数理解析研究所講究録
1108 巻 1999 年 169-197

Morally Consistent Equilibria in Normal Form Games:A Game Theoretic Approach to Moral Judgements and a Normative Justification of Nash Equilibrium[1]

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Abstract : Consider an n-person normal form game in which each player acts rationally, but subject to a constraint made by a moral judgement rule (MJ for short), which gives players the proper instructions about the set of actions that are allowed to take in their situations (i.e., the combination of a preferences profile and others' actions). The purpose of this paper is to clarify the properties and the existence conditions of equilibrium derived from each player's rational choice under the given MJ, called morally consistent equilibrium (MCE for short). We show that the set of MCEs contains the set of approval equilibria, which is a special class of (pure) Nash equilibria, and is contained in the set of (pure) Nash equilibria if MJ satisfies four axioms, i.e., anonymity, neutrality, monotonicity and effectiveness, each of which reflects ethical values of morals (Theorems 1 and 2). Moreover the set of Nash equilibria is equivalent to the set of MCEs if MJ satisfies strong neutrality and monotonicity (Theorem 3).

These results, in particular Theorem 2, have three implications. First, any morally right action of a player is incentive compatible in the sense that it is the best response strategy in all the actions available to the player if others take morally right actions. Usually most of economists and game theorists share an intuition that a deviation from morally right actions make one better off if others acts morally. However this intuition is indeed false as shown in Theorem 2. Second, morals is ineffective as a norm that conduct one to morally right actions in a society. This strongly holds for Kantian utilitarianism advocated in the literatures of moral philosophy. Third, this paper carries out a normative justification of Nash equilibrium. Nash equlibrium, although has been supported by prescriptive game theory, is justified by a normative aspect.

[1] This paper is presented in seminars of Hokkaido University, Kyoto University, Kobe University, and Kansai University in 1998. We thank the participants for their useful comments. In particular detailed and huge e-mails by Naoki Yoshihara and Manabu Toda, received after the seminars, were very useful for the revision of the earlier manuscript. Nagahisa is financially supported by Special Research fund of

1 Introduction

Morals is a system of rules about what is right or wrong, and what is good or bad to do in society. Although morals varies from country to country, culture to culture, era to era, and so forth, there has been no society in the past and the present that dispenses with morals. One of the reasons why any society retains morals might be that people think of them to give proper suggestions for good and right actions in their course of social lives.

We have no doubt about the necessity of morals, but can we immediately conclude that the resulting state of the world is good or right when everyone takes the right action in the light of morals? Apart from morals, we know the phenomenon of 'fallacy of composition' in rational choices. This shows that the rational actions of the people give rise to the irrational consequence for the whole society. The similar phenomenon to it may occur in the case of morals. The peoples' morally right actions might derive the morally wrong or improper outcome for the whole society. Even if the resulting state is morally right, it might be unacceptable from the viewpoint of their happiness or well-being. However it is obvious that which state results from morally right actions of people depends on what kind of morals prevails in the society. Thus preceding to study the consequence of morally right actions, we need to clarify the meanings of morals for the first time.

Morals is thought of as a principle of actions governing self-determination, the role of which is to indicate appropriate instructions about what people should do when they come across the questions what is good and how they behave in various situations.[2] This is a tentative definition of morals we give here. With this definition of morals we cope with the problem of rational choices constrained by morals in a game theoretic model. This paper aims at developing the above approach to morals.

Kansai University in 1997.

[2] Here we have to notice the following two facts: First, though morals restrict the range of actions which they can select, they generally have the remaining room for rational actions according to their own preferences. Morals do not necessarily limit people's free will completely. Second, one particular society has one system of morals which is accepted by all the members, so that people never have different morals in principle. In that sense, morals have same property as laws.

Consider an n-person normal form game in which each player acts rationally, but subject to a constraint made by some moral judgement rule (MJ for short). Here MJ is the system which gives players some proper suggestions about the set of actions that are allowed to take in their situations (described by the combination of the preferences profile and the others' actions). Formally it is defined by a nonempty correspondence that associates with each player standing at a situation the set of actions which are morally allowed to take. It is an formal illustration of morals, and has a role of constraint on self-determination.

The purpose of this paper, hence, is to clarify the properties and the existence conditions of equilibrium derived from each player's rational choice under the given MJ, called morally consistent equilibrium (MCE for short). An actions profile (a combination of strategies) is a MCE if and only if the action taken by each player is permitted under his/her situation according to the MJ, and is optimal in the set of such actions with respect to his/her preference. In other words, MCE is a Social equilibrium (Debreu (1952)) in the set of actions restricted by MJ. For example, take a 2-person 2-strategy game, where each player has strategies (actions) X and Y. Player 1's preference is given in descending order by: XX, YX, XY, YY; and 2's by: YX, XX, XY, YY. Here, for example, YX represents the state where player 1 chooses Y and 2 chooses X. Suppose that MJ permits player 1 to take X and Y (respectively Y only) if player 2 takes X (respectively Y). On the other hand, suppose that the moral judgement rule permits player 2 to take X (respectively X and Y) if player 1 takes Y (respectively X). Then only XX is the unique MCE under this MJ.

Which actions profile is a MCE depends on which MJ applies. In other words, we do not know what is morally right (or wrong) until we verify the contents of MJ. We adopt an axiomatic approach to MJ in this paper. We take four normative axioms which MJ should satisfy, and examine the properties of MCE under the MJ satisfying them simultaneously.

The first axiom is anonymity, which demonstrates that the mutual exchange of positions of the players has the same implication as before. That is, if the preferences profile changes correspondingly to the permutation of positions among players, the actions derived from those allowed before must also be permitted now. In other words, there is no-importance-in renaming the players.

The second axiom is neutrality. It demonstrates that if the new preferences profile is the same as the old one on the set of actions profiles after permuting the actions, the permuted action under the new profile is always judged right whenever the original action under the old profile is allowed to take. Note that neutrality is stronger than independence which is interpreted as minimum informational requirement, that is, any MJ only needs the preference orderings on that set of actions.

The third axiom is monotonicity. Take some action. If each player's preference changes to enhance the rank of that action, it must also be judged morally right whenever it is accepted before according to the MJ.

The fourth axiom is effectiveness which says that at least one action must be judged morally right for any preferences profile given a set of other players' actions according to MJ. This is a necessary, but not sufficient, condition for existence of morally consistent equilibrium under any preferences profile.

This paper shows that if MJ satisfies the above four axioms, the set of MCE coincides with that of Nash equilibria under any preferences profile. This result is interpreted in the following two ways.

First, it suggests that morals are not an effective norm as to persuade individuals to take socially desirable actions. Our stand point on moral philosophy may be called a weak version of Kantian utilitarianism, a strong version of which is defended by Hare (1981) in order to advocate his two level theory of moral judgement. Ours is weaker since the interpersonal comparisons of utility are permitted in Hare's but not in ours.

