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TIME LOCAL WELL-POSEDNESS FOR THE ZAKHAROV SYSTEM WITH THE PERIODIC BOUNDARY CONDITION

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1. Introduction and result

In this paper we consider the time local well-posedness for the one dimensional Zakharov system with the periodic boundary condition:

$$i\partial_t u+\alpha\partial_x^2 u=un,\ (t,x)\in[-T,T]\times\mathbb{T}, \tag{1.1}$$

$$\frac{1}{\beta^2}\partial_t^2 n-\partial_x^2 n=\partial_x^2(|u|^2),\ (t,x)\in[-T,T]\times\mathbb{T}, \tag{1.2}$$

$$u(0,x)=u_0(x),\ n(0,x)=n_0(x),\ \partial_t n(0,x)=n_1(x),\ x\in\mathbb{T}, \tag{1.3}$$

where α and β are real constants with $\alpha\neq 0$ and $\beta>0$, u and n are functions on the time-space $[-T,T]\times\mathbb{T}$ with values in $\mathbb{C}$ and $\mathbb{R}$, respectively, $\mathbb{T}$ is a one dimensional torus which implies the periodic boundary condition and T is a positive constant to be determined later.

The equations of (1.1)-(1.2) was presented by V. E. Zakharov [15] to understand the the propagation of Langmuir turbulence waves in an unmagnetized, completely ionized hydrogen plasma. In these equations, u is the slowly varying complex envelope of the electric field E with frequency ω,

$$E(t,x) = \mathrm{Re}(u(t,x)e^{-it\omega}),$$

n denotes the deviation of the ion density from the equilibrium, α is proportional to the differentiation of the group velocity in the wave number and β is the speed of ion acoustic wave in a plasma.

Our purpose in this paper is to show the time local well-posedness results in a weak class for the initial value problem (1.1)-(1.3). The difficulty to solve the initial value problem (1.1)-(1.3) by the integral equations is that the equation of (1.2) has space derivatives in the nonlinear term. One derivative can be regained because of the second order hyperbolicity, but it is a hard task to regain the remaining one derivative in that term. Then the usual contraction argument seems to meet with the so-called "loss of derivative".

There are a large amount of papers concerning the well-posedness of the initial value problem (1.1)-(1.3) for the $\mathbb{R}^d$ case (See[1,5,6,10,11,14]). However, to our knowledge, there are only a few papers concerning for the periodic boundary condition problem (see, e.g., [4]).

Before precisely stating our results, we prepare the following notations.

Definition 1.1. Let $\mathcal{V}$ be the space of functions f such that

$$f : \mathbb{R} \times \mathbb{T} \to \mathbb{C},$$

$$f(\cdot, x) \in \mathcal{S}(\mathbb{R}) \text{ for each } x \in \mathbb{T} \text{ and } f(t, \cdot) \in C^\infty(\mathbb{T}) \text{ for each } t \in \mathbb{R}.$$

For $s, b \in \mathbb{R}$, we define the spaces X_s and Y_s to be the completion of $\mathcal{V}$ with respect to the norms:

$$\|f\|_{X_s} = \|f\|_{(1,s,1/2)},$$

$$\|f\|_{Y_s} = \|f\|_{(2,s,1/2)},$$

where

$$\|f\|_{(1,s,b)} = \left(\sum_{n\in\mathbb{Z}} (1+|n|)^{2s} \int_{-\infty}^{\infty} \left(1 + |\tau + \alpha n^2|\right)^{2b} |\widehat{f}(\tau, n)|^2 d\tau \right)^{1/2},$$

$$\|f\|_{(2,s,b)} = \left(\sum_{n\in\mathbb{Z}} (1+|n|)^{2s} \int_{-\infty}^{\infty} (1 + ||\tau| - \beta|n||)^{2b} |\widehat{f}(\tau, n)|^2 d\tau \right)^{1/2}.$$

Let $\psi \in C_0^\infty(\mathbb{R})$ with $\psi = 1$ on $[-1, 1]$ and support$\psi \subseteq (-2, 2)$ and $\psi_\delta(t) = \psi(t/\delta)$.

We now state our theorem.

