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Inefficiency of Incomplete Markets: Nominal Asset Case

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Since Hart(1975) pointed out that an equilibrium allocation with incomplete markets needs not be Pareto efficient, many authors have discussed the issue of inefficiency of equilibria with incomplete markets from various points of view.

In all those works, however, incomplete asset markets to be considered are exclusively limited to the real asset markets. In this paper, we consider efficiency of equilibrium allocations in the incomplete markets model with *nominal* assets and a single consumption good.

As a result, we show Pareto inefficiency of equilibrium allocations generically both in all agents' utility functions without any convexity and their endowments.

1 Introduction

Since Hart[12] pointed out that an equilibrium allocation with incomplete markets needs not be Pareto efficient, many authors have discussed the issue of inefficiency of equilibria with incomplete markets from various points of view. To mention a few, Grossman[10], Diamond[5], Stiglitz[15], Geanakoplos and Polemarchakis[9], Geanakoplos, Magill, Quinzii and Drèze[7], Werner[19], Kajii[13], Citanna, Kajii and Villanacci[4], Magill and Quinzii[14] and so on.

In those works, however, incomplete asset markets to be considered are exclusively restricted to the real asset markets. As is well known, the assets are conceptually classified into two groups, that is, real assets and nominal assets. It is noteworthy about these two kinds of assets that the structure of the set of equilibrium allocations is very different among them. In a real asset model the equilibrium set is shown to be generically finite(Duffie and Shafer[6]) whereas in a nominal asset model there generically exists real indeterminacy of equilibria, which means that the set of equilibrium allocations constitutes a continuum(Cass[2][3], Werner[18]).

In this paper, we consider efficiency of equilibrium allocations in the incomplete markets model with *nominal* assets and a single consumption good. We investigate this issue from

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a genericity viewpoint with respect to all agents' convexity-free utility functions and their endowments. In contrast to a real asset model with a single consumption good, in a nominal asset model the spot prices to be necessarily considered form an obstacle to our efficiency argument. To cope with this difficulty, we can make use of Thom transversality theorem in $1-jet$ space and the concept of fibration so as to deduce some significant consequences about efficiency of the equilibrium allocations with incomplete nominal asset markets.

In section 2 we describe the model to be considered. In section 3, by using the idea of fibration and applying Thom Transversality Theorem in $1-jet$ space, we obtain two outcomes about efficiency of equilibrium allocations in a nominal asset model. One concerns generic inefficiency of the equilibrium allocations. The other one centers on the cause of inefficiency. In the final section we address the relation between our result and the property of the nominal asset prices (endogenous or exogenous).

2 The Model

We consider efficiency of equilibria in an exchange economy model in which there are two periods, with uncertainty in the second, and there exist nominal assets and a single consumption good. First and second period is each specified by $t=0$ and 1. At date 1 one of S states($s=1,\cdots,S$) occurs. We call date $t=0$, state $s=0$ so that there are $S+1$ states in all. There exist I consumers($i=1,\cdots,I$). The commodity space for each agent is $\boldsymbol{R}^{S+1}$ because of a single consumption good. Each agent i is characterized by its consumption set X^i, its utility function u^i and its initial endowments ω^i. We make assumptions on those factors as follows. For each i $(i=1,\cdots,I)$,

Assumption 1 $\quad X^i$ is $\boldsymbol{R}^{S+1}_{++}$.

Assumption 2 $\quad u^i$ satisfies

1. $u^i \in C^\infty(\boldsymbol{R}^{S+1}_{++}, \boldsymbol{R})$.
2. $du^i_{\boldsymbol{x}} \in \boldsymbol{R}^{S+1}_{++}$ for each $\boldsymbol{x} \in \boldsymbol{R}^{S+1}_{++}$.

Assumption 3 $\quad \omega^i \in \boldsymbol{R}^{S+1}_{++}$.

