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Global base change identity and Drinfeld's shtukas

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This is the text of my talk at the conference "Automorphic forms and representation theory of p-adic groups" in Kyoto, January 2003. It summarizes my preprint [7] which will be published elsewhere. In loc. cit. we propose a new approach to prove the global base change identity which arises in the comparison of the Lefschetz trace formula on moduli space of Drinfeld's shtukas and the Selberg's trace formula, without using the fundamental lemma for base change.

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1 Drinfeld's shtukas with multiples modifications

Let X be a geometrically connected, smooth and projective curve over $\mathbf{F}_q$. Let $\bar{X} = X \otimes_{\mathbf{F}_q} k$ where k is an algebraic closure of $\mathbf{F}_q$. Let σ denote the geometric Frobenius element of $\mathrm{Gal}(k/\mathbf{F}_q)$.

Let F denote the function field of X. For every closed point $x \in |X|$, let F_x be the completion of F at x and $\mathcal{O}_x$ be the ring of integers of F_x.

Let $d \geq 2$ be an integer and $G = \mathrm{GL}_d$. According to Drinfeld, one has the notion of G-shtukas with multiples modifications which we are going to review in a moment. Let $\bar{x}_1, \ldots, \bar{x}_n \in X(k)$ be n mutually distinct geometric points of X. Let $\bar{T} = \{\bar{x}_1, \ldots, \bar{x}_n\}$. A $\bar{T}$-modification is an isomorphism

$$t : \mathcal{V}'^{\bar{T}} \xrightarrow{\sim} \mathcal{V}^{\bar{T}}$$

between the restrictions $\mathcal{V}'^{\bar{T}}$ an $\mathcal{V}^{\bar{T}}$ of vector bundles of rank d $\mathcal{V}'$ and $\mathcal{V}$ over $\bar{X}$ to the $\bar{X} - \bar{T}$.

Let $\bar{x} \in \bar{T}$ and let denote $\mathcal{V}'_{\bar{x}}$ and $\mathcal{V}_{\bar{x}}$ the completions of $\mathcal{V}'$ and $\mathcal{V}$ at $\bar{x}$. These are free $\mathcal{O}_{\bar{x}}$-modules of rank d whose generic fibers are identified with

$t_x : V'_{\bar{x}} \xrightarrow{\sim} V_x$. By the theory of elementary divisors, two $\mathcal{O}_{\bar{x}}$-lattices within the same $F_{\bar{x}}$-vector spaces can be given an invariant

$$\mathrm{inv}(t_{\bar{x}}) \in \mathbb{Z}^d_+ = \{(\lambda^1, \ldots, \lambda^d) \in \mathbb{Z}^d \mid \lambda^1 \geq \cdots \geq \lambda^d\}.$$

For general reductive group G, $\mathbb{Z}^d_+$ must be replaced by the set of dominant coweights of G and and this set comes equipped with a natural partial order : $\lambda \geq \lambda'$ if and only if $\lambda - \lambda'$ is a sum of positive coroots. This partial order has geometric origin since an $\bar{x}$-modification with invariant λ can only degenerate to a $\bar{x}$-modification with some invariant $\lambda' \leq \lambda$. It will be convenient to write formally

$$\mathrm{inv}(t) = \sum_{i=1}^{n} \mathrm{inv}(t_{\bar{x}_i})\bar{x}_i.$$

We will say

$$\sum_{i=1}^{n} \mathrm{inv}(t_{\bar{x}_i})\bar{x}_i \leq \sum_{i=1}^{n} \lambda_i \bar{x}_i$$

if for every $i = 1, \ldots, n$, we have $\mathrm{inv}(t_{\bar{x}_i}) \leq \lambda_i$.