Second, our analysis is interpreted to accomplish a normative axiomatization of Nash equilibrium.

The notion of Nash equilibrium has been sustained from prescriptive point of view, which explains the justification of Nash equilibrium by the rationality aspect-in deciding the actions. On the other hand, as the result shows, Nash equilibrium-is also justified from the normative and ethical point of view with respect to moral judgement on actions.

The organization of the rest of this paper is as follows. We present the model in the next section. The purpose of this section-is to define MJ and the axioms, and to propose the strict notion of MCE. Section 3 contains a theorem and its proof. The meaning of the theorem and the direction of extensions of our analysis are also discussed with respect to game theory and moral philosophy. Section 4 is the conclusion.

2 Definitions and Notation

Consider a normal form game $G=(N, \Pi_{i\in N}X_i)$, where N is the finite set of players which consists of at least two, and X_i is player i's strategy set which consists of finite number of elements with at least two. Each element in X_i is called player i's strategy, and is denoted by x_i. For convenience sake, it is assumed that all the players have the same strategy set denoted by $X=X_i(i=1,...,n)$. x_i is interpreted as the action taken by player i, and also called player i's action. An actions profile is n-tuple of actions $x=(x_1,x_2,...,x_n)$. As a matter of convenience x is regarded as a function from N to X, and x_i is often denoted by x(i). The set of actions profiles is indicated by X^N. Then $X^N=\Pi_{i\in N}X_i$. In this paper each social state is assumed to consist of n-tuple of actions, one for each individual player. Hence each action profile is looked upon as a social state. In the following, X^N is called the set of social states if necessary.

As usual each player i is supposed to have a complete and transitive preference $\succsim_i$ on the set of social states X^N. $\succsim=(\succsim_1, \succsim_2,..., \succsim_n)$ is called a preferences profile. Let P be the set of complete and transitive preferences on X^N. As a matter of convenience we regard $\succsim$ as a function from N to P, and denote $\succsim_i$ by $\succsim(i)$. We assume that all logically possible set of preferences profiles is P^N.

Take a player i arbitrary. Given (n-1)-tuple of actions of other players $x_{-i}=(x_1,x_2,..,x_{i-1},x_{i+1},...,x_n)$ and a preferences profile $\succsim$, the combination $(x_{-i}, \succsim)$ is interpreted as a situation in which player i is put, and called i's situation simply. A moral judgement rule, MJ for short, is a mapping which associates with each player i the set of actions that i is allowed to take when he is put in the situation $(x_{-i}, \succsim)$. Formally, MJ is a nonempty-valued correspondence from $N\times X^{N-1}\times P^N$ to X. Given a MJ, for an i's situation $(x_{-i}, \succsim)$,if $x_i\in MJ(i, x_{-i}, \succsim)$ holds, then x_i is called to be morally consistent for i in the situation $(x_{-i}, \succsim)$;if not, morally inconsistent. MJ not only judges actions from the viewpoint of morals, but also enforces players not to take morally inconsistent actions.

Given a MJR, an actions profile x is a morally consistent equilibrium under a preferences profile $\succsim$ if [$x_i \in MJ(i, x_{-i}, \succsim)$ and $x \succsim(i)(y_i, x_{-i})$ for any $y_i \in MJ(i, x_{-i}, \succsim)$] are true for any player $i \in N$. Denote the set of morally consistent equilibria (MCE for short) under a preferences profile $\succsim$ by $MC(\succsim)$, and the set of (pure) Nash equilibria under $\succsim$ by $NA(\succsim)$. That is, $x \in NA(\succsim) \leftrightarrow x \succsim(i)(y_i, x_{-i})$ for any $i \in N$ and $y_i \in X$.

Any MCE x under a preferences profile $\succsim$ is a social equilibrium (Debreu (1952)) where each player i's strategy set is restricted to $MJ(i, x_{-i}, \succsim)$. In other words, whenever player has an incentive to deviate from x, his deviation is self-restrained by the judgement that the action is morally inconsistent according to MJ.

Two remarks are in order. First, from the above definition, MCE is derived from MJ. MJ indicates the possible actions to choose for each player under a given situation, so that the final decision among them depends on each player's free will. Hence MJ does not necessarily deprive freedom to choose of the players. Morals in this sense are not strong command to do something, but weak command not to do something.

Second, there is a problem on expectations of players in a normal form game. Each player selects his strategy simultaneously, so that they must make a consistent belief on others' strategy choice in the normal form game. In the same way, each player must make a consistent belief on others' strategy choice in MCE. Otherwise, MCE may not be attained through simultaneous selection of actions by players. We have two possible answers to this question. One is to presuppose that all the players have common knowledge about the obedience to MJ among the players. This presupposition is quite natural since all the members of the society have common interest in morals. The other answer is that MCE is a reference point to judge the resulting social state to be right or wrong, and is not necessarily a guide to play a real game. In this case we need no specification of players' beliefs on others' strategies.

Now we formulate axioms on MJ. Let π_i be a permutation on N. For any preferences profile $\succsim$ we define a new preferences profile $\succsim^{\pi}$ by $\succsim$ in the following; For any player $i \in N$ and any alternative x, $y \in X^N$, $x \succsim(i) y \leftrightarrow x o \pi \succsim^{\pi}(\pi(i)) y o \pi$.

Anonymity (AxiomA)

For any $\succsim \in P^N$, $i \in N$, and $x \in X^N$, $x(i) \in MJ(i, \succsim, x_{-i}) \leftrightarrow x(\pi(i)) \in MJ(\pi(i), \succsim^{\pi}, (x o \pi)_{-\pi(i)})$.

Let x_{-i} be given. For any player i, let $X^N(x_{-i}) = \{y \in X^N: y = (y_i, x_{-i}), y_i \in X\}$. Let $\rho_1, \ldots, \rho_n$ be permutations on X. For any $x = (x_1, \ldots, x_n) \in X^N$, let us denote $\rho o x = (\rho_1 o x_1, \ldots, \rho_n o x_n)$, and $(\rho o x)_{-i} = (\rho_1 o x_1, \ldots, \rho_{i-1} o x_{i-1}, \rho_{i+1} o x_{i+1}, \ldots, \rho_n o x_n)$. Let Y be a nonempty subset of $X^N \times X^N$. We say that two preferences profiles $\succsim$ and $\succsim'$ are homothetic on Y with respect to ρ if $x \succsim(j) y \leftrightarrow \rho o x \succsim'(j) \rho o y$ is true for any $(x, y) \in Y$ and $j \in N$. Let $j \in N$ be given. We say that two preferences profiles $\succsim$ and $\succsim'$ are j-homothetic on Y with respect to ρ if $x \succsim(j) y \leftrightarrow \rho o x \succsim'(j) \rho o y$ for any $(x, y) \in Y$ and $j \in N$.