Note that no convexity is assumed on the utility function. For simplicity, let $u=(u^1,\cdots,u^I)$ and $\omega=(\omega^1,\cdots,\omega^I)$ in the following. There are J nominal assets ($j=1,\cdots,J$) in the economy. Our interest is in the case of incomplete asset markets so that $J<S$. Each nominal asset j can be purchased for the price q_j at date 0 and promises to deliver a given stream of units of account $\boldsymbol{v}^j=(v^j_1,\cdots,v^j_S)$ across the states at date 1. If we see $\boldsymbol{v}^j(j=1,\cdots,J)$ as a column vector and put them together, then we obtain an $S\times J$ matrix of returns $V=[\boldsymbol{v}^1,\cdots,\boldsymbol{v}^J]$. We can assume that rank $V=J$ without loss of generality. Let p^s be a spot price of the good in state s $(s=0,1,\cdots,S)$. We make the following assumptions on the asset prices and spot prices of the good.

Assumption 4 $\boldsymbol{p} \in \boldsymbol{R}_{++}^{S+1}$, $\boldsymbol{q} \in \boldsymbol{R}_{++}^{J}$
where $\boldsymbol{p} = (p^0, p^1, \cdots, p^S)$ and $\boldsymbol{q} = (q_1, \cdots, q_J)$.

Given the asset structure V, each agent has a chance to purchase some amounts of J assets and adjust his income stream so that he can optimize his intertemporal consumptions. Let $\boldsymbol{z}^i = (z_1^i, \cdots, z_J^i) \in \boldsymbol{R}^J$ denote the number of units of the J assets purchased by agent i. $\boldsymbol{z}^i$ is called a portfolio of agent i. Then the problem he(or she) has to solve is as follows.

$$\begin{aligned} \max_{\boldsymbol{x}^i, \boldsymbol{z}^i} \quad & u^i(\boldsymbol{x}^i) \\ s.t. \quad & x_0^i = \omega_0^i - \boldsymbol{q} \cdot \boldsymbol{z}^i, \ \boldsymbol{z}^i \in \boldsymbol{R}^J \qquad \cdots\cdots(*) \\ & p^s x_s^i = \sum_{j-1}^{J} v_s^j z_j^i + p^s \omega_s^i, \ s = 1, \cdots, S. \end{aligned}$$

Note that there is only one good in the economy so that the good at date 0 is interpreted as a numeraire (i.e. $p^0 = 1$).

Now an economy with nominal assets is specified by all agents's utility functions and endowments as well as asset structure V. So let $\mathcal{E}(u, \omega; V)$ denote the economy composed of u, ω and V. Then the equilibrium of the economy $\mathcal{E}(u, \omega; V)$ is defined as follows.

Definition 1 *An asset market equilibrium for $\mathcal{E}(u, \omega; V)$ is a tuple $((\boldsymbol{x}^i, \boldsymbol{z}^i)_i, \boldsymbol{p_1}, \boldsymbol{q})$ such that*

(i) *$(\boldsymbol{x}^i, \boldsymbol{z}^i)$ is a solution of the problem $(*)$. $i = 1, \cdots, I$.*

(ii) $\sum_{i=1}^I \boldsymbol{x}^i = \sum_{i=1}^I \boldsymbol{\omega}^i$

(iii) $\sum_{i=1}^I \boldsymbol{z}^i = 0$

where $\boldsymbol{p_1} = (p^1, \cdots, p^S)$.

Next we consider efficiency of allocations. Given u and ω, a Pareto optimal allocation is defined as follows.

Definition 2 *An allocation $x = (\bar{\boldsymbol{x}}^1, \cdots, \bar{\boldsymbol{x}}^I) \in \boldsymbol{R}_{++}^{(S+1)I}$ is a Pareto optimum if*

(i) $\sum_{i=1}^I \bar{\boldsymbol{x}}^i = \sum_{i=1}^I \boldsymbol{\omega}^i$

(ii) *there does not exist $x = (\boldsymbol{x}^1, \cdots, \boldsymbol{x}^I) \in \boldsymbol{R}_{++}^{(S+1)I}$ such that $\sum_{i=1}^I \boldsymbol{x}^i = \sum_{i=1}^I \boldsymbol{\omega}^i$ and $u^i(\boldsymbol{x}^i) \geq u^i(\bar{\boldsymbol{x}}^i), i = 1, \cdots, I$ with a strict inequality for at least one i.*

Lastly we define a particular feasibility for the economy $\mathcal{E}(u, \omega; V)$.