Definition 1 (Drinfeld) *Let $\underline{x} = (\bar{x}_1, \ldots, \bar{x}_n)$ be a collection of mutually distinct k-points of X and let $\underline{\lambda}$ a collection of dominant coweights $\lambda_1, \ldots, \lambda_n \in \mathbb{Z}^d_+$. A $\underline{\lambda}$-shtuka over $\underline{x}$ is a pair $(\mathcal{V}, t)$ where $\mathcal{V}$ is a vector bundle of rank d over $\bar{X}$ and t is a $\bar{T}$-modification with $\bar{T} = \{\bar{x}_1, \ldots, \bar{x}_n\}$*

$$t : {}^\sigma\mathcal{V}^{\bar{T}} \xrightarrow{\sim} \mathcal{V}^{\bar{T}}$$

with $\mathrm{inv}(t) \leq \sum_{i=1}^{n} \lambda_i \bar{x}_i$. *Here ${}^\sigma\mathcal{V}$ denotes the pull-back of $\mathcal{V}$ by the endomorphism* $\mathrm{id}_X \otimes_{\mathbb{F}_q} \sigma$ *of* $X \otimes_{\mathbb{F}_q} k$

These data have a moduli stack

$$c'_{\underline{\lambda}} : \mathcal{S}'_{\underline{\lambda}} \to X^n - \Delta$$

where Δ is the union of all diagonals in X^n. This moduli space can be continued over the diagonals at the price of a small break of symmetry. Let $\underline{x} = (\bar{x}_1, \ldots, \bar{x}_n) \in X^n(k)$ with possibly $\bar{x}_i = \bar{x}_j$. Then a $\underline{\lambda}$-shtuka over $\underline{x}$ is a collection of vector bundles of rank d

$$\mathcal{V}_0, \mathcal{V}_1, \ldots, \mathcal{V}_d$$

over $\bar{X}$ equipped with

- a collection of modifications

$$t_1 : \mathcal{V}_1^{\bar{x}_1} \xrightarrow{\sim} \mathcal{V}_0^{\bar{x}_1}, \dots, t_n : \mathcal{V}_n^{\bar{x}_n} \xrightarrow{\sim} \mathcal{V}_{n-1}^{\bar{x}_n}$$

  such that for every $i = 1, \dots, n$, $\mathrm{inv}(t_i) \leq \lambda_i \bar{x}_i$,

- and an isomorphisme ${}^\sigma\mathcal{V}_0 \xrightarrow{\sim} \mathcal{V}_n$.

For a point $\underline{x}$ away from the diagonals Δ, this definition is equivalent to Definition 1.1. Therefore the above c'_λ can be continued in a natural way to a obtain a smooth morphism

$$c_\lambda : \mathcal{S}_\lambda \to X^n.$$

For every finite subscheme I of X, one can define the notion of an I-level structure of a shtuka. We also have a moduli space of $\underline{\lambda}$-shtukas with I-level structure

$$c^I_{\underline{\lambda}} : \mathcal{S}^I_{\underline{\lambda}} \to (X - I)^n.$$

This morphism is smooth, locally of finite type but in general not of finite type. This lack of finiteness is one of the main difficulties that Lafforgue had to overcome in his solution of Langlands' correspondence for GL_d over function fields [5]. Since we want to focus into another aspect of moduli spaces of shtukas, we prefer for the moment to avoid this difficulty by restricting ourself to the case of $\mathcal{D}$-shtukas associated to a division algebra.

Let D be a division algebra over F and let $\mathcal{D}$ be a maximal $\mathcal{O}_X$-algebra with generic fiber D. Let X' be the open of X where D is unramified. Let $G = D^\times$ as F-group. For every place $v \in |X'|$, G_v is isomorphic to GL_d. We can define the moduli space of G-shtukas in completely similar way to shtukas for GL_d and obtain a morphism

$$c^I_{\underline{\lambda},a} : (\mathcal{D} - \mathcal{S}^I_{\underline{\lambda}})/a^{\mathbf{Z}} \to (X' - I)^n$$

which is a separated, proper and smooth morphism under the assumption $I \neq \emptyset$. Here $a \in \mathbb{A}_F^\times$ is an idele with $\deg(a) \neq 0$ and the group $a^{\mathbf{Z}}$ acts freely on the moduli space of shtukas by $(\mathcal{V}, t) \mapsto (\mathcal{V} \otimes \mathcal{L}(a), \mathrm{id}_{\mathcal{L}(a)})$ where $\mathcal{L}(a)$ is the line bundle on X associated to the idele a.

