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最小コスト辺支配集合問題の近似について

Approximating Edge Dominating Sets in Weighted Graphs

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Abstract: We study approximability of the edge dominating set problem. It has been known, besides its *NP*-hardness, that a solution of size at most twice larger than the smallest one can be efficiently computed, due to its close relationship to minimum maximal matching. In general when graphs are edge weighted, however, such a nice relationship breaks down, and no edge dominating set of small weight is obtainable from any maximal matching. In this paper, after showing that weighted edge domination is as hard to approximate as weighted vertex cover is, we consider two natural strategies, one reducing edge dominating set to vertex cover and the other to edge cover, and show that weighted edge dominating set can be approximated within factors of 4 and $2\frac{2}{3}$, respectively.

Keywords: edge dominating set, vertex cover, edge cover, approximation.

1 Introduction

In an undirected graph an edge is said to *dominate* edges adjacent to it, and a set of edges is an *edge dominating set (eds)* if its edges collectively dominate all the other edges in a graph. The *edge dominating set problem (EDS)* is then that of finding a smallest eds or, if edges are weighted, an eds of minimum total weight. Yannakakis and Gavril showed that EDS (and the minimum maximal matching problem, to be explained later) is *NP*-complete even when graphs are planar or bipartite of maximum degree 3 [21]. This *NP*-completeness result was later extended by Horton and Kilakos to planar bipartite graphs, line and total graphs, perfect claw-free graphs, and planar cubic graphs [12]. On the other hand the polynomially solvable special cases have been discovered, in this order, for trees [18], claw-free chordal graphs, locally connected claw-free graphs, the line graphs of total graphs, the line graphs of chordal graphs [12], bipartite permutation graphs, cotriangulated graphs [20], and so on.

Although EDS has important applications in such areas as telephone switching networking, very little is known about its computational complexity when graphs are edge weighted and it is required to minimize the weight of an eds; in fact, only the minimum *cardinality* EDS is considered in all the polynomial time solvable cases mentioned above. In particular, while it is a simple matter to compute an eds of *size* at most twice the smallest one, as any maximal matching will do (to be explained later), such a simple construction easily fails when arbitrary weights are given on edges, and no approximability results have been reported in this case. In this paper we consider two natural strategies, one reducing EDS to vertex cover and the other to edge cover, and show that weighted EDS can be approximated within factors of 4 and $2\frac{2}{3}$, respectively.

The EDS problem is also interesting in the sense that it is closely related to several basic graph problems, and we summarize it below. Much better known and well-studied is the *vertex* dominating set problem, in which the minimum vertex set is sought in a graph such that every vertex is either in it or adjacent to one in it. The EDS problem for any G is clearly equivalent to the vertex dominating set problem for the line graph of G. The vertex dominating set problem for general graphs is, however, also equivalent to the set cover problem under the approximation preserving reduction, of which the polynomial time approximability is rather well established: the set cover can be approximated within a factor $\log n+1$ [14, 17, 5] and cannot be in a factor better than $\log n$ unless $NP \subset \mathrm{DTIME}(n^{O(\log\log n)})$ [7].

A set of edges is called a *matching* (or *independent*) if no two of them have a vertex in common. A matching is *maximal* if no other matching properly contains it, and the minimum *maximal matching problem (MM)* asks for computing a minimum maximal matching in a given graph. Notice that any maximal matching is also an eds because an edge not in it must be adjacent to some in it, and for this reason it is also called an *independent edge dominating set*. Certainly, a smallest maximal matching cannot be smaller than a smallest eds. Interestingly, given any "minimal" eds one can construct a maximal matching of no larger size in polynomial time [10], implying that the smallest size of an eds in any graph is equal to the size of its smallest maximal matching. Thus, EDS and MM are polynomially equivalent when graphs are *unweighted*. They are not, however, when graphs are weighted, and it is easy to construct such an instance of weighted EDS in which no minimum solution is a matching.