Neutrality (Axiom N)

For any $\succsim, \succsim' \in P^N$, $i \in N$, and $x \in X^N$, $\succsim$ and $\succsim'$ are homothetic on $\{x\} \times X^N(x_{-i}) \cup X^N(x_{-i}) \times \{x\}$ with respect to ρ, $x_i \in MJ(i, \succsim, x_{-i}) \leftrightarrow \rho_i o x_i \in MJ(i, \succsim', (\rho o x)_{-i})$.

We say that $\rho = (\rho_1, \ldots, \rho_n)$ is an identity if $\rho_1, \ldots, \rho_n$ are identities.

Independence (Axiom I)

For any $\succsim, \succsim' \in P^N$, $i \in N$, and $x \in X^N$, if $\succsim$ and $\succsim'$ are homothetic on $\{x\} \times X^N(x_{-i}) \cup X^N(x_{-i}) \times \{x\}$ with respect to an identity ρ, then $x_i \in MJ(i, \succsim, x_{-i}) \leftrightarrow x_i \in MJ(i, \succsim', x_{-i})$.

Obviously Axiom I is weaker than Axiom N.

Monotonicity (Axiom M)

For any $\succsim$, $\succsim' \in P^N$, $x \in X^N$, and $j \in N$, if $x \succsim(j) y \rightarrow x \succsim'(j) y$ & $x \succ(j) y \rightarrow x \succ'(j) y$ for any $y \in X^N - \{x\}$, then $x_i \in MJ(i, \succsim, x_{-i}) \rightarrow x_i \in MJ(i, \succsim', x_{-i})$ for any $i \in N$

If MJ satisfies Axiom, then Axiom M is equivalent to the following.

Axiom M*

For any $\succsim$, $\succsim' \in P^N$, $x \in X^N$, and $j \in N$, if $x \succsim(j) y \rightarrow x \succsim'(j) y$ & $x \succ(j) y \rightarrow x \succ'(j) y$ for any $y \in X^N(x_{-i}) - \{x\}$, then $x_i \in MJ(i, \succsim, x_{-i}) \rightarrow x_i \in MJ(i, \succsim', x_{-i})$ for any $i \in N$

Form now on, we will use Axiom M in the form of this stronger version. It is necessary for axiomatizing the set of Nash equilibria to strengthen Axiom N in the following.

Strong Neutrality (Axiom SN)

For any $\succsim$, $\succsim' \in P^N$, $i \in N$, and $x \in X^N$, $\succsim$ and $\succsim'$ are i-homothetic on $\{x\} \times X^N(x_{-i}) \cup X^N(x_{-i}) \times \{x\}$ with respect to ρ, then $x_i \in MJ(i, \succsim, x_{-i}) \leftrightarrow \rho_i o x_i \in MJ(i, \succsim', (\rho o x)_{-i})$.

If MJ satisfies Axiom SN, then. any player does not necessarily consider others' evaluation about his action at a given situation. Axiom SN can be interpreted as a requirement of liberalistic morals[3]. If MJ satisfies Axiom SN, Axiom M can strengthen to the following.

Axiom M**

For any $\succsim$, $\succsim' \in P^N$, $x \in X^N$, and $i \in N$, if $x \succsim(i) y \rightarrow x \succsim'(i) y$ & $x \succ(i) y \rightarrow x \succ'(i) y$ for any $y \in X^N(x_{-i})$, then

[3] To our judgement, Mill(1863) is the first person who justified liberalistic morals. He argued that no

$x_i \in MJ(i, \succsim, x_{-i}) \rightarrow x_i \in MJ(i, \succsim', x_{-i})$.

Effectiveness (Axiom E)

For any $\succsim \in P^N$, there is some $x \in X^N$ with $x \in \Pi_{i \in N} MJ(i, \succsim, x_{-i})$.

If $x \in MC(\succsim)$ then $x \in \Pi_{i \in N} MJ(i, \succsim, x_{-i})$ by definition of MCE. Hence Axiom E is weaker than the existence condition of equilibria: MCE must exist for any preferences profile.

The examples below illustrate the independence of four axioms A, N, M, and E.

Example 1 (Independence of Axiom A)

For any $\succsim \in P^N$, $i \in N$, and $x\text{-}i \in X^{N-1}$, $MJ(i,\succsim,x_{-i}) := \{\beta \in X: (\beta,x_{-i}) \sim (1) (\alpha,x_{-i})\}$ if $\exists \alpha \in X$ s.t. $(\alpha,x_{-i}) \succsim (1) y$ for any $y \in X^N$, otherwise $:= X$. This MJ satisfies all axioms except Axiom A.

Example 2 (Independence of Axiom N)

Take $\alpha \in X$ arbitrary, and fix it. For any $\succsim \in P^N$, $i \in N$, and $x_{-i} \in X^{N-1}$, $MJ(i,\succsim,x_{-i}) := \{\alpha\}$ if $x_{-i} = (\alpha,\ldots,\alpha)$, $:= X$ otherwise. This MJ satisfies all the axioms except Axiom N.

Example 3 (Independence of Axiom M)

Let the set Q of preferences profiles be such that $\succsim \in Q \leftrightarrow \exists x \in X^N$, denoted by $x(\succsim)$, with $x(\succsim) \prec (i) y$ for any $i \in N$ and $y \in X - \{x(\succsim)\}$. For any $\succsim \in P^N$, $i \in N$, and $x_{-i} \in X^{N-1}$, $MJ(i,\succsim,x_{-i}) := \{x_i\}$ if $\succsim \in Q$ and $(x_i,x_{-i}) = x(\succsim)$, $:= X$ otherwise. This MJ satisfies all the axioms except Axiom M.

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one ought to be interfere with other's liberties as possible as one can.

Example 4 (Independence of Axiom E)

For any $\succsim \in P^N$, $i \in N$, and $x_{-i} \in X^{N-1}$, $MJ(i,\succsim,x_{-i}):=\{\beta\in X: (\beta,x_{-i})\sim(i)(\alpha,x_{-i})\}$ if $\exists\alpha\in X$ s.t. $(\alpha,x_{-i})\succ(j)y$ for any $y\in X^N-(\alpha,x_{-i})$ and $j\in N-\{i\}$, otherwise $:=B(i,\succsim,x_{-i})$, where $B(i,\succsim,x_{-i})=\{\beta\in X: (\beta,x_{-i})\succsim(i)(\alpha,x_{-i})$ for any $\alpha\in X\}$.

This MJ satisfies all the axioms except Axiom E. Let us show that this MJ violates Axiom E. Take α, $\beta\in X$ arbtrary. Take social states such that $x^1=(\alpha,\alpha,\alpha,\ldots,\alpha,\alpha)$, $x^2=(\beta,\alpha,\alpha,\ldots,\alpha,\alpha)$,..., $x^n=(\beta,\beta,\beta,\ldots,\beta,\alpha)$, $x^{n+1}=(\beta,\beta,\beta,\ldots,\beta,\beta)$, $x^{n+2}=(\alpha,\beta,\beta,\ldots,\beta,\beta)$,..., $x^{2n-1}=(\alpha,\alpha,\alpha,\ldots,\alpha,\beta,\beta)$, and $x^{2n}=(\alpha,\alpha,\alpha,\ldots,\alpha,\alpha,\beta)$.