Let $\mathcal{F}_{\underline{\lambda}}$ be the intersection complex of $\mathcal{S}^I_{\underline{\lambda}}$. As usual, the restricted tensor product

$$\mathcal{H}^I = \bigotimes_{v \in |X' - I|} \mathcal{H}_v$$

where $\mathcal{H}_v$ is the unramified Hecke algebra of G_v, acts by correspondences on

$$\mathrm{R}^i (c^I_{\underline{\lambda},a})_* \mathcal{F}^I_{\underline{\lambda}}$$

which is a local system on $(X' - I)^n$ for all integer i.

Theorem 2 *We have the following equality in the Grothendieck group of local systems on $(X'-I)^n$ equipped with action of $\mathcal{H}^I$*

$$\sum_i (-1)^i [\mathrm{R}^i (c^I_{\underline{\lambda},a})_* \mathcal{F}_{\underline{\lambda}}] = \bigoplus_\pi m(\pi)\pi^I \otimes \bigotimes_{i=1}^n \mathrm{pr}_i^* \mathcal{L}_{\lambda_i}(\pi)$$

where π runs over the set of automorphic representation of $G(\mathbb{A}_F)$ where $a^{\mathbf{Z}}$ acts trivially, $m(\pi)$ its multiplicity, $\mathcal{L}_{\lambda_i}(\pi)$ is the local system on $X'-I$ such that the equality of L-functions holds

$$L(\mathcal{L}_{\lambda_i}(\pi), s) = L(\pi, \lambda_i; s)$$

where $L(\pi, \lambda_i; s)$ is the automorphic L-function associated to π and to the representation of $\hat{G}$ of highest weight λ_i.

This statement is what one can expect from the cohomology of moduli space of shtukas, according to Langlands' philosophy.

2 Outline of the proof

In order ro simplify the exposition, we will restrict ourself to the case $n=1$ and $\lambda = (\lambda^1 \geq \cdots \geq \lambda^d)$ with $\sum_j \lambda^j = 0$.

Let $\bar{x} \in (X'-I)(k)$ with $\sigma^s(x) = x$ where σ denotes the action of the geometric Frobenius on $(X'-I)(k)$. Let x be the closed point of $X'-I$ supporting $\bar{x}$.

Let $T' \subset X' - I - \{x\}$ be a finite reduced subscheme and let $\lambda'_{T'} : |T'| \to \mathbb{Z}^d_+$ be an arbitrary function. Let

$$\Phi_{T',\lambda'_{T'}} = \bigotimes_{v\in|T'|} \phi_{\lambda'(v)} \otimes \bigotimes_{v\notin|T'|} 1_v \in \mathcal{H}^I$$

where $\phi_{\lambda'(v)}$ is the characteristic function of the double coset $G(\mathcal{O}_v)\lambda'_v G(\mathcal{O}_v)$ in $G(F_v)$, and 1_v is the unit function.

One can use a similar method for counting points, due to Langlands and Kottwitz [3], in order to prove the following formula

$$\mathrm{Tr}(\sigma^s \circ \Phi_{T',\lambda'_{T'}}) = \sum_{(\gamma_0,\delta_x)} \mathrm{vol}(J_{\gamma_0,\delta_x}(F)a^{\mathbf{Z}} \backslash J_{\gamma_0,\delta_x}(\mathbb{A}_F)) \prod_{v\in|X-T'-\{x\}|} \mathbf{O}_{\gamma_0}(1_v) \prod_{v\in|X'|} \mathbf{O}_{\gamma_0}(\phi_{\lambda'(v)})\mathbf{TO}_{\delta_x}(\psi_{\lambda,\bar{x}}) \quad (1)$$

- γ_0 is a conjugacy class of $G(F)$, δ_x is a σ-conjugacy class of $G(F_x \otimes_{\mathbb{F}_q} \mathbb{F}_{q^s})$ whose norm down to $G(F_x)$ is the class of γ_0.

- $J_{(\gamma_0,\delta_x)}$ is the F-group which is an inner form of the centralizer G_{γ_0} of γ_0 such that at a place $v \neq x$, $(J_{(\gamma_0,\delta_x)})_v$ is isomorphic to $(G_{\gamma_0})_v$ and at x, $(J_{(\gamma_0,\delta_x)})_x$ is isomorphic to the twisted centralizer of δ_x. This inner form is well defined up to isomorphism.

