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Valuing Convertible Bonds with Reset Clauses via Monte Carlo Simulation*

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1 Introduction

Convertible bonds or more generally equity-linked securities have greatly evolved in the past decade. Convertible bonds are hybrid securities issued by a firm where the holder has the right to convert the bond into the common stocks of the firm according to pre-specified conditions. An essential feature of ordinary convertible bonds is that they may be converted at any time until a pre-specified maturity date into stocks at a pre-specified ratio, *i.e.*, a fixed *conversion ratio*. In recent years, however, various convertible bonds have been issued with additional conversion provisions. Among others, some Japanese bank convertibles have a *reset* clause whereby the conversion ratio is adjusted upwards, or equivalently, the conversion price adjusted downwards if the underlying stock price does not exceed pre-specified trigger prices. For investors, it has a benefit of resuscitating convertible bond prices, while for the issuer it has another benefit of prompting conversions. Hence, convertible bonds with this provision are usually issued when the outlook for the issuer is unfavorable. In fact, reset convertibles first emerged in the Japanese market after the 1996 deregulation [1]. This paper analyzes some features of a convertible bond with the reset clause via both analytic and Monte Carlo simulation approaches.

Another typical provision of convertible bonds is a *call* provision such that the issuer has the right of redemption prior to maturity. This call provision is usually subject to some kind of restriction, *e.g.*, an initial non-redemption period of 3 years for a traditional convertible debt with 5, 7 or 10 years maturity [2, pp. 1132–1133]. For the callable convertible bond, Ingersoll [3] developed a one-factor pricing model based on the value of the firm, so that he obtained optimal call and conversion strategies; see Brennan and Schwartz [4] for a general pricing algorithm. See also Brennan and Schwartz [5] for a two-factor model with interest rates as the second factor. The call provision has often been added to Japanese convertible bonds. From actual data in the market, however, the provision would not in fact be exercised under ordinary circumstances, and there are only a few cases that the bond is called [1]. It may worth noting that most recent mandatorily convertibles are non-callable [2, p. 1128]. Hence, we only deal with non-callable convertible bonds in this paper.

Assume that the underlying stock receives no dividends and that the risk-free rate of interest is constant during the period up to maturity. These assumptions are partly

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justified in Japan, because dividend yields on Japanese equity is traditionally very low and the interest rates on Japanese Government debt denominated in yen has been particularly low as well as less volatile since 1998. If the underlying stock has no credit risk of the issuer, no conversion occurs prior to maturity under these assumptions, *i.e.*, conversion may occur only at maturity.

The price of any convertible bonds can be *approximately* viewed as a sum of values of an otherwise identical non-convertible bond plus an embedded option to convert the bond into the underlying stock:

convertible bond price (V_{CB}) = straight bond value (V_{SB}) + conversion option value (V_{CO}).

In general, these two components interact with each other and so prove to difficult to separate. However, in some situations, the embedded option can be separated and easily valued. A separable case is, for example, that the convertible bond is non-callable and non-convertible until maturity. As a basic framework for pricing, we use the bond plus option valuation whether or not the underlying stock has credit risk of the issuer. We principally focus on the price of conversion option, which is essential in analyzing the price of convertible bonds under the constant interest-rate assumption.

This paper is organized as follows: First, we consider the case that the issuer has no credit risk, for which the conversion option must necessarily be the European type. In Section 2, we develop an exact formula for the conversion option value of the credit-riskless European convertible bond with the reset clause in the classical one-factor Black-Scholes-Merton framework. In Section 3, we show in the framework of Monte Calro simulation that conversion option value estimates of the American credit-risky convertible bond with the reset clause are located in a certain region defined by this formula. From estimates of the conversion probability, we also show that there exists in the latter half of the trading interval an optimal reset time for both investors and the issuer. Finally, in Section 4, we provide a few concluding remarks.

