three-dimensional isolated Gorenstein purely elliptic singularity of (0,2)-type.

Example. Let $f(x,y,z,w)$ be a quasi-homogeneous polynomial of type $(p,q,r,s:h)$ with $p+q+r+s=h$, and suppose $f(x,y,z,w) = 0$ defines an isolated singularity at the origin. Then the origin is a simple K3 singularity.

Yonemura calculate the weights of hypersurface simple K3 singularities by nondegenerate polynomials and obtained examples such that the polynomial f is quasi-homogeneous and that f=0

has a simple K3 singularity at the origin.The minimum number of parameters in the polynomial is less than or equal to 19 and is associated with the moduli of the K3 surface with singularities.

We try to impose Mathematica programs to construct the weights of simple K3 singularities.

$$f = z^2 + y^5 + x^6 + w^6 y + zyxw$$

The follwing vector means {z,y,x,w}

$$\{2,0,0,0\}, \{0,5,0,0\}, \{0,0,6,0\}, \{0,1,0,6\}$$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 5; x= 0; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 6; w= 0; f3= a z + b y + c x + d w + e;

z= 0; y= 1; x= 0; w= 6; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[4]],2], Part[S[[1]][[2]],2],Part[S[[1]][[3]],2] };

T /. e → LCM[2,5,6,15] * (-1)

{15, 6, 5, 4}

$$\{2,0,0,0\}, \{0,5,0,0\}, \{0,0,6,0\}, \{0,0,2,5\}$$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 5; x= 0; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 6; w= 0; f3= a z + b y + c x + d w + e;

z= 0; y= 0; x= 2; w= 5; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[4]],2],Part[S[[1]][[3]],2] };

T /. e → LCM[2,5,6,15] * (-1)

{15, 6, 5, 4}

$$\{2,0,0,0\}, \{0,5,0,0\}, \{0,1,0,7\}, \{0,0,2,5\}$$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 5; x= 0; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 1; x= 0; w= 7; f3= a z + b y + c x + d w + e;

z= 0; y= 0; x= 2; w= 5; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[3]],2], Part[S[[1]][[2]],2],Part[S[[1]][[4]],2] };

T /. e → LCM[2,5,14,35] * (-1)

{35, 14, 15, 8}

{2,0,0,0}, {0,5,0,0}, {0,0,6,0}, {1,1,1,1}

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 5; x= 0; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 6; w= 0; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[2]],2],Part[S[[1]][[3]],2], Part[S[[1]][[4]],2],Part[S[[1]][[1]],2] }; T /. e → LCM[2,5,6,15] * (-1)

{15, 6, 5, 4}

{2,0,0,0}, {0,5,0,0}, {0,1,0,7}, {1,1,1,1}

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 5; x= 0; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 1; x= 0; w= 7; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[2]],2],Part[S[[1]][[4]],2], Part[S[[1]][[1]],2],Part[S[[1]][[3]],2] };

T /. e → LCM[2,5,70,35] * (-1)

{35, 14, 13, 8}

{2,0,0,0}, {0,0,6,0}, {0,1,0,7}, {1,1,1,1}

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 0; x= 6; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 1; x= 0; w= 7; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[3]],2],Part[S[[1]][[4]],2] };

T /. e → LCM[2,9,6,9] * (-1)

{9, 4, 3, 2}

$$\{0,5,0,0\}, \{0,0,6,0\}, \{0,1,0,7\}, \{1,1,1,1\}$$

z= 0; y= 5; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 0; x= 6; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 1; x= 0; w= 7; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[4]],2], Part[S[[1]][[2]],2],Part[S[[1]][[3]],2] };

T /. e → LCM[210,5,6,35] * (-1)

{109, 42, 35, 24}

$$f = z^2 + y^3 + x^7 + w^4 2 + zyxw$$

The follwing vector means {z,y,x,w}

$$\{2,0,0,0\}, \{0,3,0,0\}, \{0,0,7,0\}, \{0,0,0,42\}$$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 3; x= 0; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 7; w= 0; f3= a z + b y + c x + d w + e;

z= 0; y= 0; x= 0; w= 42; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[3]],2],Part[S[[1]][[4]],2] };

