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# Positive increasing solutions to two-dimensional differential systems with singular nonlinearities

## 非線形特異項をもつ２階微分方程式系の正値増大解について

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## 0. Introduction

In this paper we consider second order differential systems with singular nonlinearities of the type

$$\begin{cases} (p(t)|y'|^{\alpha-1}y')' = \varphi(t)z^{-\lambda} \\ (q(t)|z'|^{\beta-1}z')' = \psi(t)y^{-\mu}, \quad t \geq a, \end{cases} \tag{A}$$

where $\lambda$ and $\mu$ are positive constants and $p(t)$, $q(t)$, $\varphi(t)$ and $\psi(t)$ are positive continuous functions on $[a,\infty)$, $a \geq 0$. It is assumed throughout the paper that

$$\int_a^\infty (p(t))^{-\frac{1}{\alpha}}dt = \int_a^\infty (q(t))^{-\frac{1}{\beta}}dt = \infty. \tag{0.1}$$

By a solution of (A) on an interval $J \subset [a,\infty)$ we mean a vector function $(y,z)$ which has the property that $y$ and $z$ are continuously differentiable on $J$ together with $p|y'|^{\alpha-1}y'$ and $q|z'|^{\beta-1}z'$ and satisfies the system (A) at all points of $J$. Obviously, both components of a soluton must be positive on $J$. Such a solution $(y,z)$ is said to be *singular* or *proper* according to whether its maximal interval of existence $J$ is bounded or unbounded. A solution $(y,z)$ is called increasing (or decreasing) if both of its components $y$ and $z$ are increasing (or decreasing).

We are interested in the existence and asymptotic behavior of positive increasing proper solutions of (A). More specifically, The set of all positive increasing solutions of (A) is calssified into four disjoint classess according to their asymptotic behavior as $t \to \infty$, and criteria are given for the existence and/or non-existence of solutions belonging to each of these classes.

A simple application of the fundamental theorems on ordinary differential equations shows that, for any given $y_0 > 0$, $z_0 > 0$, $y_1 \geq 0$, $z_1 \geq 0$, the system (A) has a unique positive solution $(y,z)$ determined by the initial conditions

$$y(a) = y_0, \quad (p(a))^{\frac{1}{\alpha}}y'(a) = y_1, \quad z(a) = z_0, \quad (q(a))^{\frac{1}{\beta}}z'(a) = z_1 \tag{0.2}$$

and that, because of the presence of negative exponents in (A), the solution $(y,z)$ can be continued over the entire interval $[a,\infty)$, so that it always becomes a proper solution.

Let $(y,z)$ be a positive increasing solution of (A) defined on $[a,\infty)$. Since $y'(t)$ and $z'(t)$ are positive, the functions $p(t)|y'(t)|^{\alpha-1}y'(t) = p(t)(y'(t))^\alpha$ and $q(t)|z'(t)|^{\beta-1}z'(t) = q(t)(z'(t))^\beta$ are positive and increasing on $(a,\infty)$, so that

$$\text{either} \quad \lim_{t\to\infty} p(t)(y'(t))^\alpha = \text{const} > 0 \quad \text{or} \quad \lim_{t\to\infty} p(t)(y'(t))^\alpha = \infty \tag{0.3$_\mathrm{a}$}$$

and

$$\text{either} \quad \lim_{t\to\infty} q(t)(z'(t))^\beta = \text{const} > 0 \quad \text{or} \quad \lim_{t\to\infty} q(t)(z'(t))^\beta = \infty. \tag{0.3$_\mathrm{b}$}$$

Note that $(0.3_\mathrm{a})$ and $(0.3_\mathrm{b})$ are equivalent, respectively, to

$$\text{either} \quad \lim_{t\to\infty} \frac{y(t)}{P(t)} = \text{const} > 0 \quad \text{or} \quad \lim_{t\to\infty} \frac{y(t)}{P(t)} = \infty \tag{0.4$_a$}$$

and

$$\text{either} \quad \lim_{t\to\infty} \frac{z(t)}{Q(t)} = \text{const} > 0 \quad \text{or} \quad \lim_{t\to\infty} \frac{z(t)}{Q(t)} = \infty, \tag{0.4$_b$}$$

where $P(t)$ and $Q(t)$ are given by

$$P(t) = \int_a^t (p(s))^{-\frac{1}{\alpha}} ds, \qquad Q(t) = \int_a^t (q(s))^{-\frac{1}{\beta}} ds, \qquad t \geq a. \tag{0.5}$$