- The function $\psi_{\lambda,\bar{x}} \in \mathcal{H}(G(F_x \otimes_{\mathbb{F}_q} \mathbb{F}_{q^s}))$ is defined as follows. Let $y_1, \ldots, y_r$ be the places of $F \otimes_{\mathbb{F}_q} \mathbb{F}_{q^s}$ over x. Assume the geometric point $\bar{x}$ lies over y_1. Then we define

$$\psi_{\lambda,\bar{x}} = \psi_{\lambda(y_1)} \otimes 1_{y_2} \otimes \cdots \otimes \cdots 1_{y_r}$$

where $1_{y_2}, \ldots, 1_{y_r}$ are the unit functions of $\mathcal{H}(G_{y_2}), \ldots, \mathcal{H}(G_{y_r})$ respectively. The function $\psi_{\lambda(y_1)} \in \mathcal{H}(G_{y_1})$ is the unique function whose the Satake transform is the function on $\hat{G}(\mathbb{C})$ given by

$$\hat{g} \mapsto \mathrm{Tr}(\hat{g}, V_\lambda)$$

where V_λ is the irreducible representation of $\hat{G}$ of highest weight λ.

I refer to [7] for the detailed proof of this counting point formula.

To prove the theorem, we need to transform (1) in to a sum without twisted orbital integral. Namely, we want to prove that (1) is equal to the following sum

$$\sum_{\gamma_0} \mathrm{vol}(G_{\gamma_0}(F)a^{\mathbf{Z}} \backslash G_{\gamma_0}(\mathbb{A}_F)) \prod_{v \in |X - T' - \{x\}|} \mathbf{O}_{\gamma_0}(1_v) \prod_{v \in |X'|} \mathbf{O}_{\gamma_0}(\phi_{\lambda'(v)}) \mathbf{O}_{\gamma_0}(\mathbf{b}(\psi_{\lambda,\bar{x}})) \quad (2)$$

where

$$\mathbf{b} : \mathcal{H}(G(F_x \otimes_{\mathbb{F}_q} \mathbb{F}_{q^s})) \to \mathcal{H}(G(F_x))$$

is the base change homomorphism. Once the equality (1) = (2) has been etablished, it remains to apply Selberg to obtain the equality between the sum (2) and the following

$$\mathrm{Tr}\left(\bigotimes_{v \in |X - T' - \{x\}|} 1_v \otimes \bigotimes_{v \in |T'|} \phi_{\lambda'(v)} \otimes \mathbf{b}(\psi_{\lambda,\bar{x}}), \mathrm{L}^2\left(a^{\mathbf{Z}} G(F) \backslash G(\mathbb{A}_F)\right) \right) \quad (3)$$

and the theorem follows by a standard argument.

The above strategy is well known and goes back to Langlands and Kottwitz's work on Shimura varieties [2]. For the moduli space of shtukas, this is also done by Drinfeld and Lafforgue with maybe some technical differences. The only new point in our work concerns the proof of the identity (1) = (2). Usually, one needs the fundamental lemma for base changein order to convert a twisted orbital integral into orbital integral, which is known in p-adic case due to works of Kottwitz, Clozel and Labesse. In positive characteristic, the fundamental lemma for base change was not written down except for the function associated to the minuscule coweight which is proved by a direct calculation due to Drinfeld [6], but it is known to Henniart.

Our point is that one can prove the global base change identity (1) = (2) without using local harmonic analysis but rather a combination of counting of points, local model theory, a geometric interpretation of the base change homomorphism in terms of perverse sheaves and Tchebotarev's density theorem. We hope that our method can be generalized to other situations.

3 Global base change identity

Equality (1) = (2) will be proved by counting points on two different moduli spaces called A and B.