2 Exact Analysis of Credit-Riskless Convertible Bonds

For a class of non-callable convertible bonds with the reset clause, we first consider a special case such that the issuer has no credit risk. Under the assumptions of no dividends on the underlying stock and the flat term structure of the risk-free interest rate, no conversion occurs prior to maturity, *i.e.*, the conversion option must necessarily be the European type. Hence, we can use the economic framework of the contingent claims analysis pioneered by Black and Scholes [6] and Merton [7] for valuing the conversion option: Assume that the capital market is well-defined and follows the efficient market hypothesis. Let S_t denote the underlying stock price at time t and assume a geometric Brownian motion model

$$dS_t = S_t(r\,dt + \sigma\,dW_t), \qquad 0 \le t \le T. \tag{1}$$

The interest rate r, the volatility σ and the maturity T of the convertible bond are assumed to be positive constants. The process $W \equiv \{W_t;\ 0 \le t \le T\}$ is the standard Brownian motion process under a probability measure $\mathbb{P}$ which is *risk-neutral*, *i.e.*, is chosen so that the stock has mean rate of return r. In addition, let $\{\mathcal{F}_t;\ t \geq 0\}$ be the natural filtration corresponding to W.

Let K (> 0) be the *original* conversion price, and let τ $(\in (0, T))$ be the reset time. Then, the *actual* conversion price of the reset convertible is changed to aS_τ if $S_\tau < K/a$ for a pre-specified constant $a \geq 1$, or it remains K otherwise. This implies that the conversion price is adjusted downwards from K to aS_τ when $S_\tau < K/a$. In other words, the conversion ratio is adjusted upwards from B/K to $B/(aS_\tau)$ when $S_\tau < K/a$, where B is the par value of the convertible bond.

Assume that the convertible bond receives a coupon of amount C (> 0) at time T_i $(i = 1, \dots, n)$ where $0 < T_1 < T_2 < \cdots < T_n \leq T$. For the straight bond value V_{SB}, we immediately have

$$V_{\mathrm{SB}} = C \sum_{i=1}^{n} e^{-rT_i} + Be^{-rT}. \tag{2}$$

On the other hand, for the conversion option price V_{CO}, we have

Theorem 1 *Let V_{CO} be the conversion option value of the credit-riskless, non-callable, convertible bond with the reset clause at time $t = 0$. Then,*

$$\begin{aligned} V_{\mathrm{CO}} = {} & S_0\Big(\Phi(d_1^+) - ae^{-r(T-\tau)}\Phi(d_1^-)\Big)\Phi(-d_0^+) \\ & + \int_{\ln(K/aS_0)}^{\infty} \Big(S_0 e^{-r\tau}\Phi(d_2^+(y))e^y - Ke^{-rT}\Phi(d_2^-(y))\Big)\psi(y)dy, \end{aligned} \tag{3}$$

where $\Phi(\,\cdot\,)$ is the cdf of the standard normal distribution, i.e., *for $x \in \mathbb{R}$*

$$\Phi(x) = \int_{-\infty}^{x} \phi(y)dy \quad \textit{with} \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}},$$

$$\psi(y) = \frac{1}{\sigma\sqrt{\tau}}\, \phi\left(\frac{y - (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}\right),$$

and the parameters d_0^+, $d_1^\pm$ and $d_2^\pm(y)$ are defined by

$$\begin{aligned} d_0^+ &= \frac{\ln(aS_0/K) + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}, \\ d_1^\pm &= \frac{-\ln a + (r \pm \frac{1}{2}\sigma^2)(T-\tau)}{\sigma\sqrt{T-\tau}}, \\ d_2^\pm(y) &= \frac{y + \ln(S_0/K) + (r \pm \frac{1}{2}\sigma^2)(T-\tau)}{\sigma\sqrt{T-\tau}}. \end{aligned}$$

Proof. From the definition of the reset clause and the risk-neutral pricing theory, we obtain

$$V_{\mathrm{CO}} = e^{-rT}\mathbb{E}\left[(S_T - aS_\tau)^+ \mathbf{1}_{\{S_\tau < K/a\}} + (S_T - K)^+ \mathbf{1}_{\{S_\tau \geq K/a\}} \middle| \mathcal{F}_0\right]. \tag{4}$$

For the ease of exposition, we change the variable S_τ into Y_τ by $S_\tau = S_0 e^{Y_\tau}$. Applying Itô's lemma to (1), we obtain

$$d\log S_t = \left(r - \tfrac{1}{2}\sigma^2\right)dt + \sigma dW_t,$$

from which

$$\log S_\tau - \log S_0 = \left(r - \tfrac{1}{2}\sigma^2\right)\tau + \sigma W_\tau.$$