Thus a positive increasing solution $(y, z)$ of (A) falls into one of the following four types:

(I) $\lim_{t\to\infty} \frac{y(t)}{P(t)} = \text{const} > 0$ and $\lim_{t\to\infty} \frac{z(t)}{Q(t)} = \text{const} > 0$;

(II) $\lim_{t\to\infty} \frac{y(t)}{P(t)} = \text{const} > 0$ and $\lim_{t\to\infty} \frac{z(t)}{Q(t)} = \infty$;

(III) $\lim_{t\to\infty} \frac{y(t)}{P(t)} = \infty$ and $\lim_{t\to\infty} \frac{z(t)}{Q(t)} = \text{const} > 0$;

(IV) $\lim_{t\to\infty} \frac{y(t)}{P(t)} = \infty$ and $\lim_{t\to\infty} \frac{z(t)}{Q(t)} = \infty$.

A solution of the type (I) or (IV) is called a *weakly increasing solution* or a *strongly increasing solution* of (A), respectively. Solutions of the types (II) and (III) are referred to as *semi-strongly increasing solutions* of (A).

In Section 1 the type-(I) solution of (A) is examined in Section 1, establishing a necessary and sufficient condition for the existence of such solutions. In Section 2 sufficient condtions are provided under which (A) has solutions of the types (II), (III) and (IV). The results obtained will shed light on the structure of positive increasing proper solutions of the system (A). Example illustrating the main results are presented in Section 3.

There has been an increasing interest in the study of nonlinear dfferential systems of the type (A) or their variants; see e. g. the papers [1–6]. This work can be considered as a continuation of our previous paper [2] which is concerned with the existence of positive decreasing solutions for (A).

## 1. Weakly increasing solutions

This section is devoted to the study of the existence of a weakly increasing positive solution of (A).

**THEOREM 1.** *Suppose that* (0.1) *holds. The system* (A) *possesses an increasing positive solution* $(y, z)$ *of the type* (I) *if and only if*

$$\int_b^\infty \varphi(t)(Q(t))^{-\lambda} dt < \infty \tag{1.1$_a$}$$

*and*

$$\int_b^\infty \psi(t)(P(t))^{-\mu} dt < \infty \tag{1.1$_b$}$$

*for any* $b > a$, *where* $P(t)$ *and* $Q(t)$ *are defined by* (0.5).

PROOF. (The "only if"part) Suppose that (A) has a weakly increasing solution $(y, z)$ existing on $[a, \infty)$ and satisfying (0.2). Integrating (A) from $a$ to $t$, we have

$$p(t)(y'(t))^\alpha = y_1^\alpha + \int_a^t \varphi(s)(z(s))^{-\lambda} ds, \tag{1.2$_a$}$$

$$q(t)(z'(t))^\beta = z_1^\beta + \int_a^t \psi(s)(y(s))^{-\mu} ds, \quad t \geq a. \tag{1.2$_b$}$$

Letting $t \to \infty$ in the above, we see that

$$\int_a^\infty \varphi(s)(z(s))^{-\lambda} ds < \infty \quad \text{and} \quad \int_a^\infty \psi(s)(y(s))^{-\mu} ds < \infty. \tag{1.3}$$

The desired inequalities (1.1$_a$) and (1.1$_b$) immediately follow from (1.3) combined with the fact that

$$kP(t) \leq y(t) \leq k'P(t), \quad lQ(t) \leq z(t) \leq l'Q(t), \quad t \geq b,$$

for some positive constants $k$, $k'$, $l$ and $l'$.