3.1 Situation A

The moduli space A is a scalar restriction à la Weil. Consider the s-fold product

$$(c^I_{\lambda,a})^s : (\mathcal{D} - \mathcal{S}^I_\lambda/a^{\mathbf{Z}})^s \to (X' - I)^s$$

of $c^I_{\lambda,a} : (\mathcal{D} - \mathcal{S}^I_\lambda)/a^{\mathbf{Z}} \to X' - I$. This morphism comes with an action of the symmetric group $\mathfrak{S}_s$ and of the action by correspondences of $(\mathcal{H}^I)^{\otimes s}$. Let denote

$$[A] := \sum_i (-1)^i \mathrm{R}(c^I_{\lambda,a})^s_* \mathcal{F}_\lambda^{\boxtimes s}$$

the class in the Grothendieck group of local system on $(X' - I)^s$ equipped with an action of $(\mathcal{H})^s$ and with a compatible action of $\mathfrak{S}_s$. By the Kunneth formula, $[A]$ should be

$$\bigoplus_{\pi_1,\dots,\pi_s} \prod_{i=1}^{s} m(\pi_i) \bigotimes_{i=1}^{s} \pi_i^I \otimes \bigotimes_{i=1}^{s} \mathrm{pr}_i^* \mathcal{L}_\lambda(\pi_i) \tag{4}$$

where $\pi_1, \dots, \pi_s$ are automorphic representations of G with trivial action of $a^{\mathbf{Z}}$. It's clear how $\mathfrak{S}_s$ and $(\mathcal{H}^I)^s$ should act on (4).

Assume for simplicity that the closed point x supporting $\bar{x}$ is of degree 1. Let $\underline{x} = (\bar{x}, \ldots, \bar{x})$ be the correponding point in the small diagonal of $(X' - I)^s$. By usual properties of Weil's scalar restriction, (1) is equal to

$$\mathrm{Tr}(\tau \circ \sigma \circ (1 \otimes \cdots \otimes 1 \otimes \Phi_{T', \lambda'_{T'}}), [A]_{\underline{x}}) \qquad (5)$$

where $\tau \in \mathfrak{S}_s$ is the cyclic permutation.

3.2 Situation B

Let us consider a particular collection of coweights

$$\underline{s\lambda} = \underbrace{(\lambda, \ldots, \lambda)}_{s}$$

and the associated moduli space of shtukas with "symmetric modifications"

$$c^I_{\underline{s\lambda}} : (\mathcal{D} - \mathcal{S}^I_{\underline{s\lambda}})/a^{\mathbf{Z}} \to (X' - I)^s.$$

By the very definition, for every $\tau \in \mathfrak{S}_s$, the fiber of $c^I_{\underline{s\lambda}}$ over a point $(\bar{x}_1, \ldots, \bar{x}_s)$ away from the union Δ of all diagonals, is canonically isomorphic with the fiber over $\tau(\bar{x}_1, \ldots, \bar{x}_s)$. This gives rises to a compatible action of $\mathfrak{S}_s$ on the restriction of

$$\mathrm{R}(c^I_{\underline{s\lambda}})_* \mathcal{F}_{\underline{s\lambda}}$$

to $(X' - I)^s - \Delta$. Since this direct image is a local system, we can extend canonically the action of $\mathfrak{S}_s$ over the diagonals. Let denote

$$[B] = \sum_i (-1)^i \mathrm{R}(c^I_{\underline{s\lambda}})_* \mathcal{F}_{\underline{s\lambda}}$$

the class in the Grothendieck group of local systems equipped with an action of $\mathcal{H}^I$ and a compatible action of $\mathfrak{S}_s$.

Assuming Theorem 2, $[B]$ should be

$$\bigoplus_{\pi} m(\pi)\pi^I \otimes \bigotimes_{i=1}^{s} \mathrm{pr}_i^* \mathcal{L}_\lambda(\pi) \qquad (6)$$

Let $\underline{x} = (\bar{x}, \ldots, \bar{x})$ in the small diagonal as in 3.1. We want to compute

$$\mathrm{Tr}(\tau \circ \sigma \circ \Phi_{T', \lambda'_{T'}}, [B]_{\underline{x}}) \qquad (7)$$

where τ is the cyclic permutation like in 3.1. A priori, it is not obvious how to compute this trace by counting points, since the action of the symmetric group is not concretely defined over the diagonals. This is however possible using local model theory and the geometric interpretation of the base change homomorphism in terms of perverse sheaves on the affine Grassmannian. What we get finally is (2) = (7).

3.3 Main observation

To prove (1) = (2) is now equivalent to proving (5) = (7). We can in fact prove a more general equality.