Definition 3 *For any given strictly positive S-vector $\boldsymbol{\lambda}(= (\lambda_1, \cdots, \lambda_S))$, an allocation $x = (\boldsymbol{x}^1, \cdots, \boldsymbol{x}^I) \in \boldsymbol{R}^{(S+1)I}$ is pseudo-λ-feasible if*

(i) $\sum_{i=1}^{I} \boldsymbol{x}^i = \sum_{i=1}^{I} \omega^i$

(ii) $\boldsymbol{x}_1^i \in \langle \Lambda V \rangle + \boldsymbol{\omega}_1^i, i = 1, \cdots, I$

where Λ is a $S \times S$ diagonal matrix with $\boldsymbol{\lambda}$ for its diagonal and $\langle \Lambda V \rangle$ indicates a vector subspace spanned by the columns of a $S \times J$ matrix ΛV whereas $\boldsymbol{x}_1^i$ and $\boldsymbol{\omega}_1^i$ denote $(x_1^i, \cdots, x_S^i)$ and $(\omega_1^i, \cdots, \omega_S^i)$ respectively.

Let $F_\lambda(\omega)$ denote the set of pseudo-λ-feasible allocations with respect to ω. Indeed only the intersection of $F_\lambda(\omega)$ and $\boldsymbol{R}_{++}^{(S+1)I}$ makes sense, but we will use the whole set of $F_\lambda(\omega)$ for the analytical purpose in the following. If a tuple $((\boldsymbol{x}^i, \boldsymbol{z}^i)_i, \boldsymbol{p}_1, \boldsymbol{q})$ is an asset market equilibrium for $\mathcal{E}(u, \omega; V)$, then obviously $(\boldsymbol{x}^i)_i$ is an element of $F_\lambda(\omega)$.

In the following we investigate efficiency of asset market equilibria from a genericity viewpoint with regard to u and ω. To this end, we need to specify the set of admissible u and ω.

With respect to ω, each $\boldsymbol{\omega}^i$ is restricted to $\boldsymbol{R}_{++}^{S+1}$ which is an open subset of a topological space $\boldsymbol{R}^{S+1}$ with the ordinary Euclidean topology, so that the set of admissible ω is $\boldsymbol{R}_{++}^{(S+1)I}$ which is naturally interpreted as an open subset of a product topological space $\boldsymbol{R}^{(S+1)I}$. On the other hand, an utility function of each agent is required to belong to the subset of $C^\infty(\boldsymbol{R}_{++}^{S+1}, \boldsymbol{R})$. Let U denote the subset. Given the Whitney C^∞ topology to $C^\infty(\boldsymbol{R}_{++}^{S+1}, \boldsymbol{R})$, then U has a definite structure.

Proposition 1 *U is an open subset of $C^\infty(\boldsymbol{R}_{++}^{S+1}, \boldsymbol{R})$ in the Whitney C^∞ topology.*

Proof: *See proof of Proposition 2 in Nagata[17].*

Let the I-product of U be $\mathcal{U}$, which is obviously the admissible set of u. Then it follows from the above proposition that $\mathcal{U}$ is an open subset of a product topological space $C^\infty(\boldsymbol{R}_{++}^{S+1}, \boldsymbol{R})^I$. Thus the whole set of admissible u and ω is $\mathcal{U} \times \boldsymbol{R}_{++}^{(S+1)I}$ which is called the economy space.