(The "if" part) Suppose that (1.1$_a$) and (1.1$_b$) are satisfied. Let $y_0$ and $z_0$ be any fixed constants and choose positive constants $\eta_1$ and $\zeta_1$ so large that

$$\int_a^\infty \varphi(t)\left(z_0 + \frac{1}{2}\zeta_1 Q(t)\right)^{-\lambda} dt \leq \left(1 - \frac{1}{2^\alpha}\right)\eta_1^\alpha \tag{1.4$_a$}$$

and

$$\int_a^\infty \psi(t)\left(y_0 + \frac{1}{2}\eta_1 P(t)\right)^{-\mu} dt \leq \left(1 - \frac{1}{2^\beta}\right)\zeta_1^\beta. \tag{1.4$_b$}$$

Let $U$ denote the set of all vector functions $(y,z) \in C[a,\infty) \times C[a,\infty)$ such that

$$y_0 + \frac{1}{2}\eta_1 P(t) \leq y(t) \leq y_0 + \eta_1 P(t) \tag{1.5$_a$}$$

$$z_0 + \frac{1}{2}\zeta_1 Q(t) \leq z(t) \leq z_0 + \zeta_1 Q(t), \quad t \geq a. \tag{1.5$_b$}$$

Define the mapping $\mathcal{F} : U \to C[a,\infty) \times C[a,\infty)$ by

$$\mathcal{F}(y,z) = (\mathcal{G}z, \mathcal{H}y), \tag{1.6}$$

where $\mathcal{G}$ and $\mathcal{H}$ are the integral operators given by

$$\mathcal{G}z(t) = y_0 + \int_a^t \left[(p(s))^{-1}\left(\eta_1^\alpha - \int_s^\infty \varphi(r)(z(r))^{-\lambda} dr\right)\right]^{\frac{1}{\alpha}} ds, \tag{1.7$_a$}$$

$$\mathcal{H}y(t) = z_0 + \int_a^t \left[(q(s))^{-1}\left(\zeta_1^\beta - \int_s^\infty \psi(r)(y(r))^{-\mu} dr\right)\right]^{\frac{1}{\beta}} ds, \quad t \geq a. \tag{1.7$_b$}$$

It is easy to verify that $\mathcal{F}$ maps $U$ continuously into a relatively compact subset of $U$, and so, by the Schauder-Tychonoff fixed point theorem, there exists an element $(y,z) \in U$ such that $(y,z) = \mathcal{F}(y,z)$, that is,

$$y(t) = y_0 + \int_a^t \left[(p(s))^{-1}\left(\eta_1^\alpha - \int_s^\infty \varphi(r)(z(r))^{-\lambda} dr\right)\right]^{\frac{1}{\alpha}} ds, \tag{1.8$_a$}$$

$$z(t) = z_0 + \int_a^t \left[(q(s))^{-1}\left(\zeta_1^\beta - \int_s^\infty \psi(r)(y(r))^{-\mu} dr\right)\right]^{\frac{1}{\beta}} ds, \quad t \geq a. \tag{1.8$_b$}$$

Differentiating ($1.8_a$) and ($1.8_b$) twice, we conclude that the vector function $(y, z)$ is a positive solution of (A) defined on $[a, \infty)$ and satisfying

$$\lim_{t\to\infty} (p(t))^{\frac{1}{\alpha}} y'(t) = \eta_1 > 0 \quad \text{and} \quad \lim_{t\to\infty} (q(t))^{\frac{1}{\beta}} z'(t) = \zeta_1 > 0,$$

which ensures that $(y, z)$ is of the type (I). This completes the proof.

## 2. Strongly and semi-strongly increasing solutions

We first try to find necessary conditions for the existence of strongly and semi-strongly increasing solutions for the system (A).

Let $(y, z)$ be a strongly increasing solution $(y, z)$ on $[a, \infty)$. Since

$$\lim_{t\to\infty} (p(t))^{\frac{1}{\alpha}} y'(t) = \lim_{t\to\infty} (q(t))^{\frac{1}{\beta}} z'(t) = \infty,$$

letting $t \to \infty$ in ($1.2_a$) and ($1.2_b$), we obtain

$$\int_a^\infty \varphi(t)(z(t))^{-\lambda} dt = \int_a^\infty \psi(t)(y(t))^{-\mu} dt = \infty. \tag{2.1}$$