Let $Y(1)$ be the set of social states each of which consists of one number of α and n-1 number of β. Similarly let $Y(2)$ be the set of social states each of which consists of 2 number of α and n-2 number of β. Repeating this procedure, we define $Y(3)$,..., and $Y(n-1)$. Next let $Z(1)$ be the set of social states each of which contains just one action except α and β. Similarly let $Z(2)$ be the set of social states each of which contains just two actions except α and β. Repeating this procedure, we define $Z(3),\ldots,Z(n-1)$. Let W be the set of social states remaining. By definition, each element in W does not contain α and β as its component. Let $\succsim$ be a preference profile such that:

$\succsim(1)$: $x^1,x^2,\ldots,x^{n+1},x^{n+2},\ldots, x^{2n-1},x^{2n}$, $[Y(1)],[Y(2)],\ldots,[Y(n-1)],[Z(1)],[Z(2)],\ldots,[Z(n-1)],(W)$

$\succsim(2)$: $x^2,x^3,\ldots,x^{n+2},x^{n+3},\ldots, x^{2n},x^1$, ...The rest are the same as in $\succsim(1)$...

....

$\succsim(n-1)$: $x^{n-1},x^n,\ldots,x^{2n-1},x^{2n},x^1,\ldots, x^{n-3},x^{n-2}$, ...The rest are the same as in $\succsim(1)$...

$\succsim(n)$: $x^n,x^{n+1},\ldots,x^{2n},x^1,x^2,\ldots, x^{n-2},x^{n-1}$, ...The rest are the same as in $\succsim(1)$...

where any social states in the bracket [ ] are indifferent, and we do not need to specify the ranking of social states in the bracket ( ). For example $[Y(1)]$ means that all social states in $Y(1)$ are indifferent to each other.

By definition of MJ, $MJ(i,\succsim,x_{-i}):=B(i,\succsim,x_{-i})$ for any $i\in N$ and $x_{-i}\in X^{N-1}$. Thus if MJ satisfies

Axiom E then there must exist Nash equilibrium under the profile. By definition of $\succsim$, the Nash equlibria belongs to W. If X consists of three actions then W is singleton. This is a contradiction. Thus X contains at least four actions. Take χ, $\delta \in X-\{\alpha,\beta\}$. The same procedure as in the above specifies $\succsim$ furthermore, and we can show that the Nash equilibria consist only of actions except α, β,χ, and δ. By repeating this procedures, we can finally specify $\succsim$ in which has no Nash equilibrium. This is the example illustrates the independence of Axiom E.

The examples below show the existence of MJ with the four axioms.

Example 5 For any $\succsim \in P^N$, $i \in N$, and $x_{-i} \in X^{N-1}$, $MJ(i,\succsim,x_{-i}):=X$

Example 6 For any $\succsim \in P^N$, $i \in N$, $x_{-i} \in X^{N-1}$, $MJ(i,\succsim,x_{-i}):=\{y \in X: (y,x_{-i}) \in PO(\succsim,x_{-i})\}$, where $PO(\succsim,x_{-i})$is the set of strong Pareto optimal social states on $X(x_{-i})$ under $\succsim$.

3 Main results and proofs

An action profile x is an approval equilibrium under $\succsim$ if $x \succsim(j)(y_i, x_{-i})$ for any $y_i \in X$, and i, $j \in N$. Let $AP(\succsim)$ be the set of approval equlibria for $\succsim$. Obviously $AP(\succsim)=\cap_{\pi \in \Pi} NA(\succsim o \pi)$.

Theorem 1 Suppose that MJ satisfies Axioms N and M. Then we have

$$AP(\succsim) \subset MC(\succsim) \text{ for any } \succsim \in P^N$$

Proof: Let $\succsim^* \in P^N$ be such that any player is indifferent between all social states. Since MJ is non-empty valued, we have for any $i \in N$,

(1) $x_i \in MJ(i, \succsim^*, x_{-i})$ for some $x_i \in X$;

Applying Axiom N to (1), we have

(2) $x \in \Pi_{i \in N} MJ(i, \succsim^*, x_{-i})$ for any $x \in X^N$;

Take $\succsim \in P^N$ and $y \in AP(\succsim)$ arbitrary. Comparing $\succsim$ and $\succsim^*$, and applying Axiom M to (2), we have

(3) $y \in \Pi_{i \in N} MJ(i, \succsim, y_{-i})$.

This together with $y \in AP(\succsim)$ implies $y \in MC(\succsim)$which is the desired result. □

Any deviation of a player from an approval equilibrium is not supported by anyone as well as the player himself. Is seems to be very natural that any approval equilibrium is a morally consistent equilibrium since moral judgements we defined are done by considering the interest in all individuals. However it is quite obvious that approval equilibria do not necessarily exist.

It is difficult to prove that any MCE is a Nash equilibrium, which is the most important result in the paper. The proof proceeds with a mathematical induction on the number of players and actions. Lemmas 1 and 2 complete the proof of the case with two players and two actions. This proof is relatively simple, thus it is sufficient for the readers who has few time to read untill the two lemmas.

Lemma 1 Assume that there are two players with two actions. Suppose that MJ satisfies Axiom A, N, and E. Then we have

$[(x_i, x_{-i}) \succ (i)(y_i, x_{-i}) \& (x_i, x_{-i}) \prec (j)(y_i, x_{-i}) \to \{x_i, y_i\} = MJ(i, \succsim, x_{-i})]$ for any $i, j \in N$, x_i, y_i, and $x_{-i} \in X$.

Proof: Let $N=\{1, 2\}$ and $X=\{\alpha, \beta\}$. We will show

(1) $(\alpha,\alpha) \succ (1)(\beta,\alpha) \& (\alpha,\alpha) \prec (2)(\beta,\alpha) \to \{\alpha,\beta\} = MJ(1, \succsim, \alpha)$.

By Axioms A and N, this completes the proof of Lemma 2. Suppose that $\{\alpha\}=MJ(1, \succsim, \alpha)$.

Let $\succsim^1 \in P^N$ be such that

(2) $(\beta,\beta) \succ^1(1)(\alpha,\beta) \& (\beta,\beta) \prec^1(2)(\alpha,\beta)$

Comparing $\succsim$ and $\succsim^1$, and applying Axiom N, we have

(3) $\{\beta\}=MJ(1, \succsim^1, \beta)$

Let $\succsim^2 \in P^N$ be such that

(4) $(\beta,\alpha) \succ^2(1)(\alpha,\alpha) \& (\beta,\alpha) \prec^2(2)(\alpha,\alpha)$.