Theorem 1.1. *Assume that β/α is not integer. Given s, l satisfying $0 \le s - l \le 1$ and $0 \le l + 1/2 \le 2s$. For any $(u_0, n_0, n_1) \in H^s \times H^l \times H^{l-1}$, there exist $T = T(\|u_0\|_{L^2}, \|n_0\|_{H^{-1/2}}, \|n_1\|_{H^{-3/2}}) > 0$ and a unique solution*

$(u, n, \partial_t n)$ *of the initial value problem (1.1)-(1.3) in the time interval* $[-T, T]$ *such that*

$$u \in C([-T,T] : H^s(\mathbb{T})),\ \psi_T u \in X_s,$$

$$n \in C([-T,T] : H^l(\mathbb{T})),\ \psi_T n \in Y_l,$$

$$\partial_t n \in C([-T,T] : H^{l-1}(\mathbb{T})),\ \psi_T \partial_t n \in Y_{l-1}.$$

For any $T' \in (0,T)$, *there exists* $\epsilon > 0$ *such that the map* $(\tilde{u}_0, \tilde{n}_0, \tilde{n}_1) \mapsto (\tilde{u}, \tilde{n}, \partial_t \tilde{n})$ *is Lipschitz from* $\{(\tilde{u}_0, \tilde{n}_0, \tilde{n}_1) : \|\tilde{u}_0 - u_0\|_{H^s} + \|\tilde{n}_0 - n_0\|_{H^l} + \|\tilde{n}_1 - n_1\|_{H^{l-1}} < \epsilon\}$ *to* $\|\tilde{u} - u\|_{L_t^\infty([-T',T']:H^s)} + \|\tilde{n} - n\|_{L_t^\infty([-T',T']:H^l)} + \|\partial_t \tilde{n} - \partial_t n\|_{L_t^\infty([-T',T']:H^{l-1})} + \|\psi_{T'}(\tilde{u} - u)\|_{X_s} + \|\psi_{T'}(\tilde{n} - n)\|_{Y_l} + \|\psi_{T'}(\partial_t \tilde{n} - \partial_t n)\|_{Y_{l-1}}$.

Remark 1.1. In [4], J. Bourgain showed that when $\alpha = \beta = 1$ the initial value problem (1.1)-(1.3) is time locally well-posed in a certain class. More precisely, he showed that for any (u_0, n_0, n_1) such that

$$u_0 \in H^s,$$

$$\sup_{k \in \mathbb{Z}} (1+|k|)^{s_1} |\widehat{u_0}(k)|, \sup_{k \in \mathbb{Z}} (1+|k|)^{\sigma} |\widehat{n_0}(k)|, \sup_{k \in \mathbb{Z}} (1+|k|)^{\sigma-1} |\widehat{n_1}(k)| < \infty, \tag{1.4}$$

with $\sigma < 0 < s < 1/2 < s_1 < 1$ where σ, s are sufficiently close to $0, 1/2$, respectively. There exist $T > 0$ and a unique solution $(u, n, \partial_t n)$ satisfying

$$u \in C([-T,T] : H^s),\ \sup_{k \in \mathbb{Z}} (1 + |k|)^{s_1} |\mathcal{F}_x u(t,k)| < \infty,$$

$$n \in C([-T,T]:H^{s-1}),\ \sup_{k\in\mathbb{Z}}(1+|k|)^{\sigma}|\mathcal{F}_x n(t,k)| < \infty,$$

$$\partial_t n \in C([-T,T]:H^{s-2}),\ \sup_{k\in\mathbb{Z}}(1+|k|)^{\sigma-1}|\mathcal{F}_x \partial_t n(t,k)| < \infty,$$

for $t \in [-T,T]$ and

$$|||u||| = \inf\{\|v\|_{X_s} : v(t) = u(t) \text{ on } [-T,T]\times\mathbb{T}\} < \infty.$$

Under the non resonance condition that β/α is not integer, Theorem 1.1 implies the time local well-posedness with the data in the usual Sobolev spaces $H^s \times H^l \times H^{l-1}$. We do not assume the weighted L^∞ condition in the Fourier space such as (1.4) and we shall give a slightly simpler proof than in [4].

Remark 1.2. By Theorem 1.1, we have the time local well-posedness in $L^2 \times H^{-1/2} \times H^{-3/2}$. Of course, the result in Theorem 1.1 also contain the Hamiltonian class, that is, the energy space.

Remark 1.3. When $0 \le s-l \le 1$ and $1/2 \le l+1/2 \le 2s$, the result in Theorem 1.1 hold with $\beta/\alpha \in \mathbb{R}$.

The proof of Theorem 1.1 is based on the method introduced by Bourgain [3] and Kenig-Ponce-Vega [9] to treat the KdV equation. We use respectively the two different Fourier restriction norms for the equations (1.1)-(1.2) under the non resonance condition that β/α is not integer.