3 Main Result

It has been shown that the structure of the set of equilibrium allocations is quite different between a nominal asset model and a real asset one when markets are incomplete. In the real asset case generically the set of equilibrium allocations is finite whereas the set in the nominal asset case generically expands and has partly the structure of a $S-1$ or $S-J$ dimensional manifold, depending on if the nominal asset prices are taken to be endogenous or exogenous. It is true that Pareto inefficiency of equilibrium allocations is generically obtained in the real asset case (see Magill and Quinzii[14], Nagata[17]), but we can not determine immediately whether the consequence carries over to the nominal asset case or not since the set of equilibrium allocations becomes drastically large in the latter case. To investigate this issue we first consider an auxiliary set which is substituted for the set of equilibrium allocations in our nominal asset model.

Proposition 2 *For any equilibrium $((\bar{\boldsymbol{x}}^i, \bar{\boldsymbol{z}}^i)_i, \bar{\boldsymbol{p}}_1, \bar{\boldsymbol{q}})$ for $\mathcal{E}(u,\omega;V)$, there exists a vector $\bar{\boldsymbol{\lambda}}$ of Δ_{++}^{S-1} such that the equilibrium allocation $(\bar{\boldsymbol{x}}^i)_i$ is pseudo-$\bar{\lambda}$-feasible where Δ_{++}^{S-1} designates the strictly positive $S-1$ dimensinal simplex in $\boldsymbol{R}^S$.*

Proof: *It is sufficient to show that $(\bar{\boldsymbol{x}}^i)_i$ satisfies that $\sum_{i=1}^I \bar{\boldsymbol{x}}^i = \sum_{i=1}^I \omega^i$ and $\bar{\boldsymbol{x}}^i_{\mathbf{1}} \in \langle \bar{\Lambda} V \rangle + \omega^i_{\mathbf{1}}, i = 1, \cdots, I$. Set $c = \sum_{s=1}^S \frac{1}{\bar{p}^s}$, $\tilde{z}^i = \frac{1}{c}\bar{z}^i$, $i = 1, \cdots, I$, $\tilde{\boldsymbol{p}}_1 = (\frac{1}{\bar{p}^1}, \cdots, \frac{1}{\bar{p}^S})$ and $\tilde{\boldsymbol{q}} = c\bar{\boldsymbol{q}}$. Then it is easily seen that $((\bar{\boldsymbol{x}}^i, \tilde{\boldsymbol{z}}^i)_i, \tilde{\boldsymbol{p}}_1, \tilde{\boldsymbol{q}})$ is also an equilibrium for the economy. Now we set $\bar{\boldsymbol{\lambda}} = c(\frac{1}{\bar{p}^1}, \cdots, \frac{1}{\bar{p}^S})$ which is obviously an element of Δ_{++}^{S-1}. Considering the property of the new equilibrium, the following equations hold. $\sum_{i=1}^I \bar{\boldsymbol{x}}^i = \sum_{i=1}^I \omega^i$, $\bar{\boldsymbol{x}}^i_{\mathbf{1}} = \bar{\Lambda} V \cdot \tilde{\boldsymbol{z}}^i + \omega^i_{\mathbf{1}}, i = 1, \cdots, I$.*

It is obvious from the above proposition that all the equilibrium allocations for $\mathcal{E}(u,\omega;V)$ are included in the union of $F_\lambda(\omega)$ over $\lambda \in \Delta_{++}^{S-1}$ (i.e.$\bigcup_{\lambda \in \Delta_{++}^{S-1}} F_\lambda(\omega)$). We will make use of this union instead of the set of equilibrium allocations itself in the following.

Each factor of the union (i.e. $F_\lambda(\omega)$) has a definite structure with the following assumption.