Comparing $\succsim$ and $\succsim^2$, and applying Axiom N, we have

(5) $\{\beta\}=MJ(1, \succsim^2, \alpha)$.

Let $\succsim^3 \in P^N$ be such that

(4) $(\alpha,\beta) \succ^3(1)(\beta,\beta) \& (\alpha,\beta) \prec^3(2)(\beta,\beta)$.

Comparing $\succsim^1$ and $\succsim^3$, and applying Axiom N, we have

(5) $\{\alpha\}=MJ(1, \succsim^3, \beta)$.

Let a permutation π on N be such that $\pi(1)=2$ and $\pi(2)=1$. With this π, let us define profiles $\succsim^{2\pi}$ and $\succsim^{3\pi}$ corresponding to $\succsim^2$ and $\succsim^3$ respectively. By Axiom A we have

(6) $(\alpha, \beta) \succ^{2\pi}(2)(\alpha,\alpha) \& (\alpha, \beta) \prec^{2\pi}(1)(\alpha,\alpha) \& \{\beta\}=MJ(2, \succsim^{2\pi}, \alpha)$.

(7) $(\beta, \alpha) \succ^{3\pi}(2)(\beta,\beta) \& (\beta, \alpha) \prec^{3\pi}(1)(\beta,\beta) \& \{\alpha\}=MJ(2, \succsim^{3\pi}, \beta)$

Let $\succsim^* \in P^N$ be such that

$\succsim^*(1)$: (α,α), (β,β), (β,α), (α,β)

$\succsim^*(2)$: (β,α), (α,β), (α,α), (β,β).

Applying Axiom N, we have $\{\beta\}=MJ(1, \succsim^*, \beta)$ (by the comparison of $\succsim^1$ and $\succsim^*$)、$\{\beta\}=MJ(2, \succsim^*, \alpha)$ (by the comparison of $\succsim^{2\pi}$ and $\succsim^*$)、$\{\alpha\}=MJ(2, \succsim^*, \beta)$ (by the comparison of $\succsim^{3\pi}$ and $\succsim^*$)、及び$\{\alpha\}=MJ(1, \succsim^*, \alpha)$ (by the comparison of $\succsim$ and $\succsim^*$) . However theses contradict Axiom E. If $\{\beta\}=MJ(1, \succsim, \alpha)$, then the same procedure leads us to a contradiction. Hence we must recognize $\{\alpha、\beta\}=MJ(1, \succsim, \alpha)$. This completes the proof of (1). □

Lemma 2 Assume that there are two players with two actions. Suppose that MJ satisfies Axiom A, N, M, and E. Then we have

$MC(\succsim) \subset NA(\succsim)$ for any $\succsim \in P^N$

Proof: Let $N=\{1, 2\}$ and $X=\{\alpha, \beta\}$. Take $(\alpha,\alpha) \in MC(\succsim)$. Suppose $(\alpha,\alpha) \prec (1)(\beta,\alpha)$. By Lemma1 and Axioms N and M, we have $\beta \in MJ(1,\succsim,\alpha)$ no matter what preference player 2 have. However this contradicts $(\alpha,\alpha) \in MC(\succsim)$. Thus we have $(\alpha,\alpha) \succsim(1)(\beta,\alpha)$. Similarly we have $(\alpha,\alpha) \succsim(2)(\alpha, \beta)$. These complete $(\alpha,\alpha) \in NA(\succsim)$ which is the desired result.□

Lemma 3 Assume that there are two actions. Suppose that MJ satisfies Axioms A, N,M, and E. Then we have

$MC(\succsim) \subset NA(\succsim)$ for any $\succsim \in P^N$.

Proof: We use an induction argument on the number of players. Lemma 2 shows that it is true for the case of n=2. Assuming that Lemma 3 is true for n−1, we consider the case of n. Let $X=\{\alpha、\beta\}$. Suppose that

(1) $x \in MC(\succsim)$.

Without loss of generality, let $x=(\beta,\beta,...,\beta)$. By letting $y=(\alpha,\beta,...,\beta)$, and by noting Axioms A and N, it is sufficient to show

(2) $x \succsim(1) y$.

Let $X^*=\{y \in X^N: y_n=\beta\}$. Consider a game $G^{-n}=(N-\{n\},X^{N-1})$. For $\succsim^{N-1} \in P^{N-1}$, we define $\succsim^N \in P^N$ such that

(3) $x \succsim^{N-1}(i) y \leftrightarrow (x,\alpha) \succsim^N(i)(y,\alpha)$ for any $i \in N-\{n\}$, $x, y \in X^{N-1}$;

(4) $(x.\alpha) \sim^N(i)(x,\beta)$ for any $i \in N$, $x \in X^N$; and

(5) $\succsim^N(n) \cap X^* \times X^* = \succsim(n) \cap X^* \times X^*$.

$\succsim^N$ is uniquely determined. A moral judgement rule MJ^{N-1} of game G^{-n} is given by

(6) For any $i \in N-\{n\}$, $\succsim^{N-1} \in P^{N-1}$, and $x_{-\{i,n\}} \in X^{N-2}$,

$MJ^{N-1}(i, \succsim^{N-1}, x_{-\{i,n\}}) := MJ(i, \succsim^N, x_{-i})$, where $x_{-i}=(x_{-\{i,n\}}, \beta)$.

It is obvious that MJ^{N-1} satisfies Axioms A, N, and M.

Let us show that MJ^{N-1} satisfies Axiom E. Take $\succsim^{N-1} \in P^{N-1}$ be arbitrary. Since MJ satisfies Axiom E, there is some $y \in X^N$ such that $y \in \Pi_{i \in N} MJ(i, \succsim^N, y_{-i})$. Letting $z=(y_{-n},\beta)$, (4) and Axiom N imply $z \in \Pi_{i \in N} MJ(i, \succsim^N, y_{-i})$. This with (6) implies $z_{-n} \in \Pi_{i \in N-\{n\}} MJ^{N-1}(i, \succsim^{N-1}, y_{-\{i,n\}})$, which is the desired result. Let $\succsim^{N-1} \in P^{N-1}$ be such that

(7) For any $i, j \in N-\{n\}$, and $z, w \in X(x_{-j})$, where $x=(\beta,\beta,..,\beta)$,

$z_{-n} \succsim^{N-1}(i) w_{-n} \leftrightarrow z \succsim(i) w$.

By definition, $\succsim^N \in P^N$ is given by

(8) $\succsim^N(i) \cap X(x_{-j}) \times X(x_{-j}) = \succsim(i) \cap X(x_{-j}) \times X(x_{-j})$ for any $i \in N$, $j \in N-\{n\}$, where $x=(\beta,\beta,...,\beta)$.