2. Proof of Theorem 1.1

First, we rewrite equations (1.1)-(1.3) as following integral equations:

$$u(t) = S(t)u_0 - i\int_0^t S(t-\tau)(un)(\tau)d\tau,$$

$$n(t) = \partial_t V(t)n_0 + V(t)n_1 + \int_0^t V(t-\tau)\partial_x^2(|u|^2)(\tau)d\tau,$$

where $S(t) = e^{it\partial_x^2}$ and $V(t) = \sin\beta t(-\partial_x^2)^{1/2}/\beta(-\partial_x^2)^{1/2}$.

We first state the two lemmas concerning the estimates of the linear and nonlinear part of the Schrödinger and Wave equations on the function spaces we consider.

Lemma 2.1. *For any $s \in \mathbb{R}$, we have*

$$\|S(t)u_0\|_{X_s} \leq c\|u_0\|_{H^s},$$

$$\|\partial_t V(t)n_0\|_{Y_s} \leq c\|n_0\|_{H^s},$$

$$\|V(t)n_1\|_{Y_s} \leq c\|n_1\|_{H^{s-1}}.$$

Lemma 2.2. *For any $s \in \mathbb{R}$, we have*

$$\left\|\psi(t)\int_0^t S(t-\tau)F(\tau)d\tau\right\|_{X_s} \tag{2.1}$$

$$\leq c\|F\|_{(1,s,-1/2)} + c\left(\sum_{k\in\mathbb{Z}}(1+|k|)^{2s}\left(\int\frac{|\widehat{F(\tau,k)}|}{1+|\tau+\alpha k^2|}d\tau\right)^2\right)^{1/2},$$

$$(2.2)\qquad \left\| \psi(t)\int_0^t V(t-\tau)\partial_x F(\tau)d\tau \right\|_{Y_s}$$
$$\leq c\|F\|_{(2,s,-1/2)} + c\left(\sum_{k\in\mathbb{Z}}(1+|k|)^{2s}\left(\int \frac{|\widehat{F(\tau,k)}|}{1+\||\tau|-\beta|k|\|}d\tau\right)^2\right)^{1/2}.$$

Lemma 2.1 and 2.2 follow from a direct calculation. For the proof of that lemmas, see [3,9]. The following lemma plays to use the contraction argument in the proof of Theorem 1.1.

Lemma 2.3. *For any $s \in \mathbb{R}, 0 < \epsilon \ll 1, \delta \in (0,1]$ and $0 < b < 1/2$, it follows*

$$\|\psi_\delta F\|_{(i,s,1/2)} \leq c_\epsilon \delta^{-\epsilon}\|F\|_{(i,s,1/2)},$$

$$\|\psi_\delta F\|_{(i,s,b)} \leq c\delta^{1/2-b+\epsilon}\|F\|_{(i,s,1/2)},$$

for $i = 1$ and 2.

Lemma 2.3 is proved by using the Leibniz rules for fractional derivatives [8, Theorem A.12], Hölder and Sobolev inequality with respect to time variable. We give the following lemma needed to estimate the nonlinear terms appearing in the right hand side of (2.1) and (2.2). The following lemma plays a key role in our proof.

Lemma 2.4. *Assume β/α is not integer. For $0 \leq s - l \leq 1$ and $0 \leq l + 1/2 \leq 2s$, we have*

$$(2.3)\qquad \left\| \frac{(1+|k|)^s}{(1+|\tau+\alpha k^2|)^{1/2}} \sum_{k_1\in\mathbb{Z}}\int_{-\infty}^{\infty} \frac{f(\tau_1,k_1)}{(1+|k_1|)^s(1+|\tau_1+\alpha k_1^2|)^{1/2}}\right.$$

$$\times \frac{g(\tau-\tau_1,k-k_1)}{(1+|k-k_1|)^l(1+||\tau-\tau_1|-\beta|k-k_1||)^{1/2}}d\tau_1\Bigg\|_{L^2_\tau l^2_k}$$

$$\leq c\|f\|_{L^2_\tau l^2_k}\|g\|_{L^2_\tau l^2_k},$$

$$\text{(2.4)}\qquad \left\|\frac{k(1+|k|)^l}{(1+||\tau|-\beta|k||)^{1/2}}\sum_{k_1\in\mathbb{Z}}\int_{-\infty}^{\infty}\frac{f(\tau_1,k_1)}{(1+|k_1|)^s(1+|\tau_1+\alpha k_1^2|)^{1/2}}\right.$$