Hence, Y_τ can be written as

$$Y_\tau = (r - \tfrac{1}{2}\sigma^2)\tau + \sigma W_\tau.$$

Clearly, $Y_\tau \sim N((r - \frac{1}{2}\sigma^2)\tau,\ \sigma^2\tau)$ and its pdf is given by $\psi(\cdot)$. Then, the tower property of conditional expectations yields

$$\begin{aligned}
V_{\text{CO}} &= e^{-rT}\mathbb{E}\left[\mathbb{E}\left[(S_T - aS_0e^{Y_\tau})^+\mathbf{1}_{\{Y_\tau<\ln(K/aS_0)\}} + (S_T-K)^+\mathbf{1}_{\{Y_\tau\geq\ln(K/aS_0)\}}\middle|\,\mathcal{F}_\tau\right]\middle|\,\mathcal{F}_0\right] \\
&= e^{-rT}\int_{-\infty}^{\ln(K/aS_0)} \mathbb{E}\left[(S_T - aS_0e^y)^+|\,S_\tau\right]\psi(y)dy \\
&\qquad + e^{-rT}\int_{\ln(K/aS_0)}^{\infty} \mathbb{E}\left[(S_T - K)^+|\,S_\tau\right]\psi(y)dy \\
&= e^{-r\tau}\left(\int_{-\infty}^{\ln(K/aS_0)} C_{\text{BS}}(S_0e^y, T, aS_0e^y)\psi(y)dy + \int_{\ln(K/aS_0)}^{\infty} C_{\text{BS}}(S_0e^y, T, K)\psi(y)dy\right),
\end{aligned}$$

where $C_{\text{BS}}(S_t, T, K)$ is the Black-Scholes pricing formula for the European call option at time t with the exercise price K and the maturity T, *i.e.*,

$$C_{\text{BS}}(S_t, T, K) \equiv e^{-r(T-t)}\mathbb{E}\left[(S_T - K)^+|\,S_t\right] = S_t\Phi(d^+) - Ke^{-r(T-t)}\Phi(d^-),$$

with

$$d^\pm = \frac{\ln(S_t/K) + (r \pm \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}.$$

Hence,

$$\begin{aligned}
V_{\text{CO}} &= S_0\left(\Phi(d_1^+) - ae^{-r(T-\tau)}\Phi(d_1^-)\right)\int_{-\infty}^{\ln(K/aS_0)} e^{y-r\tau}\psi(y)dy \\
&\quad + \int_{\ln(K/aS_0)}^{\infty}\left(S_0e^{-r\tau}\Phi(d_2^+(y))e^y - Ke^{-rT}\Phi(d_2^-(y))\right)\psi(y)dy,
\end{aligned}$$

which leads to (3). □

Remark 1 For two special cases when $\tau = 0$ and $\tau = T$, we can easily obtain explicit forms of V_{CO}: From (4),

$$\lim_{\tau\to 0} V_{\text{CO}} = \begin{cases} C_{\text{BS}}(S_0, T, aS_0), & S_0 < K/a \\ C_{\text{BS}}(S_0, T, K), & S_0 \geq K/a, \end{cases}$$

and

$$\lim_{\tau\to T} V_{\text{CO}} = C_{\text{BS}}(S_0, T, K).$$

Remark 2 To compute the conversion option value V_{CO}, it is necessary to execute a numerical integration on the infinite interval $[\ln(K/aS_0), \infty)$. This integration is, however, reduced to an approximate integration on a bounded interval as follows: For sufficiently small $\varepsilon > 0$, define

$$M_\varepsilon = \inf\left\{y \;\middle|\; \min\{\Phi(d_2^+(y)), \Phi(d_2^-(y))\} > 1 - \varepsilon\right\}.$$

The the second term in the right-hand side of (3) can be approximated by

$$\int_{\ln(K/aS_0)}^{M_\varepsilon} \Big(S_0 e^{-r\tau}\Phi(d_2^+(y))e^y - Ke^{-rT}\Phi(d_2^-(y))\Big)\psi(y)dy + S_0\Phi(d_3^+) - Ke^{-rT}\Phi(d_3^-), \quad (5)$$

where $d_3^\pm = (-M_\varepsilon + r \pm \frac{1}{2}\sigma^2)/\sigma\sqrt{\tau}$. This approximation can be easily implemented by using mathematical softwares, producing accurate values quickly for practical purposes.