Another basic NP-complete graph problem closely related to either EDS or MM is the *vertex cover (VC)* problem, the problem of finding a minimum vertex set in G s.t. every edge of G is incident to some vertex in the set. For any edge set $F \subseteq E$ let $V(F)$ denote the set of vertices to which edges of F are incident. Then, F is an eds for G iff $V(F)$ is a vertex cover for G. As pointed out in [21], using this fact and that any vertex cover must contain at least one vertex from every edge of a maximal matching, we see that $V(F)$ provides a VC of the size at most twice the minimum VC for any maximal matching F. For some class of graphs (unweighted) VC is known to be polynomially solvable, and in particular, it is so if a minimum vertex cover is as small as a maximum matching for any graph in it. One of the best known among those with this property is perhaps that of bipartite graphs, and such a class of graphs, called *semipartite*, in which the equality holds was studied by Gavril in [9], who described a polynomial time algorithm for recognizing if a graph is semipartite or not.

The *connected vertex cover* problem is a variant of VC, in which a vertex cover C is sought in a connected graph s.t. C induces a connected subgraph. This problem is also NP-complete and as hard to approximate as VC is [8]. As stated above, enforcing the "independence" property on EDS solutions does not alter (increase) their sizes, but the connectivity condition certainly does (just consider a path of length 5). When an eds is required to be connected, and if a smaller one is desired, it must form a tree (assuming G is connected), and the problem is called *tree cover*. As in the relationship between EDS and VC, an edge set F is a tree cover iff $V(F)$ is a connected vertex cover. Moreover, since F is a tree, $|F| = |V(F)| - 1$, and hence, a smallest tree cover provides a smallest connected vertex cover, and vice versa. The tree cover was shown approximable within a factor of 2 when graphs are unweighted, and when weighted, within a factor of $r_{\mathrm{St}} + r_{\mathrm{WVC}}(1 + 1/k)$, where $r_{\mathrm{St}}(r_{\mathrm{WVC}})$ is the performance ratio of any polynomial time algorithm for the Steiner tree (weighted vertex cover, resp.) problem [1].

2 Preliminaries

For a vertex set S let $\delta(S)$ denote the set of edges incident to a vertex in S. When S is an edge set, we let $\delta(S) = \delta(\cup_{e \in S} e)$ where edge e is a set of two vertices; then, $\delta(S)$ also denotes the set of edges in S and those dominated by S. When S is a singleton set $\{s\}$, $\delta(\{s\})$ is abbreviated to $\delta(s)$.

2.1 Unweighted EDS

As stated in Introduction the EDS problem can be approximated within a factor of 2 in a graph with unit edge weights: just compute any maximal matching, and this is because

1. for any eds there exists a maximal matching of no larger size, and
2. no two maximal matchings can differ in their sizes by a factor larger than 2.

Once arbitrary (nonnegative) weights are associated with edges and total weights are compared, however, neither of these holds. In fact the minimum weight of an eds does not only equal to that of a maximal matching, but also the latter could be arbitrarily larger than the former; to see it, consider a simple path of length 4, in which both of the internal edges are given small weights while external ones are both given some large weights.

2.2 Approximation Hardness

Yannakakis and Gavril proved the *NP*-hardness of EDS by reducing VC to it [21]. Although their reduction can be made to preserve approximation quality within some constant factor, and hence, (unweighted) EDS is *MAX SNP*-hard implying non-existence of polynomial time approximation schema (unless *P*=*NP*) [19, 2], it does not exclude better approximation of EDS than that of VC. On the other hand, it is quite straightforward to see that approximation of EDS is as hard as that of VC once arbitrary weights are allowed:

Theorem 1 *The weighted VC problem can be approximated as good as the weighted EDS problem can be.*