$$\times \frac{h(\tau-\tau_1,k-k_1)}{(1+|k-k_1|)^s(1+|\tau-\tau_1-\alpha(k-k_1)^2|)^{1/2}}d\tau_1\Bigg\|_{L^2_\tau l^2_k}$$

$$\leq c\|f\|_{L^2_\tau l^2_k}\|h\|_{L^2_\tau l^2_k}.$$

Proof of Lemma 2.4. The proof uses the following two identities:

$$(\tau+\alpha k^2)-(\tau-\tau_1\pm\beta(k-k_1))-(\tau_1+\alpha k_1^2)=\alpha(k-k_1)(k+k_1\mp\frac{\beta}{\alpha}),$$

$$(\tau\pm\beta k)-(\tau-\tau_1-\alpha(k-k_1)^2)-(\tau_1+\alpha k_1^2)=\alpha k(k-2k_1\pm\frac{\beta}{\alpha}),$$

which implies

(2.5)

$$\max\left\{|\tau+\alpha k^2|,|\tau-\tau_1\pm\beta(k-k_1)|,|\tau_1+\alpha k_1^2|\right\}\geq\frac{|\alpha|}{3}|k-k_1||k+k_1\mp\frac{\beta}{\alpha}|,$$

(2.6)

$$\max\left\{|\tau\pm\beta k|,|\tau-\tau_1-\alpha(k-k_1)^2|,|\tau_1+\alpha k_1^2|\right\}\geq\frac{|\alpha|}{3}|k||k-2k_1\pm\frac{\beta}{\alpha}|.$$

Using these inequalities, we are able to overcome the difficulty of derivative lose. In this paper, we prove only (2.3) and (2.4) in the regions that

$$\text{(2.7)}\qquad |\tau+\alpha k^2|\geq|\tau_1+\alpha k_1^2|,||\tau-\tau_1|-\beta|k-k_1||,$$

$$(2.8) \qquad ||\tau| - \beta|k|| \geq |\tau - \tau_1 - \alpha(k-k_1)^2|, |\tau_1 + \alpha k_1^2|,$$

respectively.

We first prove (2.3) restricted to the region of (2.7). In that region by Schwarz inequality we have that the left-hand side of (2.3) is bounded by

$$\sup_{\tau,k}\left(\frac{(1+|k|)^{2s}}{(1+|\tau+\alpha k^2|)}\sum_{k_1}\int\frac{1}{(1+|\tau_1+\alpha k_1^2|)(1+|k_1|)^{2s}(1+|k-k_1|)^{2l}}\right.$$
$$\left.\times\frac{1}{(1+||\tau-\tau_1|-\beta|k-k_1||)}d\tau_1\right)^{1/2}\|f\|_{L^2_\tau l^2_k}\|g\|_{L^2_\tau l^2_k}$$
$$=\sup_{\tau,k} I_{\tau,k}\|f\|_{L^2_\tau l^2_k}\|g\|_{L^2_\tau l^2_k}.$$

In order to prove (2.3) it suffices to show that $\sup_{\tau,k} I_{\tau,k} < \infty$. Integrating with respect to τ_1 and dividing the integral region into two regions that $k_1 = k$ and $k \neq k_1$. In $k_1 = k$, we have that the contribution of $k_1 = k$ to $I^2_{\tau,k}$ is bounded by

$$\frac{1}{(1+|\tau+\alpha k^2|)} < c.$$

We next consider the region of $k_1 \neq k$. Then by (2.5), (2.7) and β/α is not integer, we first note that

$$|\tau + \alpha k^2| \geq c(1+|k-k_1|)(1+|k+k_1 \pm \beta/\alpha|).$$

First in the regions of $|k_1| < |k|/2$ or $|k|/3 < |k_1|$ it follows that $|k-k_1|, |k+k_1 \pm \beta/\alpha| \geq c|k|$, then we have that the contribution of the region $k_1 \neq k$

to $I^2_{\tau,k}$ is bounded by

$$\begin{aligned}
&c\sum_{k_1}\frac{1}{(1+|k_1|)^{2s}(1+|k|)^{2l-2s+2}(1+|\tau+\alpha k_1^2\pm\beta(k-k_1)|)^{1-\epsilon}}\\
\leq& c\sum_{k_1}\frac{1}{(1+|\tau+\alpha k_1^2\pm\beta(k-k_1)|)^{1-\epsilon}}\\
\leq& c
\end{aligned}$$

for $\epsilon\in(0,1/2)$, $s\geq 0$, $l\geq -1/2$ and $s-l\leq 1$. Next in the region of $|k|/2\leq|k_1|\leq 3|k|/2$, we have that the contribution of the region $k_1\neq k$ to $I^2_{\tau,k}$ is bounded by

$$\begin{aligned}
&c\sum_{k_1}\frac{1}{(1+|k-k_1|)^{2l+1}(1+|\tau+\alpha k_1^2\pm\beta(k-k_1)|)^{1-\epsilon}}\\
\leq& c,
\end{aligned}$$

for $l\geq -1/2$.