By Axiom N, (1), and (8), we have $x \in \Pi_{i \in N} MJ(i, \succsim^N, x_{-i})$. Thus by (6),

(9) $x_{-n} \in \Pi_{i \in N-\{n\}} MJ^{N-1}(i, \succsim^{N-1}, x_{-\{i,n\}})$.

Now we show

(10) For any $i \in N-\{n\}$, $(\alpha, x_{-\{i,n\}}) \succ^{N-1}(i) x_{-n}$, we have $\alpha \notin MJ(1, \succsim^{N-1}, x_{-\{1,n\}})$, where α is i-component

We have $(\alpha, x_{-\{i,n\}}, \beta) \succ^{N}(i)(x_{-n}, \beta)$. By (1) and (8), we have $\alpha \notin MJ(i, \succsim^{N}, x_{-i})$. This with (6) means (10).

(9) and (10) mean $x_{-n} \in MC(\succsim^{N-1})$. By the hypothesis of the mathematical induction, x_{-n} is a Nash equilibrium of game G^{-n}. Thus we have

(11) $x_{-n} \succsim^{N-1}(1)(\alpha, x_{-\{1,n\}})$.

By (3) and (11), we have $(x_{-n}, \alpha) \succsim^{N}(1)(\alpha, x_{-\{1,n\}}, \alpha)$. This with (4) means $(x_{-n}, \beta) \succsim^{N}(1)(\alpha, x_{-\{1,n\}}, \beta)$, i.e., $x \succsim^{N}(1) y$. Moreover (8) means $x \succsim(1) y$, which is the desired result.□

Lemma 4 Suppose that MJ satisfies Axioms N and M. Then we have the following.

Let $\succsim \in P^N$, $i \in N$, $x_i, y_i \in X$, and $x_{-i} \in X^{N-1}$ be given.

If (i) $(x_i, x_{-i}) \succ (j)(y_i, x_{-i})$ for any $j \in N$, and (ii) $y_i \in MJ(i, \succsim, x_{-i})$, then we have $x_i \in MJ(i, \succsim, x_{-i})$.

Proof: Let $\succsim \in P^N$ be such that

(1) $(x_i, x_{-i}) \succ (j)(y_i, x_{-i})$ for any $j \in N$ and;

(2) $y_i \in MJ(i, \succsim, x_{-i})$.

Let $\succsim^* \in P^N$ be such that

(3) $(x_i, x_{-i}) \sim^*(j)(y_i, x_{-i})$ for any $j \in N$; and

(4) $(z_i, x_{-i}) \succsim^*(j)(y_i, x_{-i}) \leftrightarrow (z_i, x_{-i}) \succsim (j)(y_i, x_{-i})$ for any $z_i \in X-\{x_i\}$ and $j \in N$.

It follows from (1), (2), (3), (4), and Axiom M that

(5) $y_i \in MJ(i, \succsim^*, x_{-i})$.

Let permutations $\rho_1, \ldots, \rho_n$ on X be such that

(6) $\rho_1, \ldots, \rho_{i-1}, \rho_{i+1}, \ldots, \rho_n$ are identities, and $\rho_i(x_i) = y_i$, $\rho_i(y_i) = x_i$, $\rho(z_i) = z_i$ for any $z_i \neq x_i, y_i$.

Let $\succsim' \in P^N$ be such that

(7) For any $x, y \in X^N$, $j \in N$, $x \succsim^*(j) y \leftrightarrow \rho o x \succsim'(j) \rho o y$.

By (5), (7), and Axiom N, we have

(8) $x_i \in MJ(i, \succsim', x_{-i})$.

Since (3) menas $\succsim' = \succsim^*$, we have

(9) $x_i \in MJ(i, \succsim^*, x_{-i})$.

Comparing $\succsim$ and $\succsim^*$, applying (9) to Axiom N, we have $x_i \in MJ(i, \succsim, x_{-i})$. □

Theorem 2 Suppose that MJ satisfies Axioms A, N, M, and E. Then we have

$$MC(\succsim) \subset NA(\succsim) \text{ for any } \succsim \in P^N$$

Proof: We use an induction argument on the number of actions. Lemma 3 shows that this is true for the case with two actions. Assuming that Theorem 2 is true for the case with n-1 actions, we consider the case with n actions. Suppose

(1) $x \in MC(\succsim)$.

Take $\alpha \in X$ arbitrary, and fix it hereafter. Let permutations $\rho_1, \ldots, \rho_n$ on X be given by

$\rho_i(x_i) \neq \alpha$ if $x_i = \alpha$、 $\rho_i(x_i) = x_i$ otherwise, for any $i \in N$、 where x_i is i-component of x in (1).

Let $\succsim^\rho \in P^N$ be given by

(2) $x \succsim(i) y \leftrightarrow \rho o x \succsim^\rho(i) \rho o y$ for any $i \in N$, and $x, y \in X^N$.

Applying Axiom N to (1), we have

(3) $\rho o x \in MC(\succsim^\rho)$.

Now partition X^N into n+1nonempty subsetsY^0, Y^1,...,Y^n as follows. : $Y^k(0 \leq k \leq n)$ is the set of all actions profiles $y \in X^N$ for which k components satisfy $y_i = \alpha$. A subset P* of preferences profiles is defined by

(4) $\succsim^* \in P^* \leftrightarrow$ (i) $x \succ^*(j) y$ for any $x \in Y^{k-1}$, $y \in Y^k$ $(0 \leq k \leq m)$, and $j \in N$; (ii) $x \sim^*(j) x'$ for any x, $x' \in Y^k$ $(1 \leq k \leq m)$, and $j \in N$.

Consider a game $G^0=(N,(Y^0)^N)$. We denote generic preferences profile in the game by $\succsim^0$, and the set of the profiles P^0 respectively. For any $\succsim^0 \in P^0$, we define $\succsim^{0*} \in P^N$ by

(5) $\succsim^{0*} \in P^*$; and

(6) $\succsim^{0*}(i) \cap Y^0 \cap Y^0 = \succsim^0(i)$ for any $i \in N$.

By (5) and (6), $\succsim^{0*}$ is uniquely determined. A moral judgement rule MJ^0 of the game is defined by

(7) For any $\succsim^0 \in P^0$, $i \in N$, $x_{-i} \in (Y^0)^{N-1}$, $MJ^0(i, \succsim^0, x_{-i}) := MJ(i, \succsim^{0*}, x_{-i}) - \{\alpha\}$.

By (5) and Lemma 4, MJ^0 is well defined. It is obvious that MJ^0 satisfies Axioms A, N, and M. Let us show that MJ^0 satisfies Axiom E. Take $\succsim^0 \in P^0$ arbitrary. Corresponding to the $\succsim^0$, there is a unique $\succsim^{0*} \in P^N$, and moreover there is some $x \in X^N$ with $x \in \Pi_{i \in N} MJ(i, \succsim^{0*}, x_{-i})$ since MJ satisfies Axiom E. If $x \in Y^k$ $(k \neq 0)$, then (5) and Lemma 4 imply that there is some $y^{k-1} \in Y^{k-1}$ with $y^{k-1} \in \Pi_{i \in N} MJ(i, \succsim^{0*}, y_{-i})$. Repeating the same procedure by k times, we can find some $z \in Y^0$ with $z \in \Pi_{i \in N} MJ(i, \succsim^{0*}, z_{-i})$. This shows that MJ^0 satisfies Axiom E.