Figure 1 illustrates values of the conversion option embedded in a convertible bond with the reset clause as a function of the reset time τ, where $a = 1$, $T = 5$ and $K = 1000, 1050, 1100$. Assume $S_0 = 1000$ and $\sigma = 0.3$ for the underlying stock, and also assume $r = 0.02$ for the risk-free interest rate. To compute these values, we used the truncation approximation (5) with $M_\varepsilon = 4.8$ for which $\varepsilon < 10^{-6}$ holds. The two extreme values when $\tau = 0$ and $\tau = T$ were directly computed by using the Black-Scholes formula. From Figure 1, we see that the maximum value of V_{CO} is attained when the reset time is in the center of the trading interval if $S_0 = K$. The lower the conversion ratio, the earlier the reset time that gives the maximum value of V_{CO}. Also, we see that the lower the conversion ratio, the more rapidly the value of V_{CO} decreases as τ tends to T.

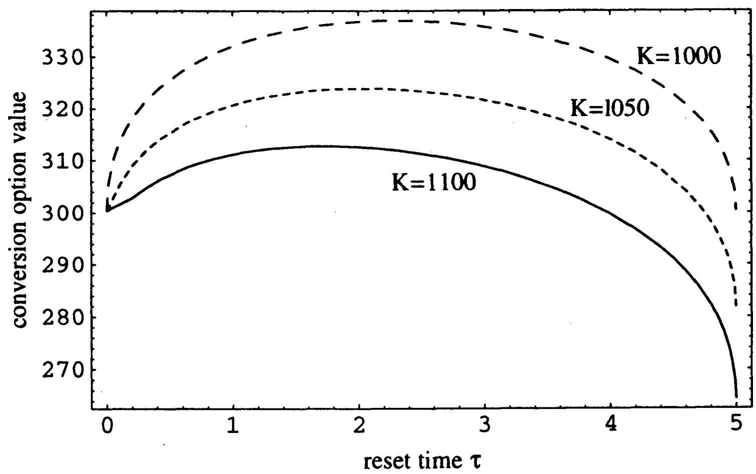

Figure 1: Conversion Option Values of Credit-Riskless Reset Convertible Bonds

3 Simulation Analysis of Credit-Risky Convertible Bonds

As shown in Hull [8, pp. 634–635], the impact of default risk on contracts where the holder has early exercise decisions is a little bit tricky: It is well known that a riskless American call option on a stock with no dividends would *not* be exercised early. However, if the underlying stock has default risk, the holder might choose to exercise the option prior to maturity rather than hold. The same motivation of early exercise can be found in credit-risky convertible bonds. The higher the underlying stock, the more in-the-money convertible bonds, and hence the payoff from conversion exercise becomes greater than the present value of the expected value of holding, due to credit risk of the issuer.

We use Monte Carlo simulation to analyze features of the credit-risky and non-callable convertible bond with the reset clause, since its analysis is quite difficult. Various approaches have been developed to account for credit risk in valuing convertible bonds without reset clauses; see Nyborg [9] for a review. Most of these approaches commonly used the total value of the firm as a stochastic factor in their models. While such approaches are self-consistent, they contain many parameters and can be impractical. As an approach similar to this paper, Tsiveriotis and Fernandes [10] used a single-factor model based on stock price and numerically solved a system of two coupled Black-Scholes equations that govern the convertible bond price and its *cash-only part*, respectively.

The first step in Monte Carlo simulation is to generate sample paths of the process $S \equiv \{S_t;\, 0 \le t \le T\}$. Let N be the total number of sample paths and let M be the number of time steps in the following discrete-time version of (1):

$$S_{ij} = S_{i,j-1}(1 + r\Delta t + \sigma\sqrt{\Delta t}\xi_{ij}), \qquad j = 1, \dots, M, \quad i = 1, \dots, N, \tag{6}$$

where $\Delta t = T/M$ and S_{ij} is the simulated stock price at time $t_j \equiv j\Delta t$ ($j = 0, \dots, M$) in the i-th sample path starting from $S_{i0} = S_0$ ($i = 1, \dots, N$). The variables $\{\xi_{ij}\}$ are iid standard normal random numbers. As a credit-risk dynamics, assume that defaults may occur depending on the stock price at that time. More specifically, assume that the issuer defaults during the time interval $[t_{j-1}, t_j)$ with probability $\lambda(S_{i,j-1})\Delta t$ given that it survives until time $t_{j-1} < T$, where $\lambda(S_t) \ge 0$ is the instantaneous default rate. For the case that $\lambda(\cdot)$ is independent of S, this assumption clearly implies that the time of default, say D, is exponentially distributed with parameter λ, *i.e.*, $\mathbb{P}\{D > t\} = e^{-\lambda t}$ for $t \ge 0$. Consequently, a procedure of generating sample paths of the process S in our simulations can be summarized as follows:

$S_{ij} := 0$ for $j = 1, \dots, M$, $i = 1, \dots, N$. $\Delta t := T/M$. $i := 1$.
While $i \le N$:
  $S_{i0} := S_0$. $j := 1$.
  While $j \le M$:
    Generate ν_{ij} randomly from $U(0, 1)$.
    If $\nu_{ij} > \lambda(S_{i,j-1})\Delta t$:
      Generate ξ_{ij} randomly from $N(0, 1)$.
      $S_{ij} := S_{i,j-1}(1 + r\Delta t + \sigma\sqrt{\Delta t}\xi_{ij})$.
    Else: $j := M$.
    $j := j + 1$.
  $i := i + 1$.
Return (S_{ij} for $j = 0, \dots, M$, $i = 1, \dots, N$).

Note that the stock price is defined as zero when the default occurs, since we primarily focus on either the conversion option value or the conversion probability.

To compute the conversion option value, we adopt the Grant-Vora-Week (GVW) method [11], which is originally developed for American put options. The GVW method

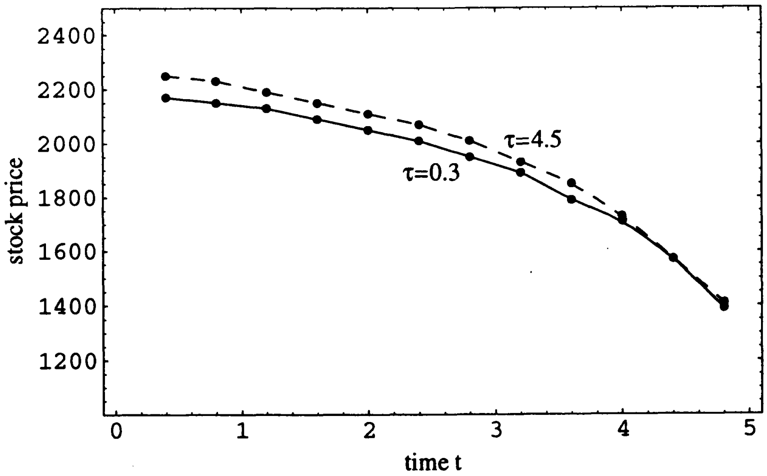

Figure 2: Locus of Critical Prices of Credit-Risky Reset Convertible Bonds

consists of two procedures for obtaining (i) the *locus of critical prices* or *optimal exercise boundary* $S^*(t)$ $(0 \leq t \leq T)$ at which the payoff upon exercise is equal to the discounted expected value of holding, and (ii) the option value. The first procedure is done by backward-moving recursions of dynamic programming, and the second one by forward-moving simulations terminated by the stopping time

$$\theta = \inf_{0 \leq t \leq T} \left\{ t \;\middle|\; S_t \geq S^*(t) \right\}.$$

The second procedure in the modified GVW method for valuing the conversion option in the reset convertible bond corresponds to computing the expected value

$$\begin{aligned} &\mathbb{E}\left[e^{-r\theta}\left\{(S_\theta - aS_\tau)^+ \mathbf{1}_{\{S_\tau < K/a\}} + (S_\theta - K)^+ \mathbf{1}_{\{S_\tau \geq K/a\}}\right\} \mathbf{1}_{\{\theta \leq T\}} \middle| \mathcal{F}_0\right] \\ &\quad + e^{-rT}\mathbb{E}\left[\left\{(S_T - aS_\tau)^+ \mathbf{1}_{\{S_\tau < K/a\}} + (S_T - K)^+ \mathbf{1}_{\{S_\tau \geq K/a\}}\right\} \mathbf{1}_{\{\theta > T\}} \middle| \mathcal{F}_0\right]. \end{aligned}$$