Next we prove (2.4) restricted to the region of (2.8). In the same argument as in the proof of (2.6), it suffices to show that

$$\begin{aligned}
&\sup_{\tau,k} II_{\tau,k}\\
=&\sup_{\tau,k}\left(\frac{(1+|k|)^{2l}|k|^2}{(1+||\tau|-\beta|k||)}\sum_{k_1}\int\frac{1}{(1+|\tau_1+\alpha k_1^2|)(1+|k_1|)^{2s}(1+|k-k_1|)^{2s}}\right.\\
&\left.\times\frac{1}{(1+|\tau-\tau_1-\alpha(k-k_1)^2|)}d\tau_1\right)^{1/2}\\
<&\infty.
\end{aligned}$$

We can assume $k\neq 0$. First in the regions that $|k_1|\leq 2|k|/5$ or $2|k|/3\leq|k_1|$, integrating with respect to τ_1 and using (2.6),(2.8) we have that the contribution of the region $k \neq 0$ to $II^2_{\tau,k}$ is bounded by

$$
\begin{aligned}
&c \sum_{|k_1| \leq 2|k|/5 \text{ or } 2|k|/3 \leq |k_1|} \frac{(1+|k|)^{2l+1}}{(1+|k_1|)^{2s}(1+|k-k_1|)^{2s}(1+|k-2k_1 \pm \beta/\alpha|)} \\
&\times \frac{1}{(1+|\tau-\alpha k^2+2\alpha k k_1|)^{1-\epsilon}} \\
\leq & c + c \sum_{|k_1| \leq 2|k|/5 \text{ or } 2|k|/3 \leq |k_1|} \frac{1}{(1+|k_1|)^{2s}(1+|k|)^{2s-2l+1-\epsilon}} \\
&\times \frac{1}{(1+|k_1-\gamma(\tau,k)|)^{1-\epsilon}},
\end{aligned}
$$

for $\epsilon \in (0, 1/2)$, $s \geq 0$ and $s - l \geq 0$ where $\gamma = \gamma(\tau, k)$ is the solution of the following linear equation with respect to k_1,

$$
\tau - \alpha k^2 + 2\alpha k k_1 = 0, \text{ i.e., } \gamma = \frac{k}{2} - \frac{\tau}{2\alpha k}.
$$

Then, we divide the sum with respect to k_1 into the following three regions:

$$
|k_1| \leq 2|k|/5, 2|k|/3 \leq |k_1| < 3|k|/2 \text{ and } 3|k|/2 \leq |k_1|.
$$

In the regions that $|k_1| \leq 2|k|/5$ or $2|k|/3 \leq |k_1| < 3|k|/2$, $II^2_{\tau,k}$ restricted to that regions is bounded by

$$
\begin{aligned}
&c \sum_{k_1} \frac{1}{(1+|k_1|)^{4s-2l+1-\epsilon}(1+|k_1-\gamma(\tau,k)|)^{1-\epsilon}} \\
\leq & c,
\end{aligned}
$$

for $s \geq 0, s - l \geq -1/2 + \epsilon/2$ and $2s - l > -1/2 + \epsilon$. In the same way as above, in the region of $3|k|/2 \leq |k_1|$, $II^2_{\tau,k}$ restricted to the above region is bounded by

$$
\begin{aligned}
&c \sum_{k_1} \frac{1}{(1+|k_1|)^{4s-2l-\epsilon}(1+|k-2k_1 \pm \beta/\alpha|)(1+|k_1-\gamma(\tau,k)|)^{1-\epsilon}} \\
\leq & c,
\end{aligned}
$$

for $s \geq 0$, $2s - l > -1/2 + \epsilon$ and $2s - l \geq -1/2 + \epsilon/2$. Next, in the region of $2|k|/5 < |k_1| < 2|k|/3$, we have that $|k|/3 < |k - k_1| < 5|k|/3$. Then we have that $II^2_{\tau,k}$ restricted to the above integral region is bounded by