Proof. Let $G = (V, E)$ be an instance graph for VC with weight $w_u \in \mathbb{Q}_+$ for each vertex $u \in V$. Let s be a new vertex not in V and construct a new graph $G' = (V \cup \{s\}, E \cup E')$ by attaching s to G: s is connected to each vertex of G by a new edge, and so $E' = \{\{s, u\} : u \in V\}$. Assign a weight $w'(e)$ to each edge e of G' s.t. $w'(e) = w_u$ if $e = \{s, u\} \in E'$, and $w'(e) = w_u + w_v$ if $e = \{u, v\} \in E$. By the way w' is determined, if F, an eds for G', contains $\{u, v\} \in E$ it can be replaced by two edges, $\{u, s\}$ and $\{v, s\}$, without increasing the weight of F, and so, F can be assumed to be contained entirely in E'. But then, there exists a one-to-one correspondence between vertex covers in G and eds's in G'; namely, $V(F) - s$ in G and F in G', and the weights are preserved between them. □

3 Reducing to Vertex Cover

Given an edge weighted graph G, EDS can be thought of as a problem of finding an edge set D of minimum weight such that $V(D)$ forms a vertex cover in G. We first present a heuristic which computes an eds by reducing the problem to VC. Although it is quite simple, both in its description and analysis, we see at least that the weighted EDS is approximable within a constant factor.

Given $G = (V, E)$ with edge weight $w(e) \geq 0$ for each $e \in E$, construct a vertex weight assignment $w^v : V \to \mathbb{Q}_+$ s.t.

$$w^v(u) \stackrel{\text{def}}{=} \min\{w(e) : e \in \delta(u)\}, \qquad \forall u \in V.$$

Let D^* be a minimum eds for G with w, and C^* be a minimum vertex cover for G with w^v. Since $V(D^*)$ is a vertex cover for G, $w^v(C^*) \leq w^v(V(D^*))$. From the way w^v is defined, $w^v(u) \leq \min\{w(e) : e \in \delta(u) \cap D^*\}$ for each $u \in$

$V(D^*)$, and hence, $w^v(V(D^*)) = \sum_{u \in D^*} w^v(u) \leq 2w(D^*)$. We thus have

$$w^v(C^*) \leq 2w(D^*). \quad (1)$$

Suppose C is a vertex cover for G s.t.

$$w^v(C) \leq rw^v(C^*) \quad (2)$$

for some $r \geq 1$. Construct an eds D_C from C by including into it a minimum weight edge e in $\delta(u)$ for each $u \in C$. Then, $w(D_C) \leq w^v(C)$, and combining (1) and (2) with this, we have

$$w(D_C) \leq rw^v(C^*) \leq 2rw(D^*).$$

More formally, the algorithm VCover suggested by the reasoning above is described as:

1. Let $w^v(u) \stackrel{\text{def}}{=} \min\{w(e) : e \in \delta(u)\}$ for each $u \in V$.
2. Find a vertex cover C in G with weight w^v.
3. Pick an edge $e' \in E$ for each $u \in C$ s.t. $w(e') = \min\{w(e) : e \in \delta(u)\}$;
4 Output the set of edges picked in step 3.

Theorem 2 *The algorithm* VCover *computes an EDS of which weight is at most* $2r_{\mathrm{WVC}}$ *times the optimal weight, where* r_{WVC} *is the performance ratio of any polynomial time algorithm for the weighted vertex cover problem.*

Step 2 of VCover computes a vertex cover in G with weight w^v, which can be implemented by any approximation algorithm for the VC problem. For instance, when G is unweighted, i.e., $w \equiv 1$, vertex weight w^v introduced by w is also constantly 1. Thus, the Gavril's procedure can be used to compute an approximate vertex cover, which simply constructs a maximal matching M and returns $V(M)$. But then, in Step 3 we may simply pick edges of M (and output M), and this way, VCover reduces to the maximal matching heuristic when G is unweighted.

In general any approximation algorithm with performance ratio bounded by 2 can be used to implement Step 2, such as those in [11, 4], giving the performance ratio of 4 for VCover. When G is of special type, however, such as bipartite and planar, more appropriate procedures are available for Step 2; namely, the optimal algorithm based on maximum matching for the former case and the approximation scheme for the latter [16, 3], substituting 1 and $1+\epsilon$ (for any $\epsilon > 0$), respectively, for the multiplicative factor of r in (2):

Corollary 3 *When G belongs to the class of perfect graphs such as bipartite graphs,* VCover *approximates the EDS problem with performance ratio of 2. When G is a planar graph,* VCover *approximates within a factor arbitrarily close to 2.*

Remark: There exists a PTAS for the EDS problem when graphs are planar [3] and λ-precision unit disk graphs[13].

