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Vanishing Theorems in Hyperasymptotic Analysis

Hideyuki Majima
Department of Mathematics
Faculty of Science
Ochanomizu University
majima@math.ocha.ac.jp

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In the former paper, we proved the following: Theorem[Commutative case] Let σ be a rational number ≥ 1 and

$$\{\mathcal{S}(R, a_h, b_h)\,(h = 1, \ldots, N)\}$$

be a good open sectorial covering for σ of $\mathcal{D}(R, \infty) = \{\, z \mid +\infty > |z| > R\,\}$.. For $h = 1, \ldots, N$, let $U_{h-1,h}(z)$ be an $m \times n$ matricial function defined in $\mathcal{S}_{h-1,h}(R)$ and, for some non-zero constant $\kappa_{h-1,h}$ with $\arg \kappa_{h-1,h} = -\frac{a_h + b_{h-1}}{2\sigma}$, $\exp(-\kappa_{h-1,h} z^{\frac{1}{\sigma}})$ is asymptotically developable to the formal power-series 0 and, for a complex number $\mu_{h-1,h}$,

$$z^{\mu_{h-1,h}} \exp(\kappa_{h-1,h} z^{\frac{1}{\sigma}}) U_{h-1,h}(z)$$

is asymptotically developable to a formal power-series matrix $\sum_{s=0}^{\infty} U_s^{h-1,h} z^{-s}$ in the sector $\mathcal{S}_{h-1,h}(R)$.

Then, there exist a positive number $R''(\geq R)$, a formal power-series matrix $\widehat{V}(z) = \sum_{r=0}^{\infty} T_r z^{-r}$ and $m \times n$ matricial functions V_h defined in $\mathcal{S}_h(R'')$ $(h = 1, \ldots, N)$ such that

(i) the relation

$$U_{h-1,h}(z) = -V_{h-1}(z) + V_h(z)$$

holds for $z \in \mathcal{S}_{h-1,h}(R'') = \mathcal{S}(R'', a_{h-1}, b_{h-1}) \cap \mathcal{S}(R'', a_h, b_h)$.

(ii) V_h is aymptotically developable to the formal power-series matrix $\widehat{V}(z)$ in $\mathcal{S}_h(R'')$, and for any sufficiently large number r,

$$\begin{aligned} T_r &= \sum_{(h-1,h)} \sum_{s=0}^{M-1} \sigma U_s^{h-1,h} (\kappa_{h-1,h})^{(s-r)\sigma + \mu_{h-1,h}} \Gamma((r-s)\sigma - \mu_{h-1,h}) \\ &+ O\{\Gamma((r-M)\sigma - \Re\mu_{h-1,h})\} \end{aligned}$$

provided $1 \leq M < r$.

This theorem can be used to study the stucture of divergent pwer-series solutions to the non-homogeneous differential equations associated to linear ordinary differential equations, for example, Bessel equations, Whittaker equations, Weber equations and so on. In this talk, we will give a refined version of the above theorem. The result will be published as a joint-work with C. J. Howls, and A. B. Olde Daalhuis.

References

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