Theorem 3 *For all $\xi \in \pi_1((X'-I)^s)$ and $\phi \in \mathcal{H}^I$ and for the cyclic permutation $\tau \in \mathfrak{S}_r$, we have*

$$\mathrm{Tr}(\tau \circ \xi \circ (\underbrace{1 \otimes \cdots \otimes 1}_{s-1} \otimes \Phi), [A]_{\underline{x}}) = \mathrm{Tr}(\tau \circ \xi \circ \Phi, [B]_{\underline{x}}) \tag{8}$$

Heuristically, assuming Theorem 2, equality (8) can be proved as follows. In comparing (6) with (4) one can observe that (6) consists essentially in the diagonal terms $\pi_1 = \cdots = \pi_s$ of (4), up to multiplicity. But the non-diagonal terms of (4) are permuted around by τ and therefore don't contribute to the trace. The diagonals terms of (4) give now the same trace as (6) according to the following general linear algebra lemma which is implicit in papers of Saito and Shintani on base change.

Lemma 4 *Let V be a finite dimensional vector space over some field K. Let f be any endomorphism of V. Then*

$$\mathrm{Tr}(f, V) = \mathrm{Tr}(\tau \circ (\underbrace{1 \otimes \cdots \otimes 1}_{s-1} \otimes f), V^{\otimes s})$$

where τ is the cyclic permutation.

3.4 Tchebotarev's density theorem

The rigourous proof of Theorem 3 makes essential use of Tchebotarev's density theorem. Let $U = (X'-I)^s - \Delta$ be the complement of the union of all diagonals. Let $\tilde{U} = U/\langle\tau\rangle$ the free quotient of U by the cyclic group $\langle\tau\rangle = \mathbb{Z}/s\mathbb{Z}$ generated by τ. One has the exact sequence of fundamental groupoids

$$1 \to \pi_1(U) \to \pi_1(\tilde{U}) \to \mathbb{Z}/s\mathbb{Z} \to 1.$$

Any closed point $\tilde{u} \in |\tilde{U}|$ gives rises to a conjugacy class $\mathrm{Frob}_{\tilde{u}}$ of $\pi_1(\tilde{U})$. A closed point $\tilde{u} \in |\tilde{U}|$ is called **cyclic** if the image of $\mathrm{Frob}_{\tilde{u}}$ in $\mathbb{Z}/s\mathbb{Z}$ is the generator τ. By Tchebotarev's theorem, it is enough to prove

$$\mathrm{Tr}(\mathrm{Frob}_{\tilde{u}} \circ (\underbrace{1 \otimes \cdots \otimes 1}_{s-1} \otimes \Phi), [A]) = \mathrm{Tr}(\mathrm{Frob}_{\tilde{u}} \circ \Phi, [B])$$

for all cyclic closed points $\tilde{u} \in |\tilde{U}|$ and for all $\Phi \in \mathcal{H}^I$.

Since we are away from the diagonals one can compute the above traces by counting points without using local model theory. The nice feature of cyclic points is that in the expressions of traces of cyclic points on $[A]$ and $[B]$, there are no twisted orbital integrals. The expressions we get for the traces of cyclic points on $[A]$ and on $[B]$, are in fact identical.

Note that even outside the diagonals, if we take $\mathrm{Frob}_{\bar{u}}^2$ instead of $\mathrm{Frob}_{\bar{u}}$, the expressions we gets for the traces on $[A]$ and $[B]$ are no longer identical due to the apperance of twisted orbital integrals on both side. Therefore our proof relies heavily on Tchebotarev's theorem.

The proof in the case $n > 1$ is a little more complicated since the closed points of $X^{ns}/(\mathbb{Z}/s\mathbb{Z})$ are not as nice as those of $X^s/(\mathbb{Z}/s\mathbb{Z})$. For that case, we made essential use of a theorem of Drinfeld asserting that the representations of $\pi_1((X'-I)^{ns})$ on $[A]$ and on $[B]$ factor through $\pi_1(X'-I)^{ns}$. Consequently, instead of closed points $X^{ns}/(\mathbb{Z}/s\mathbb{Z})$ we can take collections of n cyclic closed points of $X^s/(\mathbb{Z}/s\mathbb{Z})$. We refer again to [7] for more details.

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