4 Reducing to Edge Cover

The second algorithm for EDS reduces it to (a variant of) the edge cover problem, which is known to be solvable in time complexity of the maximum matching problem (e.g.,[15]). In what follows, for any real vector x indexed by elements of T and for a subset S of T, $x(S)$ means $\sum_{t \in S} x_t$.

4.1 LP Relaxation

Let us consider the following linear program defined on $G = (V, E)$ with edge weights w_e, and call a feasible solution for it a *fractional edge dominating set (fractional eds)*. Clearly, any 0-1 feasible solution here corresponds to an eds.

$$\begin{array}{llll} & \text{Min} & \sum_{e \in E} w_e x_e & \\ & \text{subject to:} & & \\ \text{(EDS)} & & x(\delta(e)) \geq 1 & e \in E \\ & & x_e \geq 0 & e \in E \end{array}$$

The dual LP of (EDS) is then written as:

$$\begin{array}{llll} & \text{Max} & \sum_{e \in E} y_e & \\ & \text{subject to:} & & \\ \text{(D)} & & y(\delta(e)) \leq w_e & e \in E \\ & & y_e \geq 0 & e \in E \end{array}$$

Thus, both primal and dual variables, x and y, are indexed by edges.

Although we do not use (D) in approximation of weighted EDS, it follows directly from these formulations that the size of any maximal matching is at most twice the smallest size of an eds by setting $w_e = 1, \forall e \in E$. Let $\tilde{x}$ be the incidence vector of any maximal matching. Then, it is feasible to (EDS) but not to (D) in general. However, when $w_e \geq 1$ for all $e \in E$, $\tilde{y} = \frac{1}{2}\tilde{x}$ becomes feasible to (D), and hence, $\sum_{e \in E} \tilde{y}_e = \frac{1}{2}\sum_{e \in E} \tilde{x}_e$ bounds from below the optimal value of (EDS).

4.2 Fractional Edge Cover

Assume in this section that a graph G has no isolated vertices. We introduce another linear program defined on $G = (V, E)$ with edge weights w_e, of which feasible solutions are called *fractional edge covers*.

$$\begin{array}{llll} & \text{Min} & Z = \sum_{e \in E} w_e x_e & \\ & \text{subject to:} & & \\ \text{(EC)} & & x(\delta(u)) \geq 1 & u \in V \\ & & x_e \geq 0 & e \in E \end{array}$$

It is easy to see that the incidence vector of any edge cover for G satisfies all the constraints in (EC), thus feasible to it. It may not, however, have an integral optimal solution in general, to which the simplest example attesting is a triangle with unit weights: The optimal solution for (EC) has $x_e = 1/2, \forall e \in E$, with its total weight $3/2$, while the weight of an integral solution must be at least 2. Thus, the minimum weight of an integral solution could become as large as $4/3$ times that of a fractional one, but it does no more[1]:

Theorem 4 *For any G let Z and Z_I denote, respectively, the optimal cost of (EC) and that of an edge cover for G. Then, $Z_I/Z \leq 4/3$.*

Proof. The edge cover polytope P_{EC} is the convex hull of the incidence vectors of edge covers. Due to the result of Edmonds and Johnson [6], P_{EC} can be described by the set of linear inequalities in (EC), plus the following ones:

$$x(\delta(U)) \geq \lceil \frac{|U|}{2} \rceil, \quad U \subseteq V \qquad (3)$$