Combining (2.1) with the inequalities

$$y(t) \geq kP(t), \quad z(t) \geq lQ(t), \quad t \geq b$$

$b > a$, $k$ and $l$ being positive constants, we conclude that

$$\int_b^\infty \varphi(t)(Q(t))^{-\lambda} dt = \infty, \tag{2.2$_a$}$$

and

$$\int_b^\infty \psi(t)(P(t))^{-\mu} dt = \infty. \tag{2.2$_b$}$$

Let us turn to a semi-strongly increasing solution $(y, z)$ of the type (II):

$$\lim_{t\to\infty} (p(t))^{\frac{1}{\alpha}} y'(t) = \text{const} > 0, \quad \lim_{t\to\infty} (q(t))^{\frac{1}{\beta}} z'(t) = \infty.$$

We need the function $\Psi : [b, \infty) \to \mathbb{R}$, $b > a$, defined by

$$\Psi(t) = \int_b^t \left[ (q(s))^{-1} \int_b^s \psi(r)(P(r))^{-\mu} dr \right]^{\frac{1}{\beta}} ds, \quad t \geq b. \tag{2.3}$$

We claim that

$$\int_b^\infty \psi(t)(P(t))^{-\mu} dt = \infty \tag{2.4$_a$}$$

and

$$\int_c^\infty \varphi(t)(\Psi(t))^{-\lambda} dt < \infty, \quad c > b. \tag{2.4$_b$}$$

In fact, we have from ($1.2_a$) and ($1.2_b$)

$$\int_a^\infty \varphi(t)(z(t))^{-\lambda} dt < \infty, \quad \int_a^\infty \psi(t)(y(t))^{-\mu} dt = \infty. \tag{2.5}$$

The second inequality in (2.5) together with the inequality $y(t) \geq kP(t)$, $t \geq b$, holding for some $k > 0$ and $b > a$, implies that ($2.4_a$) is true. To derive ($2.4_b$), we integrate ($1.2_b$) to obtain

$$z(t) = z_0 + \int_a^t \left[ (q(s))^{-1} \left( z_1^\beta + \int_a^s \psi(r)(y(r))^{-\mu} dr \right) \right]^{\frac{1}{\beta}} ds, \quad t \geq a. \tag{2.6}$$

For any fixed $b$, $c$ with $c > b > a$, we obtain by L'Hospital's rule

$$\lim_{t\to\infty} \frac{\int_a^t \left[(q(s))^{-1}\left(z_1^\beta + \int_a^s \psi(r)(y(r))^{-\mu}dr\right)\right]^{\frac{1}{\beta}} ds}{\int_b^t \left[(q(s))^{-1}\int_b^s \psi(r)(y(r))^{-\mu}dr\right]^{\frac{1}{\beta}} ds} = 1,$$

$$\lim_{t\to\infty} \frac{\int_b^t \left[(q(s))^{-1}\int_b^s \psi(r)(y(r))^{-\mu}dr\right]^{\frac{1}{\beta}} ds}{\int_b^t \left[(q(s))^{-1}\int_b^s \psi(r)(P(r))^{-\mu}dr\right]^{\frac{1}{\beta}} ds} = \kappa^{-\frac{\mu}{\beta}}$$

where $\kappa = \lim_{t\to\infty} y(t)/P(t) > 0$. It follows therefore from (2.6) that

$$z(t) \le m\Psi(t), \qquad t \ge c, \tag{2.7}$$

for some constant $m > 0$. Using (2.7) in the first inequality in (2.5), we conclude that ($2.4_b$) holds true as claimed.

A similar argument applies to a semi-strongly increasing solution $(y, z)$ of the type (III), leading to the conclusion that

$$\int_b^\infty \varphi(t)(Q(t))^{-\lambda}dt = \infty \tag{$2.8_a$}$$

and

$$\int_c^\infty \psi(t)(\Phi(t))^{-\mu}dt < \infty, \qquad c > b, \tag{$2.8_b$}$$

where

$$\Phi(t) = \int_b^t \left[(p(s))^{-1}\int_b^s \varphi(r)(Q(r))^{-\lambda}dr\right]^{\frac{1}{\alpha}} ds, \qquad t \ge b. \tag{2.9}$$

Our next task is to derive sharp sufficient conditions for the existence of strongly and semi-strongly increasing solutions of (A). This, however, seems to be difficult to attain, and we are content to give simple conditions under which (A) actually possesses the three types of increasing solutions in question.