T /. e → LCM[2,3,7,42] * (-1)

{21, 14, 6, 1}

$$\{2,0,0,0\}, \{0,3,0,0\}, \{0,0,7,0\}, \{0,0,6,6\}$$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 3; x= 0; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 7; w= 0; f3= a z + b y + c x + d w + e;

z= 0; y= 0; x= 6; w= 6; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[4]],2],Part[S[[1]][[3]],2] };

T /. e → LCM[2,3,7,42] * (-1)

{21, 14, 6, 1}

$\{2,0,0,0\},\{0,3,0,0\},\{0,0,0,43\},\{0,0,6,6\}$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 3; x= 0; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 0; w= 43; f3= a z + b y + c x + d w + e;

z= 0; y= 0; x= 6; w= 6; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[3]],2],Part[S[[1]][[4]],2] };

T /. e → LCM[2,3,258,43] * (-1)

{129, 86, 37, 6}

$\{2,0,0,0\},\{0,3,0,0\},\{0,0,7,0\},\{1,1,1,1\}$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 3; x= 0; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 7; w= 0; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[2]],2],Part[S[[1]][[3]],2], Part[S[[1]][[4]],2],Part[S[[1]][[1]],2] };

T /. e → LCM[2,3,7,42] * (-1)

{21, 14, 6, 1}

$\{2,0,0,0\},\{0,3,0,0\},\{0,0,0,43\},\{1,1,1,1\}$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 3; x= 0; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 0; w= 43; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[2]],2],Part[S[[1]][[3]],2], Part[S[[1]][[1]],2],Part[S[[1]][[4]],2] };

T /. e → LCM[2,3,258,43] * (-1)

{129, 86, 37, 6}

$\{2,0,0,0\}, \{0,0,7,0\}, \{0,0,0,43\}, \{1,1,1,1\}$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 0; x= 7; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 0; w= 43; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[2]],2],Part[S[[1]][[1]],2], Part[S[[1]][[3]],2],Part[S[[1]][[4]],2] };

T /. e → LCM[2,602,7,43] * (-1)

{301, 201, 86, 14}

$\{0,3,0,0\}, \{0,0,7,0\}, \{0,0,0,43\}, \{1,1,1,1\}$

z= 0; y= 3; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 0; x= 7; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 0; w= 43; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[3]],2],Part[S[[1]][[4]],2] };

T /. e → LCM[903,3,7,43] * (-1)

{452, 301, 129, 21}

$$f = z^2 + xy^4 + wx^5 + w^7x + zyxw$$

The follwing vector means $\{z,y,x,w\}$

$\{2,0,0,0\}, \{0,4,1,0\}, \{0,0,5,1\}, \{0,0,1,7\}$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 4; x= 1; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 5; w= 1; f3= a z + b y + c x + d w + e;

z= 0; y= 0; x= 1; w= 7; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[4]],2],Part[S[[1]][[3]],2] };

T /. e → LCM[2,34,17,17] * (-1)

{17, 7, 6, 4}

$\{2,0,0,0\}, \{0,4,1,0\}, \{0,0,5,1\}, \{0,0,3,4\}$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 4; x= 1; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 5; w= 1; f3= a z + b y + c x + d w + e;

z= 0; y= 0; x= 3; w= 4; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[4]],2],Part[S[[1]][[3]],2] };

T /. e → LCM[2,34,17,17] * (-1)

{17, 7, 6, 4}

$\{2,0,0,0\}, \{0,4,1,0\}, \{0,0,1,8\}, \{0,0,3,4\}$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 4; x= 1; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 1; w= 8; f3= a z + b y + c x + d w + e;

z= 0; y= 0; x= 3; w= 4; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[4]],2],Part[S[[1]][[3]],2] };

T /. e → LCM[2,5,5,10] * (-1)

{5, 2, 2, 1}

$\{2,0,0,0\}, \{0,4,1,0\}, \{0,0,5,1\}, \{1,1,1,1\}$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 4; x= 1; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 5; w= 1; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[4]],2],Part[S[[1]][[3]],2] };