Assumption 5 $(J+1)I > S+1$

Lemma 1 *Under assumption 5, for any λ in Δ_{++}^{S-1} and any ω in $\boldsymbol{R}_{++}^{(S+1)I}$, $F_\lambda(\omega)$ constitutes a $(J+1)I - (S+1)$ dimensional linear submanifold in $\boldsymbol{R}^{(S+1)I}$.*

Proof: *For any λ in Δ_{++}^{S-1}, obviously the $S \times S$ matrix Λ is nonsingular so that rank $\Lambda V = J$ by the assumption that rank $V = J$. Thus we can apply the way of proving in proof of proposition 5 in Nagata[17] to have the desired result.*

The union $\bigcup_{\lambda \in \Delta_{++}^{S-1}} F_\lambda(\omega)$ itself is not tractable for a genericity analysis since it is not necessarily a manifold. Therefore, we are going to process this set so that we can facilitate our genericity analysis.

Let $\boldsymbol{e}$ be a unit S-vector(i.e. $\boldsymbol{e} = (1, \cdots, 1)$) and consider the pseudo-e-feasible set $F_e(\omega)$ which is obviously an $(J+1)I - (S+1)$ dimensional linear submanifold in $\boldsymbol{R}^{(S+1)I}$. Note that for any λ in Δ_{++}^{S-1}, $F_\lambda(\omega)$ and $F_e(\omega)$ are diffeomorphic. In addition, considering the specifying conditions of those sets, there exists a smooth map $G : F_e(\omega) \times \Delta_{++}^{S-1} \to \boldsymbol{R}^{(S+1)I}$ such that $G(\cdot, \lambda) : F_e(\omega) \to \boldsymbol{R}^{(S+1)I}$ is a into-diffeomorphism and $G(F_e(\omega), \lambda) = F_\lambda(\omega)$ for any λ in Δ_{++}^{S-1}. Now Let $\boldsymbol{F}(\omega)$ be the disjoint union of $F_\lambda(\omega)$, $\lambda \in \Delta_{++}^{S-1}$. Then, by using the idea of fibration, we can make it a manifold.

Proposition 3 *For any ω in $\boldsymbol{R}_{++}^{(S+1)I}$, $\boldsymbol{F}(\omega)$ constitutes a $(J+1)I - 2$ dimensional manifold.*

Proof: *See Appendix.*

We shall now turn to the characterization of efficiency of allocations. To this end, we consider the first order necessary conditions for Pareto optimal allocations. Considering assumption 2, it is easily shown that if an allocation $\bar{x} = (\bar{x}^1, \cdots, \bar{x}^I) \in \boldsymbol{R}_{++}^{(S+1)I}$ is Pareto optimal, then the following equation holds.

$$du^1_{\bar{x}^1} / \sum_{s=0}^{S+1} du^1_{s,\ \bar{x}^1} = \cdots = du^I_{\bar{x}^I} / \sum_{s=0}^{S+1} du^I_{s,\ \bar{x}^I}$$

Let $A(u)$ be the set of allocations which satisfy the above condition for any given u. Note that no feasibility is required to $A(u)$. With regard to the property of $A(u)$, the following proposition is obtained by altering Thom Transversality Theorem to a product functional form and using it in $1 - jet$ space.

Proposition 4 *There exists a dense subset $\mathcal{U}^*$ of $\mathcal{U}$ such that for any u in $\mathcal{U}^*$, $A(u)$ constitutes an $S + I$ dimensional submanifold in $\boldsymbol{R}_{++}^{(S+1)I}$.*

Proof: *See proof of Proposition 4 in Nagata[17].*

Finally we consider the relation between $A(u)$ and $\bigcup_{\lambda \in \Delta_{++}^{S-1}} F_\lambda(\omega)$. In order to obtain a fruitful consequence, we need some device. First fix an $A(u)$ for any given u of $\mathcal{U}^*$ and consider the inclusion map $\iota : A(u) \to \boldsymbol{R}^{(S+1)I}$. Then the image of ι, i.e. $A(u)$, can be seen as a $(S + I)$ dimensional submanifold of $\boldsymbol{R}^{(S+1)I}$. Next, pick an arbitrary ω out of $\boldsymbol{R}_{++}^{(S+1)I}$ and define the following set. $N_1^+(\omega) = \{y \in \boldsymbol{R}_{++}^{(S+1)I} \mid \|y - \omega\| < 1\}$. By using this set as a parameter space, we define the map $\psi : \boldsymbol{F}(\omega) \times N_1^+(\omega) \to \boldsymbol{R}^{(S+1)I}$ by $\psi(x^*, y) = x + y$ where $x^* = \{(x, \lambda) \mid x \in F_\lambda(\omega),\ \lambda \in \Delta_{++}^{S-1}\}$.