**THEOREM 2.** *Suppose that* (0.1) *holds. If* ($1.1_a$) *and* ($2.4_a$) *are satisfied, then the system* (A) *has positive increasing solutions of the type* (II). *In fact, in this case all positive increasing solutions of* (A) *are of the type* (II).

**THEOREM 3.** *Suppose that* (0.1) *holds. If* ($1.1_b$) *and* ($2.8_a$) *are satisfied, then the system* (A) *has positive increasing solutions of the type* (III). *In fact, in this case all positive increasing solutions of* (A) *are of the type* (III).

The proof of Theorem 2 and 3 be omitted.

**THEOREM 4.** *Suppose that* (0.1) *holds. The system* (A) *has positive increasing solutions of the type* (IV) *if in addition to* ($2.2_a$) *and* ($2.2_b$) *the following conditions are satisfied:*

$$\int_c^\infty \varphi(t)(\Psi(t))^{-\lambda}dt = \infty \tag{$2.10_a$}$$

$$\int_c^\infty \psi(t)(\Phi(t))^{-\mu}dt = \infty, \tag{$2.10_b$}$$

*for any $c > b$ and $\Psi(t)$* and *$\Phi(t)$ are defined by* (2.3) *and* (2.9), *respectively. In this case all positive solutions of* (A) *are of the type* (IV).

Proof. Let $(y, z)$ be a positive increasing solution of (A). The case that $(y, z)$ is of the type (I) is excluded by Theorem 1. That $(y, z)$ can be neither of the types (II) and (III) follows from the fact that $(2.10_a)$ and $(2.10_b)$ are inconsistent with $(2.4_b)$ and $(2.8_b)$ which are necessary conditions for the existence of solutions of these types. It follows that $(y, z)$ must be a type-(IV) solution of (A). This completes the proof.

Remark. A question arises as to how fast a strongly increasing positive solution of (A) grows as $t \to \infty$. Let $(y, z)$ be one such solution. The procedure of deriving (2.7) for the second component $z$ of a semi-strongly increasing solution also applies to the first component $y$, implying that

$$y(t) \le n\Phi(t) \quad \text{and} \quad z(t) \le m\Psi(t), \quad t \ge c, \tag{2.11}$$

for some positive constants $m$ and $n$. Using (2.11) in

$$y(t) \ge \int_a^t \left[(p(s))^{-1} \int_a^s \varphi(r)(z(r))^{-\lambda} dr\right]^{\frac{1}{\alpha}} ds,$$

$$z(t) \ge \int_a^t \left[(q(s))^{-1} \int_a^s \psi(r)(y(r))^{-\mu} dr\right]^{\frac{1}{\beta}} ds, \quad t \ge a,$$

(see (2.6)), we obtain

$$y(t) \ge m^{-\frac{\lambda}{\alpha}} \int_c^t \left[(p(s))^{-1} \int_c^s \varphi(r)(\Psi(r))^{-\lambda} dr\right]^{\frac{1}{\alpha}} ds, \tag{2.12$_a$}$$

$$z(t) \ge n^{-\frac{\mu}{\beta}} \int_c^t \left[(q(s))^{-1} \int_c^s \psi(r)(\Phi(r))^{-\mu} dr\right]^{\frac{1}{\beta}} ds, \quad t \ge c. \tag{2.12$_b$}$$

The inequalities (2.11), $(2.12_a)$ and $(2.12_b)$ provide estimates for the growth order of $y$ and $z$ as $t \to \infty$.