T /. e → LCM[2,34,17,17] * (-1)

{17, 7, 6, 4}

$\{2,0,0,0\},\{0,4,1,0\},\{0,0,1,8\},\{1,1,1,1\}$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 4; x= 1; w= 0; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 1; w= 8; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[4]],2],Part[S[[1]][[3]],2] };

T /. e → LCM[2,5,5,10] * (-1)

{5, 2, 2, 1}

$\{2,0,0,0\},\{0,0,5,1\},\{0,0,1,8\},\{1,1,1,1\}$

z= 2; y= 0; x= 0; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 0; x= 5; w= 1; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 1; w= 8; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[2]],2],Part[S[[1]][[1]],2], Part[S[[1]][[3]],2],Part[S[[1]][[4]],2] };

T /. e → LCM[2,78,39,39] * (-1)

{39, 17, 14, 8}

$\{0,4,1,0\},\{0,0,5,1\},\{0,0,1,8\},\{1,1,1,1\}$

z= 0; y= 4; x= 1; w= 0; f1= a z + b y + c x + d w + e;

z= 0; y= 0; x= 5; w= 1; f2= a z + b y + c x + d w + e;

z= 0; y= 0; x= 1; w= 8; f3= a z + b y + c x + d w + e;

z= 1; y= 1; x= 1; w= 1; f4= a z + b y + c x + d w + e;

S= Solve[{f1,f2,f3,f4}=={0,0,0,0},{a,b,c,d}];

T= { Part[S[[1]][[1]],2],Part[S[[1]][[2]],2], Part[S[[1]][[4]],2],Part[S[[1]][[3]],2] };

T /. e → LCM[39,39,39,39] * (-1)

{20, 8, 7, 4}

$$f = z^2 + y^3 + \ldots + zyxw$$

The number in following vector shows {z,y,x,w} Define the weight of x and w

$$xx[hxx\_] \quad := \quad 1/(hxx + 3)$$

$$xy[hxy\_] \quad := \quad (1 - (1/3)) * (1/(hxy + 2))$$

$$xz[hxz\_] \quad := \quad (1 - (1/2)) * (1/(hxz + 2))$$

$$ww[iww\_] \quad := \quad 1/(iww + 3)$$

$$wy[iwy\_] \quad := \quad (1 - (1/3)) * (1/(iwy + 2))$$

$$wz[iwz\_] \quad := \quad (1 - (1/2)) * (1/(iwz + 2))$$

$$xww[hxww\_] \quad := \quad (1 - ww[iww]) * (1/(hxww + 2))$$

$$xwy[hxwy\_] \quad := \quad (1 - wy[iwy]) * (1/(hxwy + 2))$$

$$xwz[hxwz\_] \quad := \quad (1 - wz[iwz]) * (1/(hxwz + 2))$$

$$wxx[iwxx\_] \quad := \quad (1 - xx[hxx]) * (1/(iwxx + 2))$$

$$wxy[iwxy\_] \quad := \quad (1 - xy[hxy]) * (1/(iwxy + 2))$$

$$wxz[iwxz\_] \quad := \quad (1 - xz[hxz]) * (1/(iwxz + 2))$$

calculation

```
Do[If[xx[hxx]+ww[iww]==1/6 && xx[hxx]>=ww[iww],
   Print[{1/2,1/3,xx[hxx],ww[iww]}]],{hxx,1,9,1},{iww,1,39,1}];
```