Lemma 2 *ψ is a smooth map and a submersion.*

Proof: *See Appendix.*

Then we claim the following.

Proposition 5 *For almost all $y \in N_1^+(\omega)$, $\psi(\cdot, y) : \boldsymbol{F}(\omega) \to \boldsymbol{R}^{(S+1)I}$ is transversal to $A(u)$.*

Proof: *$\psi : \boldsymbol{F}(\omega) \times N_1^+(\omega) \to \boldsymbol{R}^{(S+1)I}$ is transversal to $A(u)$ since it is shown in the above proposition to be a submersion. Thus, by applying The Transversality Theorem (see Guillemin and Pollack[11], p.68) to ψ, we obtain the desired result.*

ψ has another remarkable property as follows.

Proposition 6 *For each $y \in N_1^+(\omega)$, $\psi(\boldsymbol{F}(\omega), y) = \bigcup_{\lambda \in \Delta_{++}^{S-1}} F_\lambda(\omega + y)$.*

Proof: *Since $\boldsymbol{F}(\omega)$ is the disjoint union of $F_\lambda(\omega), \lambda \in \Delta_{++}^{S-1}$, obtaining its image by $\psi(\cdot, y)$ requires us to consider all the $F_\lambda(\omega)$ with regard to $\lambda \in \Delta_{++}^{S-1}$. Recall that $F_\lambda(\omega)$ is the set of $x(=(\boldsymbol{x}^1, \cdots, \boldsymbol{x}^I) \in \boldsymbol{R}^{(S+1)I})$ which satisfies (1) $\sum_{i=1}^I \boldsymbol{x}^i = \sum_{i=1}^I \boldsymbol{\omega}^i$ and (2) $\boldsymbol{x}_{\mathbf{1}}^i \in \langle \Lambda V \rangle + \boldsymbol{\omega}_{\mathbf{1}}^i, i = 1, \cdots, I$. Thus for any $\lambda \in \Delta_{++}^{S-1}$ and $y \in N_1^+(\omega)$ the set $\{(x+y) \in \boldsymbol{R}^{(S+1)I} \mid x \in F_\lambda(\omega)\}$ is obviously equal to the set $\{x \in \boldsymbol{R}^{(S+1)I} \mid x \in F_\lambda(\omega + y)\}$. Since $\psi(\cdot, y)$ substantially transforms any x into $x + y$, our claim immediately follows.*

To obtain our final consequences, we need another assumption on the numbers of agents, assets and states.

Assumption 6 $\quad S > J + 1, \;\; I > J$

Note that this assumption is not required in the case of real asset economy(see Nagata[17]) although the assumption itself is not so harmful.

Now we are in a position to state our main results.

Theorem 1 *Under assumptions 1~6, for almost all u and ω, each equilibrium allocation of the economy $\mathcal{E}(u, \omega; V)$ with nominal assets is Pareto inefficient.*

Proof: *Let u and ω be respectively arbitrary elements of $\mathcal{U}^*$ and $\boldsymbol{R}_{++}^{(S+1)I}$. Let y be an element of $N_1^+(\omega)$ such that $\psi(\cdot, y)$ is transversal to $A(u)$. Note that from proposition 5 such an y is an element of a dense set of $N_1^+(\omega)$. Now suppose that $\psi(\boldsymbol{F}(\omega), y) \cap A(u) \neq \emptyset$. Then for any $x \in \psi(\boldsymbol{F}(\omega), y) \cap A(u)$, we have*

$$d\psi_{z^*}(T_{z^*}(\boldsymbol{F}(\omega)) + T_x(A(u)) = \boldsymbol{R}^{(S+1)I}$$

where T designates a tangent space and z^* is an element of $\psi^{-1}(\cdot, y)(x)$. However, by proposition 3 and 4 and assumption 6, we have the following inequality.