Let x denote any fractional edge cover. To prove the claim it suffices to show that $\frac{4}{3}x$ belongs to P_{EC}, or in other words, it satisfies (3). If $|U| = 1$ this is trivial, so consider the case when $|U| \geq 2$. Notice that, since $x(\delta(u)) \geq 1, \forall u \in V$,

$$\sum_{u \in U} x(\delta(u)) = 2x(\delta(U)) - x(\bar{\delta}(U)) \geq |U|,$$

where $\bar{\delta}(U) = \{e : e \in \delta(U) \text{ but not induced by } U\}$. Thus, $x(\delta(U)) \geq |U|/2$ for any $U \subseteq V$, to which (3) reduces when $|U|$ is even. Moreover, this implies that $\frac{4}{3}x(\delta(U)) \geq \frac{2}{3}|U| = |U|/2 + |U|/6$, and since $\lceil |U|/2 \rceil = |U|/2 + 1/2$ when $|U|$ is odd, $\frac{4}{3}x(\delta(U)) \geq \lceil |U|/2 \rceil$ for any U with $|U| \geq 3$. □

4.3 Algorithm

Let us fix an optimal solution x for (EDS). The vertex set V is divided into V_+ and V_-, depending on the sign of $x(\delta(u)) - \frac{1}{2}$, such that $V_+ = \{u \in V : x(\delta(u)) \geq \frac{1}{2}\}$ and $V_- = \{u : x(\delta(u)) < \frac{1}{2}\}$. Since $x(\delta(e)) = x(\delta(u)) + x(\delta(v)) - x_e$ for an edge $e = \{u, v\}$, if both u and v are in V_-, $x(\delta(e)) < 1 - x_e < 1$, which implies non-existence of an edge between any two vertices of V_-:

Lemma 5 *V_+ is a vertex cover in G.*

So, if an edge set covers all the vertices in V_+, it is automatically an eds. This is not exactly the standard edge cover problem but is easily reducible to it. For any vertex subset X of $G = (V, E)$, let X' be a set of new vertices, a copy of X, and, for each $u \in X$ and $u' \in X'$, attach a new edge $\{u, u'\}$ with its weight equal to zero. Let $G' = (V \cup X', E \cup E')$ denote the graph constructed this way from G, where E' is the set of new edges. Then, if F' is a minimum weight edge cover for G', $F' \cap E$ must be an edge set of

[1] the author believes that this fact has been long known but was unable to locate it in the literature.

minimum weight in G covering all the vertices in $V - X$.

The second approximation algorithm for weighted EDS called ECover is now described simply as:

1. Compute an optimal solution x for (EDS).
2. Compute V_+.
3. Compute and output a minimum weight edge set covering all the vertices in V_+.

Theorem 6 *The algorithm* ECover *computes an eds, of which weight is at most $2\frac{2}{3}$ times larger than the optimal weight.*

Proof. From the argument above ECover clearly produces an eds.

Let x be an optimal fractional eds in $G = (V, E)$. Recall the graph $G' = (V \cup V'_-, E \cup E')$ used in ECover, which is constructed from G by attaching disjointly to it new edges with zero weights. Define $\bar{x} \in \mathbb{R}^{E \cup E'}$ on the edge set of G' by setting $\bar{x}_e = x_e$ if $e \in E$ and $\bar{x}_e = 1/2$ otherwise. Similarly, let $\bar{w}$ denote the edge weight vector defined on G' s.t. $\bar{w}_e = w_e$ if $e \in E$ and $\bar{w}_e = 0$ otherwise. The optimal edge covers, fractional one and integral one, for G' under the weight $\bar{w}$ are denoted by y and y_I, respectively.

Now, $\bar{x}(\delta(u)) \geq 1/2$ for $u \in V \cup V'_-$, and $2\bar{x}$ is a fractional edge cover for G' since it satisfies all the constrains in (EC). Also, the weight of $\bar{x}$ is that of a fractional eds x for G, i.e., $\bar{w}^T \bar{x} = w^T x$, and hence,

$$\bar{w}^T y \leq 2\bar{w}^T \bar{x} = 2w^T x.$$

The eds computed by ECover is a minimum edge cover for G' under $\bar{w}$ less extra edges in E'. So, its weight is exactly $\bar{w}^T y_I$. Since $\bar{w}^T y_I \leq \frac{4}{3}\bar{w}^T y$ by Theorem 4, we conclude that it is no larger than $\frac{8}{3}w^T x$, that is, at most $2\frac{2}{3}$ times the minimum cost of (EDS) on G. □