Figures 2 and 3 respectively illustrate simulation results for the locus of critical prices and the conversion option value embedded in a risky convertible bond with the reset clause, where $a = 1$, $T = 5$ and $K = 1100$. Assume $S_0 = 1000$ and $\sigma = 0.3$ for the underlying stock, and $r = 0.02$ for the risk-free interest rate. We used the constant default rate satisfying $\mathbb{P}\{D > T\} = e^{-\lambda T} = 0.93$, *i.e.*, $\lambda = -\ln(0.93)/5 \approx 0.0145$. To compute the locus of critical prices in the modified GVW method, we used $M = 50$ and $N = 500,000$, together with a standard antithetic variance reduction technique of coupling a pair of sample paths generated by (6) and

$$S'_{ij} = S'_{i,j-1}(1 + r\Delta t - \sigma\sqrt{\Delta t}\xi_{ij}), \qquad j = 1, \ldots, M, \quad i = 1, \ldots, N.$$

For this locus of critical prices, we replicated the run 10 times, where we generated $N = 500,000$ sample paths in each run, using a different random number seed each time. Taking arithmetic averages over these runs, we finally obtained the mean and 99% confidence interval of the conversion option value. The results are, however, simply marked by bullets in Figure 3, because the width of each confidence interval is so narrow that it cannot be distinguished from the point estimate. Together with simulation results,

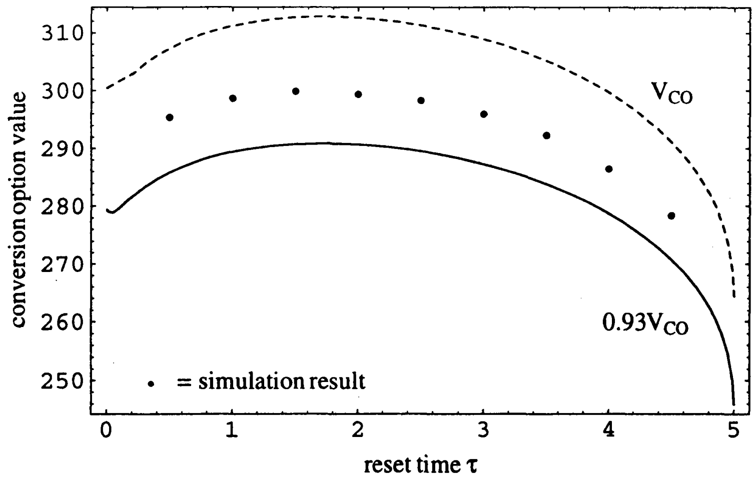

Figure 3: Conversion Option Values of Credit-Risky Reset Convertible Bonds

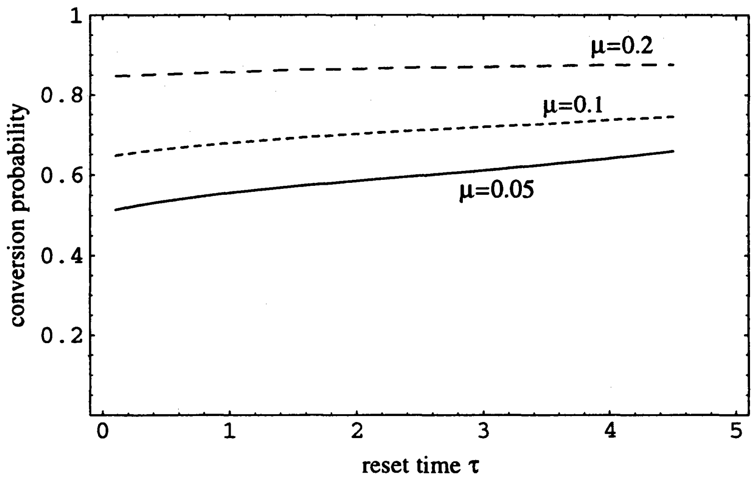

Figure 4: Conversion Probabilities of Credit-Risky Reset Convertible Bonds

Figure 3 also shows two curves indicating $V_{\rm CO}$ and $\mathbb{P}\{D > T\}V_{\rm CO}$ for the credit-riskless convertible bond with the same parameters.