$$\begin{aligned} dim\boldsymbol{R}^{(S+1)I} - (dimT_{z^*}(\boldsymbol{F}(\omega)) + dimT_x(A(u))) &\geq (S+1)I - ((J+1)I - 2 + S + I) \\ &= (I-1)(S-(J+1)) - (J-1) \\ &> (I-1)(S-(J+1)) - (I-1) \\ &= (I-1)(S-(J+1)-1) \\ &\geq 0, \end{aligned}$$

which is a contradiction to the previous equation. Thus $\psi(\boldsymbol{F}(\omega), y) \cap A(u) = \emptyset$, which implies that $\bigcup_{\lambda \in \Delta_{++}^{S-1}} F_\lambda(\omega + y) \cap A(u) = \emptyset$ by proposition 6. Noting that $\bigcup_{\lambda \in \Delta_{++}^{S-1}} F_\lambda(\omega + y)$ includes the whole set of equilibrium allocations for $\mathcal{E}(u, \omega + y; V)$ and that an equilibrium allocation can not be Pareto efficient unless it belongs to $A(u)$, it turns out that any equilibrium allocation for $\mathcal{E}(u, \omega + y; V)$ is Pareto inefficient. Since ω is arbitrarily taken from $\boldsymbol{R}_{++}^{(S+1)I}$ and u and y are respectively arbitrary elements of the dense sets, our claim

4 Concluding Remarks

We have investigated generic inefficiency of equilibrium allocations with incomplete markets of nominal assets with respect to all agents' utility functions (u) and endowments (ω) in the one good-two period exchange economy model. The admissible sets of u and ω considered here are the same as the ones in a real asset model considered before(See Nagata[17]). As a result, we have obtained similar results to the ones in the real asset model at the cost of an additional assumption on the numbers of agents, assets and states(See assumption 6). The reason why the additional assumption is required is that a nominal asset model generically yields much larger set of equilibrium allocations than a real asset model. More specifically, the fact that the dimension of the pseudo-feasible set which includes all the equilibrium allocations increases by $S-1$ in the nominal asset case needs the additional assumption to ensure the invalidity of Pareto efficiency.

In this connection it is worthy to note the following point. In a nominal asset model the structure of the set of equilibrium allocations is different between endogenous asset prices and exogenous asset prices. That is to say, generically the set consists partly of a $S-1$ dimensinal manifold in the former case(Geanakoplos and Mas-colell[8]) whereas it partly constitutes an $S-J$ dimensinal manifold in the latter case(Balasko and Cass[1]). The difference, however, does not make any change in our results. Our results hold in both cases because the pseudo-feasible set which is combined with $A(u)$ to prove the impossibility of Pareto efficiency includes any equilibrium allocation regardless of whether it is based on endogenous asset prices or exogenous asset prices. Indeed, the pseudo-feasible set is derived from all agents' budget constraints of each state at date 1 which are specified independently of asset prices(See definition 3). Obviously, the asset prices have something to do only with every agent's budget constraint at date 0.

Appendix (1) Proof of proposition 3.

We are going to give a smooth manifold structure as well as a topological structure to $\boldsymbol{F}(\omega)$ in such a way that it becomes a total space on Δ_{++}^{S-1} as a base space. Let $\pi : \boldsymbol{F}(\omega) \to \Delta_{++}^{S-1}$ be a projection defined by $\pi(x^*) = \lambda$ where $x^* = (x, \lambda \mid x \in F_\lambda(\omega), \lambda \in \Delta_{++}^{S-1})$. Let $\{(U_\alpha, \varphi_\alpha)\}_{\alpha\in A}$ be the atlas of Δ_{++}^{S-1}. Consider $\pi^{-1}(U_\alpha)$ for any chart $\varphi_\alpha : U_\alpha \to \boldsymbol{R}^{S-1}$ and difine the map $\tilde{\varphi}_\alpha : \pi^{-1}(U_\alpha) \to \boldsymbol{R}^{(J+1)I-(S+1)} \times \boldsymbol{R}^{S-1}$ by