Corresponding to $\succsim^P \in P^N$ of (2), let $\succsim^0 \in P^*$ be such that

(8) $\succsim^P(i) \cap Y^0 \times Y^0 = \succsim^0(i) \cap Y^0 \times Y^0$.

By (3), (7), and Axiom M, we have

(9) $\rho o x \in \Pi_{i\in N} MJ^0(i, \succsim^0, (\rho o x)_{-i})$.

On the other hand, (3), (7), and (8) implies

(10) $((\rho o x)_i, (\rho o x)_{-i}) \succsim^0(i)\ (y_i, (\rho o x)_{-i})$ for any $y_i \in MJ^0(i, \succsim^0, (\rho o x)_{-i})$ and $i\in N$.

By (9) and (10), we have

(11) $\rho o x \in MC(\succsim^0)$.

Therefore $\rho o x$ is a MCE of game G^0. By the hypothesis of the induction, we have

(12) $((\rho o x)_i, (\rho o x)_{-i}) \succsim^0(i)\ (y_i, (\rho o x)_{-i})$ for any $y_i \in X-\{\alpha\}$ and $i\in N$.

This with (8) implies

(13) $((\rho o x)_i, (\rho o x)_{-i}) \succsim^\rho(i)\ (y_i, (\rho o x)_{-i})$ for any $y_i \in X-\{\alpha\}$ and $i\in N$.

By (2), we have

(14) $(x_i, x_{-i}) \succsim(i)\ (\rho_i^{-1}(y_i), x_{-i}))$ for any $y_i \in X-\{\alpha\}$ and $i\in N$.

Thus $(x_i, x_{-i}) \succsim(i)\ (y_i, x_{-i})$ for any $y_i \in X-\{\alpha\}$ is true for any $i\in N$. Taking $\beta\in X$ which is different from α, the same argument leads is to $(x_i, x_{-i}) \succsim(i)\ (y_i, x_{-i})$ for any $y_i \in X-\{\beta\}\alpha\}$ is true for any $i\in N$. Thus we have $x\in NA(\succsim)$, which is the desired result. □

Theorem 2 does not hold if at least one of the four axioms is not satisfied. This fact is easily identified by the examples 1 to 4. In the case of example 4, consider the preferences profile $\succsim$ in the following: The preference of each player is a linear ordering; Players 2, . . .,n have the completely opposite orderings to that of player 1. (Hence players 2, . . .,n have the same linear orderings.)

Note that the reverse of Theorem 2 does not necessarily hold as example 6 shows. Comparing example 5 with example 6, the set of morally consistent equilibria changes according to MJ. It is equal to the set of Nash equilibria in example 5, and is less than that in example 6. What we show in the paper is that it includes the set of all approval equilibria and is included in that of Nash equilibria. There remains to be an open question about complete characterization of the set of equilibria with respect to each and every MJ. When we strengthen the neutrality axiom to the strict one, we have the following characterization.

Theorem 3 Suppose that MJ satisfies Axioms SN and M. Then we have

$$MC(\succsim)=NA(\succsim) \text{ for any } \succsim\in P^N.$$

Proof : By repeating the same procedure as in Theorem 1 , we have $\supset$. Let us show $\subset$. Suppose on the contrary that $x\in MC(\succsim)\&x\notin NA(\succsim)$, then we have

(1) there is some $i\in N$ and $y_i\notin X$ such that $(x_i,x_{-i})\prec(i)\ (y_i,x_{-i})\&\ y_i\notin MJ(i, x_{-i},\succsim)$.

By Lemma 4, let $\succsim^*\in P^N$ be such that

(2) $(x_i,x_{-i})\prec(j)\ (y_i,x_{-i})$ for any $j\in N$; and

(3) $\succsim^*(i)=\succsim(i)$.

By (3), $x\in MC(\succsim)$, and Axiom SN, we have $x\in MC(\succsim^*)$, i.e., $x_i\in MJ(i, x_{-i}, \succsim^*)$. Applying Lemma 4 to this and (2) , we have $y_i\in MJ(i, x_{-i}, \succsim^*)$. However this contradicts $x\in MC(\succsim^*)$, (1), and (3). This completes the proof. □

Theorem 3 does not hold if either Axiom SN or M is lacked. Example 2 illustrates that Axiom SN is necessary for Theorem 3. The example below shows that Axiom M is also necessary.

Example 7

For any $\succsim \in P^N$, $i \in N$, and $x_{-i} \in X^{N-1}$, $MJ(i, \succsim, x_{-i}) := \{\alpha \in X: (\alpha, x_{-i}) \precsim(i)(\beta, x_{-i})$ for any $\beta \in X\}$.

Theorem 2 is interpreted in the following three aspects. First, Theorem 2 demonstrates that the morally consistent action is a best response strategy in the limited set of possible actions so long as the other players select in the same way. In this sense the morally consistent actions are regarded to be incentive compatible. This is the most important interpretation of our theorem since most economists and game theorists seem commonly to hold the intuitive idea that the deviation from the morally preferable actions is a best response strategy. This intuition does not hold whenever the above four axioms are imposed on MJ. Thus the result is drastic so as to revolutionize our ideas and provides a new viewpoint.

Second, morals, especially welfaristic morals, are not necessarily effective as a norm to induce socially desirable actions. At least the following three reasons should be discussed. At least three reasons should be discussed.

(1) MJ in Theorem 2 is essentially the same as that in example 5[4]. It is amoral since any individual can do any action as right in any circumstance. As long as MJ satisfies the four axioms in Theorem 2, it is nonsense as a system of morals. As long as MJ satisfies the four axioms in Theorem 2, it is nonsense as a system of morals. The justification of this critics depends on that of the axioms, which must be sufficiently remarked. In particular, Axiom N (neutrality) is confronted with a lot of criticism. What result raises by relaxing Axiom N to Axiom I (independence) is an open question.

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[4] For the sake of avoiding misunderstandings, MJ satisfying the four axioms is said not to be necessarily equivalent to MJ in example 5. We only say that the set of MCEs derived from MJ satisfying the axioms

It is clear from the definition of MCE that the information used in MJ is not the cardinal but the ordinal preferences of individuals. There might be no doubt about the possibility that the deficiency of informational basis makes it ineffective (as is well known in the discussion of independence axioms in the theory of social choice). The properties of MCE under the assumption of cardinal preferences as an informational basis must be scrutinized in the future. This line of research is also critical to the study of justification of modern utilitarianism a la Harsanyi (1955) and Hare (1981).