Figure 2 shows that the locus $S^*(t)$ is a concave and decreasing function of t, and it is almost insensitive to τ, in particular, when the reset time is close to maturity. We see from Figure 3 that the simulation results are located between these two curves, which certainly justifies the theoretical result that the proportional impact of default risk on the price of an American option is less than that for a similar European option; see Hull [8, p. 635]. As in the riskless cases, the bond-holder can expect high returns when the reset time is in the former half of the trading interval. In addition, Figure 3 indicates that a certain approximation for $V_{\rm CO}$ of the credit-risky convertible bond could be developed by combining those of the credit-riskless one.

Figure 4 illustrates the conversion probabilities of a risky convertible bond with the reset clause as a function of the reset time τ, by smoothing simulation results. To compute the conversion probabilities, we used $N = 500,000$ sample paths and the antithetic variance reduction technique. Taking account for the risk premium of the stock, we assumed instead of (1) that the process S is governed by the stochastic differential equation

$$dS_t = S_t(\mu dt + \sigma d\hat{W}_t), \qquad 0 \leq t \leq T,$$

where μ ($> r$) is the return rate of the underlying stock and $\hat{W} \equiv \{\hat{W}_t;\ 0 \leq t \leq T\}$ is a $\hat{\mathbb{P}}$-Brownian motion for a probability measure $\hat{\mathbb{P}}$ in the real world. All the conditions and parameters except for r and μ are the same as in the simulations used for illustrating Figure 3. From Figure 4, we see that the conversion probability is a non-decreasing function of τ, *i.e.*, the later the reset time, the higher the conversion probability. This immediately means that the best reset time for the issuer is $\tau = T$. Combining this fact with the preference of the holder, we can conclude that there exists in the latter half of the trading interval an optimal reset time for both investors and the issuer. The curves of conversion probabilities are relatively flat with respect to τ, which certainly reflects the insensitivity of the critical prices shown in Figure 2.

Remark 3 We have also done Monte Carlo simulations for credit-riskless reset convertible bonds, in order to obtain the conversion probability. Observing simulation results carefully, we found that sample paths with no experience of reset are certainly higher than the conversion price when the reset time is close to maturity. On the other hand, sample paths with experience of reset are, roughly speaking, either higher or lower than the conversion price with equal probability $\frac{1}{2}$ during the period until maturity. Hence, if we let $\pi(\tau)$ be the conversion probability of the convertible bond with reset time τ and let π_0 be that without reset clause, then we can propose for credit-riskless reset convertible bonds

$$\lim_{\tau \to T} \pi(\tau) \approx \pi_0 + \tfrac{1}{2}(1 - \pi_0) \tag{7}$$

as a heuristic approximation. In fact, when $a = 1$, $T = 1$, $S_0 = K = 1000$, $r = 0.01$, $\sigma = 0.3$ and $\mu = 0.05$, the simulation result is $\pi_0 = 0.455$ and hence $\pi_0 + \frac{1}{2}(1 - \pi_0) = 0.7275$, while $\pi(0.999) = 0.724$ for which (7) almost holds.

4 Concluding Remarks

Since convertible bonds are complex securities with several embedded options, there exists in their modelling a trade-off between incremental gain in accuracy and computational complexity. The model analyzed in this paper is relatively simple so that it does not cover some important features of non-callable convertible bonds with reset clauses traded in the actual market.

As noted in Section 1, the reset clause has the benefits for both investors and the issuer. However, it also exposes holders of the underlying stock to the risk of diluting the value of their stocks, since the upward revision of the conversion ratio will cause the increase of the number of latent stocks in the firm. The higher the conversion ratio, the lower the stock price. As a future subject, it is important to take account of this *dilution effect* into modelling in a simple way.

As another subject for future studies, we need to consider a two-factor model when the issuer has credit risk: There exists a credit spread, due to credit risk of the issuer. The lower the credit quality of the issuer, the higher the default probability, and hence the higher the credit spread. In this sense, spreads and interest rates have the same impact on convertible bonds. Even if interest rates have the flat term structure as assumed in this paper, credit spreads bundled with interest rates may be assumed to have a term structure. To obtain accurate prices for risky convertible bonds, it is necessary to develop a two-factor model, which has stock returns as one factor and interest rates plus spreads as the other factor; cf. the two-factor Brennan-Schwartz model [5] with the value of the firm as one factor and interest rates as the other factor. Note that this two-factor model should be applied only to evaluating the straight bond value in risky convertible bonds, because the risk-neutral pricing approach is independent of the credit risk; see Tsiveriotis and Fernandes [10].

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