{1/2, 1/3, 1/7, 1/42} {1/2, 1/3, 1/8, 1/24} {1/2, 1/3, 1/9, 1/18}
{1/2, 1/3, 1/10, 1/15} {1/2, 1/3, 1/12, 1/12}

```
Do[If[xx[hxx]+wxx[iwxx]==1/6 && xx[hxx]>=wxx[iwxx],
   Print[{1/2,1/3,xx[hxx],wxx[iwxx]}]],{hxx,1,9,1},{iwxx,1,34,1}];
```
{1/2, 1/3, 1/7, 1/42} {1/2, 1/3, 1/8, 1/24} {1/2, 1/3, 1/9, 1/18}
{1/2, 1/3, 1/11, 5/66} {1/2, 1/3, 1/12, 1/12}

```
Do[If[xx[hxx]+wy[iwy]==1/6 && xx[hxx]>= wy[iwy],
   Print[{1/2,1/3,xx[hxx],wy[iwy]}]],{hxx,1,9,1},{iwy,1,26,1}];
```
{1/2, 1/3, 1/7, 1/42} {1/2, 1/3, 1/8, 1/24} {1/2, 1/3, 1/9, 1/18}
{1/2, 1/3, 1/10, 1/15} {1/2, 1/3, 1/12, 1/12}

```
Do[If[xx[hxx]+wz[iwz]==1/6 && xx[hxx]>=wz[iwz],
    Print[{1/2,1/3,xx[hxx],wz[iwz]}]],{hxx,1,9,1},{iwz,1,19,1}];
{1/2, 1/3, 1/7, 1/42} {1/2, 1/3, 1/8, 1/24} {1/2, 1/3, 1/9, 1/18} {1/2,
1/3, 1/12, 1/12}


Do[If[xy[hxy]+ww[iww]==1/6 && xy[hxy]>=ww[iww],
    Print[{1/2,1/3,xy[hxy],ww[iww]}]],{hxy,1,8,1},{iww,1,27,1}];
{1/2, 1/3, 2/15, 1/30} {1/2, 1/3, 1/9, 1/18} {1/2, 1/3, 2/21, 1/14} {1/2,
1/3, 1/12, 1/12}


Do[If[xy[hxy]+wxy[iwxy]==1/6 && xy[hxy]>=wxy[iwxy],
    Print[{1/2,1/3,xy[hxy],wxy[iwxy]}]],{hxy,1,8,1},{iwxy,1,24,1}];
{1/2, 1/3, 2/15, 1/30} {1/2, 1/3, 1/9, 1/18} {1/2, 1/3, 1/12, 1/12}


Do[If[xy[hxy]+wy[iwy]==1/6 && xy[hxy]>=wy[iwy],
    Print[{1/2,1/3,xy[hxy],wy[iwy]}]],{hxy,1,8,1},{iwy,1,18,1}];
{1/2, 1/3, 2/15, 1/30} {1/2, 1/3, 1/9, 1/18} {1/2, 1/3, 1/12, 1/12}


Do[If[xy[hxy]+wz[iwz]==1/6 && xy[hxy]>=wz[iwz],
    Print[{1/2,1/3,xy[hxy],wz[iwz]}]],{hxy,1,8,1},{iwz,1,13,1}];
{1/2, 1/3, 2/15, 1/30} {1/2, 1/3, 1/9, 1/18} {1/2, 1/3, 2/21, 1/14} {1/2,
1/3, 1/12, 1/12}


Except the case of xz included in the case of xx
Do[If[xww[hxww]+ww[iww]==1/6 && xww[hxww]>=ww[iww],
    Print[{1/2,1/3,xww[hxww],ww[iww]}]],{hxww,1,12,1},{iww,1,33,1}];
{1/2, 1/3, 5/36, 1/30} {1/2, 1/3, 5/42, 1/21} {1/2, 1/3, 5/48, 1/16} {1/2,
1/3, 1/12, 1/12}


Do[If[xwy[hxwy]+wy[iwy]==1/6 && xwy[hxwy]>=wy[iwy],
    Print[{1/2,1/3,xwy[hxwy],wy[iwy]}]],{hxwy,1,12,1},{iwy,1,22,1}];
{1/2, 1/3, 5/36, 1/30} {1/2, 1/3, 5/42, 1/21} {1/2, 1/3, 5/54, 2/27} {1/2,
1/3, 1/12, 1/12}


Do[If[xwz[hxwz]+wz[iwz]==1/6 && xwz[hxwz]>=wz[iwz],
```

```
Print[{1/2,1/3,xwz[hxwz],wz[iwz]}]],{hxwz,1,12,1},{iwz,1,16,1}];
```
{1/2, 1/3, 5/36, 1/36} {1/2, 1/3, 5/48, 1/16} {1/2, 1/3, 1/12, 1/12}

The case of xz is now constructing