(2) Theorem 2 also gives a necessary condition for existence of MCE. Given a preferences profile, MCE exists if and only if (pure) Nash equilibrium exists. Since the set of (pure) Nash equilibria may be empty when the set of strategies is finite, some limitation might be imposed on MJ. Nonexistence of MCE will occur when MJ gives amoral judgement so that each player's naive rationality is accepted.

One of the escaping routes to overcome this impossibility is to restrict the domain of MJ, i.e., the set of admissible preferences profiles. More concretely, we may construct the domain by gathering profiles under which Nash equilibrium exists. This idea is based on the idea that not moral judgement but rational decision-making of each player is to blame in the situation where Nash equilibrium does not exist. Since the assumption of unrestricted domain has been used thoroughly in Theorem 2, this direction of research is of great worth.

(3) Theorem 2 also shows that it is possible for MJ to resolve the prisoners' dilemma in which Nash equilibrium is not Pareto optimal. We will refer to repeated games for the resolution. It is well known that the dilemma is resolved in the repeated games if the players take strategies such as trigger or tit for tat against others' betrayal in the long run. These strategies may be interpreted as the process in which tacit morals emerge spontaneously to self-constrain the actions[5]. One suggested

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is equal to that from MJ in example 5.

[5] According to the Folk Theorem (see, for example, Fudenberg and Maskin (1986)), any individually rational payoff vector appears in equilibrium. This shows that any action deviating voluntarily from the

direction of research is that the spontaneous morals should be examined by our normative approach[6].

Second, Theorem 2 accomplishes a normative justification of Nash equilibrium. Furthermore Theorem 3 gives an axiomatization of Nash equilibrium[7]. The notion of Nash equilibrium has been sustained from prescriptive point of view, which explains the justification of Nash equilibrium by the rationality aspect in deciding the actions[8]. On the other hand, as the result shows, Nash equilibrium is also justified from the normative and ethical point of view with respect to moral judgement on actions.

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initial payoff vector never occur, i. e., such morals corresponding to the payoffs emerge spontaneously.

[6] We have a conjecture that this problem can not be resolved. The basic idea underlying the axioms except for effectiveness sheds light on some symmetry between the two situations, and requires the consistent application of MJ in the symmetric situations. Even though any symmetry does not hold in each stage of the whole game, some symmetry can be found in the sequence of (infinite or finitely many) games.

[7] Other axiomatization of Nash equilibrium appeared in the literature. See, for example, Peleg, Potters, and Tijs(1996), Patrone, Pieri, Tijs and Torre (1998).

[8] The justifications of Nash equilibrium can mainly be divided into five groups. The first is a justification by a pre-play communication; If the players have chances to communicate their strategies with each other and get into an agreement to the future strategies before the actual game starts, then the n-tuple of strategies must be a Nash equilibrium. See, for example, Binmore (1990), Aumann and Brandenburger (1996) for more detailed discussion about the notion of Nash equilibrium from the prescriptive point of view. The second is a self-fulfilling prophecy. If all the players share the theory to prospect others' actions, the resulting action from the theory must be a Nash equilibrium (see Myerson(1991)). The third depends on a focal point. When the players know how to play the game from the idea of the structure of the game, they come to know how the others play. Such a standard, called a focal point, gives rise to a particular combination of strategies in equilibrium (see Schelling (1960)). The fourth is given by learning. The players often learn an equilibrium in the repeated situation. When the players learn others' strategy choices and maximize their payoffs, they are on the equilibrium (see for example Kalai and Lehrer(1993) and so forth). The fifth is a justification by evolutionary games. See, for the earlier example, Taylor and Jonker (1978). Irrespective of the voluntary action of payoff maximization, the strategies giving lower payoffs may be weeded out after a long period of time, whereas those giving higher payoffs may survive. The outcome of their adjustment behavior must coincide with Nash equilibrium derived by their tentative maximizing behavior.

5 Conclusion

It is reasonable that the morals discussed in this paper should be interpreted as an external and implicit constraint which governs individual action by its normative appeal to the members. Note that our morals are different from the judgements people hold in order to observe the principle of justice since all the axioms are imposed not on the standard for judgement of the social outcomes but on the individual actions. Also note that they could not be interpreted as an implicit or explicit contract. This is because the contracts generally differ from person to person according to the personality, capability, information at hand, and so forth, whereas the requirements of morals must be consistent with each other by their own nature[9].

What is the most questionable here is the relationship between our morals and the libertarian rights. Intuitively speaking, rights in general have their bases on the law, and correspond to duties in the most cases. The rights are caught in the perspective of general contract between the state and the people as well as among them. It is obviously understood this perspective goes beyond our framework. Contrarily limiting the discussion to the narrower description, the morals characterized by the four axioms might be interpreted as the normative requirements by the society to the individual actions with respect to exercising the libertarian rights. In this view, how to judge among libertarian rights should be determined by the aggregated information on the preferences of the parties. The similar approach to libertarian rights is well known in the theory of social choice, and is quite different from ours in the motivation as well as in the formulation of morals[10].

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[9] It is not only fundamental but also the merits of contracts that the contents of contracts differ from person to person. It is said to be the most beneficial point to the contracts that their contents are determined depending on individual matters of affairs in comparison with, for example, the market trades.

[10] There are two different approaches to the theory of libertarian rights in the theory of social choice; The first is the classical Sen's theory of rights, and the second the game-form approach to rights by Gaertner, Pattanaik, and Suzumura(1992) which intends to overcome the deficits in Sen's original definition. Since the right in Sen's sense is that of rejecting undesirable outcome for the holder to be attained, it can not be derived from our definition of morally rights actions.

The traditional ethics discusses in general the normative justification of individual actions. Even though each individual action is morally right, the consequence of the actions in the society may not accepted because of uncertainty in their interdependence at least from the view of economics. The reason is probably that, contrary to general equilibrium theory and game theory in economics, ethics has no systematic model of interactions among individuals and of the consequence of their interdependence. It will be of great pleasure that our analysis would serve to give some light on such trials.

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On the other hand, the theory of rights in Gaertner, Pattanaik, and Suzumura(1992) is closely related to our framework in that both use the game-form as a fundamental basis of analysis. In spite of the formal similarity of the framework, there exists a large gulf between the two. Though Gaertner, Pattanaik, and Suzumura(1992) have no idea to judge the libertarian claim of individual rights to be good or bad depending on the preferences profile in the society, our theory has no motivation about the confirmation of the rights system through social choice, and does not focus on the extended Paretian libertarian paradox. Noticing that Nash equillibrium is not necessarily Pareto optimal, the similar proposition to liberal padradox holds true by Theorem 1when we reinterpret MJ to be the standard of rights. While it is far from Sen's original paradox and Gaertner, Pattanaik, and Suzumura(1992)'s, Theorem 1 contains the paradox in its nature since it is derived as a corollary.

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