## 3. Examples

**EXAMPLE 1.** Consider the differential system

$$\begin{cases} (e^{-\alpha t}|y'|^{\alpha-1}y')' = ke^{\gamma t}z^{-\lambda} \\ (e^{-\beta t}|z'|^{\beta-1}z')' = le^{\delta t}y^{-\mu}, \quad t \ge 0, \end{cases} \tag{3.1}$$

where $\alpha$, $\beta$, $\lambda$ and $\mu$ are as in (A), and $k > 0$, $l > 0$, $\gamma$ and $\delta$ are constants. The functions $P(t)$ and $Q(t)$ defined by (0.5) can be taken to be $P(t) = Q(t) = e^t$. Since

$$(1.1_a) \Longleftrightarrow \gamma < \lambda \quad \text{and} \quad (1.1_b) \Longleftrightarrow \delta < \mu,$$

we see from Theorem 1 that all positive increasing solutions $(y, z)$ of (3.1) satisfy

$$\lim_{t\to\infty} e^{-t}y(t) = \text{const} > 0, \quad \lim_{t\to\infty} e^{-t}z(t) = \text{const} > 0 \tag{3.2}$$

if $\gamma < \lambda$ and $\delta < \mu$. Theorems 2 and 3 imply that all positive increasing solutions $(y, z)$ of (3.1) have the property that

$$\lim_{t\to\infty} e^{-t}y(t) = \text{const} > 0, \quad \lim_{t\to\infty} e^{-t}z(t) = \infty \tag{3.3}$$

or

$$\lim_{t\to\infty} e^{-t}y(t) = \infty, \quad \lim_{t\to\infty} e^{-t}z(t) = \text{const} > 0 \tag{3.4}$$

according to whether $\{\gamma < \lambda$ and $\delta \geq \mu\}$ or $\{\gamma \geq \lambda$ and $\delta < \mu\}$.

A simple computation shows that the function $\Phi(t)$ defined by (2.9) is asymptotic as $t \to \infty$ to a positive constant multiple of

$$e^{\frac{\alpha+\gamma-\lambda}{\alpha}t} \quad \text{if} \quad \gamma > \lambda \quad \text{or} \quad t^{\frac{1}{\alpha}} e^t \quad \text{if} \quad \gamma = \lambda,$$

and that the function $\Psi(t)$ defined by (2.3) is asymptotic as $t \to \infty$ to a positive constant multiple of

$$e^{\frac{\beta+\delta-\mu}{\beta}t} \quad \text{if} \quad \delta > \mu \quad \text{or} \quad t^{\frac{1}{\beta}} e^t \quad \text{if} \quad \delta = \mu.$$

These results can be used to examine the validity of the conditions ($2.10_a$) and ($2.10_b$), and as a result it is shown that all positive increasing solutions $(y, z)$ of the system (3.1) satisfy

$$\lim_{t\to\infty} e^{-t} y(t) = \lim_{t\to\infty} e^{-t} z(t) = \infty \tag{3.5}$$

if one of the following sets of conditions holds:

$$\{\gamma > \lambda, \quad \delta > \mu, \quad \alpha\beta \geq \lambda\mu\},$$
$$\{\gamma = \lambda, \quad \delta = \mu, \quad \alpha \geq \mu, \ \beta \geq \lambda\}.$$

Let us now consider the system

$$\begin{cases} (e^{-\alpha t}|y'|^{\alpha-1}y')' = \alpha 2^\alpha e^{(\alpha+2\lambda)t} z^{-\lambda} \\ (e^{-\beta t}|z'|^{\beta-1}z')' = \beta 2^\beta e^{(\beta+2\mu)t} y^{-\mu}, \quad t \geq 0. \end{cases} \tag{3.6}$$

Since (3.6) is a special case of (3.1) with

$$k = \alpha 2^\alpha, \quad l = \beta 2^\beta, \quad \gamma = \alpha + 2\lambda \ \text{ and } \ \delta = \beta + 2\mu,$$

we see from the above result that all of its positive increasing solutions $(y, z)$ are strongly increasing, that is, satisfy (3.5) provided $\alpha\beta \geq \lambda\mu$. A concrete example of such solutions is $(y, z) = (e^{2t}, e^{2t})$, which satisfies (3.6) for any positive values of $\alpha$, $\beta$, $\lambda$ and $\mu$. A natural question arises: In case $\alpha\beta < \lambda\mu$ does (3.6) have semi-strongly increasing solutions satisfying (3.3) and/or (3.4)? It would be of interest to develop general theorems on the coexistence of strongly and semi-strongly increasing solutions for the system (A).

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