$$\tilde{\varphi}_\alpha(x^{ast}) = (\phi_\beta \circ G^{-1}(x,\lambda),\ \varphi_\alpha(\lambda))$$

where ϕ_β is an appropriate chart $\phi_\beta : W_\beta \to \boldsymbol{R}^{(J+1)I-(S+1)}$ of the atlas $(W_\beta, \phi_\beta)_{\beta\in B}$ for the manifold $F_e(\omega)$. Let a subset Z of F_ω be called an open set if for any α, $\tilde{\varphi}_\alpha(Z \cap \pi^{-1}(U_\alpha))$ is open in $\boldsymbol{R}^{(J+1)I-(S+1)} \times \boldsymbol{R}^{S-1}$. Then it is easily seen that those sets constitutes a system of open sets which gives a topological structure to F_ω.

Next, consider the property of the collcetion $(\pi^{-1}(U_\alpha, \tilde{\varphi}_\alpha)_{\alpha\in A}$. Suppose that for $\alpha, \alpha^{prime} \in A, \pi^{-1}(U_\alpha) \cap \pi^{-1}(U_{\alpha^{prime}}) \neq \emptyset$. Then for any point (a, b) of $\tilde{\varphi}_\alpha(\pi^{-1}(U_\alpha) \cap \pi^{-1}(U_{\alpha^{prime}}))$ we have

$$\begin{aligned}\tilde{\varphi}_{\alpha'} \circ \tilde{\varphi}_\alpha^{-1}(a, b) &= \tilde{\varphi}_{\alpha'}((G(\phi_\beta^{-1}(a), \lambda)) = (\phi_\beta \circ G^{-1}(G(\phi_\beta^{-1}(a), \lambda), \lambda), \varphi_{\alpha'}(\varphi_\alpha^{-1}(b))) \\ &= (a, \varphi_{\alpha'} \circ \varphi_\alpha^{-1}(b))\end{aligned}$$

where $\lambda = \varphi_\alpha^{-1}(b)$. Thus $\tilde{\varphi}_{\alpha'} \circ \tilde{\varphi}_\alpha^{-1} : \tilde{\varphi}_\alpha(\pi^{-1}(U_\alpha) \cap \pi^{-1}(U_{\alpha^{prime}})) \to \tilde{\varphi}_{\alpha'}(\pi^{-1}(U_\alpha) \cap \pi^{-1}(U_{\alpha'}))$ is a diffeomorphism. Since it is obvious that $\pi^{-1}(U_\alpha)_{\alpha\in A}$ covers $\boldsymbol{F}(\omega)$, which implies that $\boldsymbol{F}(\omega)$ is regarded as a smooth manifold the dimension of which is $(J+1)I - (S+1) + (S-1) = (J+1)I - 2$. Q.E.D.

(2) Proof of lemma 2.

It suffices for us to check the claim locally. Let $(\pi^{-1}(U_\alpha, \tilde{\varphi}_\alpha) \times (N_1^+(\omega), i))$ be a chart including any given point (x^*, y) of $\boldsymbol{F}(\omega) \times N_1^+(\omega)$ where i is the identity map. Let $\tilde{\varphi}_\alpha(x^*) = (a, b)$ where obviously $a \in \boldsymbol{R}^{(J+1)I-(S+1)}$ and $b \in \boldsymbol{R}^{S-1}$. Then we have the following local parameterization of ψ at (x^*, y).

$$G(\phi_\beta^{-1}(a), \varphi_\alpha^{-1}(b)) + y$$

which is obviously differentiable at (a, b, y) since G is a smooth map, thus ψ is also differntiable at (x^*, y). In addition, it is obvious from the form of the local parameterization that ψ is a submersion. Q.E.D.

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