It is further observed that the set of constraints in (EC) is in the form of $\{Ax \geq \vec{1}, x \geq 0\}$, where A is the $V \times E$ incidence matrix of G. So, if G is a bipartite graph A is totally unimodular, which implies that all the vertices of the polyhedron $\{x : Ax \geq \vec{1}, x \geq 0\}$ are integral, and in particular that the minimum weight of an edge cover coincides with that of a fractional edge cover in G. Since G' is bipartite if so is G, no 4/3 factor blow up is incurred in converting a fractional edge cover to an integral one in this case.

Corollary 7 *The performance ratio of* ECover *is at most 2 when G is bipartite.*

4.4 Integrality Gap

Given that weighted EDS is as hard to approximate as weighted VC is and that no polynomial time algorithm with a constant performance ratio less than 2 is known for the latter, it should be seen rather satisfactory if the former is shown to be approximable with a factor of 2. Unfortunately though, it turns out that, as long as our algorithm design and analysis are based on (EDS) using its optimal cost as a lower bound for the optima of weighted EDS, we need to relinquish such hope. This is so because (EDS) introduces the *integrality gap* (i.e., the ratio between the integer and fractional optima) larger than 2.

Consider the complete graph on $5n$ vertices, and take n subgraphs $G_1, \ldots, G_n$, each isomorphic to K_5, vertex disjointly in it. Assign to each edge of G_i a weight of 1, while, to any edge not in any of these subgraphs, some huge weight. Let $x_e = 1/7$ if e is in some G_i and $x_e = 0$ otherwise. Then, it can be verified that $x(\delta(e)) \geq 1$ for all e, and hence, x is a feasible solution for (EDS), of cost $\frac{10}{7}n$. On the other hand, any integral solution must cover all but one vertices in the graph. Being prohibited to pick an edge outside of G_i's, an integral solution of small cost would choose 3 edges from each of G_i's but one, and the total cost for it would be $3n - 1$. Thus, the integrality gap of formulation (EDS) could become at least arbitrarily close to 21/10.

On the other hand it was shown that the integrality gap of (EDS) is at most 2 when G is a bipartite graph (Corollary 7), and in fact it could be as large as arbitrarily close to 2. Let G be a complete bipartite graph $K_{k,k}$ with unit weights. Then, $x_e = \frac{1}{2k-1}, \forall e \in E$ is a feasible solution with its weight totaling to $\frac{k^2}{2k-1}$. Any integral solution must contain k edges since it has to cover all the vertices in at least one of the two vertex classes. So, the integrality gap must be at least $\frac{k(2k-1)}{k^2} = 2 - \frac{1}{k}$.

Lastly, it is pointed out that such an example as given above for the integrality gap of (EDS) larger than 2 can be eliminated if (EDS) is augmented by additional valid inequalities for the edge dominating set polytope. An edge can dominate at most 4 edges of any simple cycle C if it is an chord of C, and it can at most 3 otherwise. Therefore, the following set of inequalities is valid for the EDS polytope:

$$\begin{aligned} x(\delta(C)) &\geq \lceil \frac{|C|}{4} \rceil, \quad C\text{: a simple cycle in } G \\ (\text{or} &\geq \lceil \frac{|C|}{3} \rceil, \quad C\text{: a simple chordless cycle}) \end{aligned}$$

For any complete subgraph S on $2k+1$ vertices, these valid inequalities force any fractional solution x to have $x(\delta(S)) \geq \lceil \frac{2k+1}{4} \rceil$, which equals to $\frac{k}{2}+1$ if k is even, and to $\frac{k+1}{2}$ if k is odd. If edges are weighted as in the example above, any fractional solution has its weight at least $\frac{k+1}{2}$ per S, while any reasonable integral solution uses only $k+1$ edges from each S.

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