$$
\begin{aligned}
&c \sum_{2|k|/5<|k_1|\leq 2|k|/3} \frac{1}{(1+|k_1|)^{4s-2l-1}(1+|k-2k_1 \pm \beta/\alpha|)} \\
&\qquad \times \frac{1}{(1+(1+|k|)|k_1 - \gamma(\tau,k)|)^{1-\epsilon}} \\
\leq &c,
\end{aligned}
$$

for $\epsilon \in (0, 1/2)$ and $2s - l \geq 1/2$. □

Remark 2.1. When $0 \leq s - l \leq 1$ and $1/2 \leq l + 1/2 \leq 2s$, Lemma 2.4 holds with for any $\beta/\alpha \in \mathbb{R}$.

The following proposition is an immediate consequence of Lemma 2.4.

Proposition 2.5. *Assume that β/α is not integer. For any $0 \leq s - l \leq 1$ and $0 \leq l + 1/2 \leq 2s$, we have*

$$\|un\|_{(1,s,-1/2)} \leq c\|u\|_{(1,s,1/2)}\|n\|_{(2,l,1/2)}, \tag{2.9}$$

$$\|\partial_x(|u|^2)\|_{(2,l,-1/2)} \leq c\|u\|^2_{(1,s,1/2)}. \tag{2.10}$$

Proof of Proposition 2.5. We define

$$f(\tau, k) = (1+|k|)^s(1+|\tau + \alpha k^2|)^{1/2}|\widehat{u}(\tau, k)|,$$

$$g(\tau, k) = (1+|k|)^l(1+||\tau| - \beta|k||)^{1/2}|\widehat{n}(\tau, k)|,$$

$$h(\tau,k)=(1+|k|)^s(1+|\tau-\alpha k^2|)^{1/2}|\widehat{u}(-\tau,-k)|.$$

From Lemma 2.4, the proof of the proposition follows. □

Remark 2.2. Proposition 2.5 holds when the right hand side of (2.9)-(2.10) are replaced by

$$\|u\|_{(1,s,b)}\|n\|_{(2,l,1/2)}+\|u\|_{(1,s,1/2)}\|n\|_{(2,l,b)},$$

$$\|u\|_{(1,s,b)}\|u\|_{(1,s,1/2)},$$

respectively, with $1/2-1/8<b<1/2$.

Remark 2.3. For the proof of the second terms of the right hand side of (2.1) and (2.2), we obtain the similar result to Proposition 2.5.

Proof of Theorem 1.1. For $T\in(0,1)$, we define

$$\Phi(u,n)=S(t)u_0-i\psi(t)\int_0^t S(t-\tau)(\psi_T u\psi_T n)(\tau)d\tau,$$

$$\Psi(u)=\partial_t V(t)n_0+V(t)n_1+\psi(t)\int_0^t V(t-\tau)\partial_x^2(|\psi_T u|^2)(\tau)d\tau.$$

From Lemma 2.1-2.3, Proposition 2.5 and Remark 2.2, we have that

$$\|\Phi(u,n)\|_{X_s}\le c\|u_0\|_{H^s}+cT^\mu\|u\|_{X_s}\|n\|_{Y_l},$$

$$\|\Psi(u)\|_{Y_l}\sim\|\partial_t\Psi(u)\|_{Y_{l-1}}\le c(\|n_0\|_{H^l}+\|n_1\|_{H^{l-1}})+cT^\mu\|u\|_{X_s}^2.$$

for some $\mu > 0$. Similar argument to above, we have that

$$\|\Phi(\tilde{u},\tilde{n}) - \Phi(u,n)\|_{X_s} \leq cT^{\mu}(\|u\|_{X_s} + \|n\|_{Y_l} + \|\tilde{u}\|_{X_s} + \|\tilde{n}\|_{Y_l})$$
$$\times (\|\tilde{u} - u\|_{X_s} + \|\tilde{n} - n\|_{Y_l}),$$

$$\|\Psi(\tilde{u}) - \Psi(u)\|_{Y_l} \sim \|\partial_t\Psi(\tilde{u}) - \partial_t\Psi(u)\|_{Y_{l-1}} \leq cT^{\mu}(\|u\|_{X_s} + \|\tilde{u}\|_{X_s})\|\tilde{u} - u\|_{X_s}.$$

Thus, we have that $\Phi \times \Psi \times \partial_t\Psi$ is a contraction map. Then, we obtain the unique local existence results in $X_s \times Y_l \times Y_{l-1}$ by the contraction argument. □

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