数理解析研究所講究録 1462 巻 2006 年 131-144

Comparison of insider's optimal strategies, three different types of side information RIMS symposium, the 7th workshop on Stochastic Numerics, June 27-29, 2005

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Caroline's paper [16] deals with the following problem on a financial market. This one is built with a riskless asset and d risky assets, which are driven by a $m-$dimensional Brownian motion and a $n-$dimensional Poisson process.

The traders have different information on the market, we say that they are in an asymmetric information situation:

(i) initial strong information : at the early beginning, on time $t=0$, the insider knows $L\in\mathcal{F}_{T+\varepsilon}$, a value which will be revealed only after the end of the period T.

(ii) progressive strong information : the insider knows $L+\varepsilon_t$ at time t, he knows something more than a non insider trader, and this information is more and more precise as time evolves; for instance, the variance of random variable ε_t goes to 0 when t goes to T.

(iii) weak information : he knows (or bets) the real law of L.

In each case, L could be the price of a risky asset after time T. The purpose is to obtain optimal strategies in each case and to compare them, analytically if possible, numerically with simulations if not.

1 THE MODEL

On a filtered probability space $(\Omega,\mathcal{A},\mathrm{F}_T:=(\mathcal{F}_t),t\in[0,T],\mathbb{P})$, we consider W, a m-dimensional Brownian motion, and N, a n-dimensional Poisson process, with intensity κ.

$d = m + n$. On such a space, there exists:

$$\left\{ \begin{array}{l} \text{riskless asset (bond)} \; : dB(t) = B(t)r(t)dt, \; B(0) = 1; \\ d \text{ risky assets} \; : dP_t^i = P_{t-}^i [b_t^i dt + \sum_{j=1}^{d} \sigma_j^i(t) d(W^*, N^*)_j^*(t)]. \end{array} \right\}.$$

The filtration F_T is generated by the process (W, N), completed and càd. We suppose that r, b, σ are F_T-predictable and such that the SDE admits a unique strong solution. Moreover, σ is invertible, strictly non degenerate, $\mathcal{F}_t$ is the public information at time t, and we denote $\mathcal{G}_t$ the insider's information at time t.

On such a market, the agents can invest and their aim is to optimize the utility of their terminal wealth in the set of **admissible** portfolios π, i.e. π is predictable with respect to the agents' own filtration, namely G_T,

$$\int_0^T ||\sigma_t^* \pi_t||^2 dt < \infty,$$

and discounted associated wealth *(self-financing hypothesis)*

$$dB_t^{-1} X_t^\pi = B_t^{-1} \langle \pi_t, b_t - r_t \mathbf{1}_d \rangle dt + B_t^{-1} \pi_t^i (\sigma_j^i dW_t^j + \sigma_k^i dN_t^k), \tag{1}$$

X has to be bounded below, initial wealth $X_0 \in L_1^+(\mathcal{G}_0)$, $X_T \geq 0$.

Henceforth, we suppose there is no Poisson component in the drivers, for sake of simplicity, but the results are quite analogous in case of a Poisson component.

In the non informed case, with logarithmic utility of the terminal wealth, we get as optimal wealth and optimal portfolio:

$$\begin{aligned} B_t^{-1} \hat{X}_t &= X_0 (Y_t^0)^{-1} \; ; \; \hat{\pi}_t = \hat{X}_t (\sigma_t^*)^{-1} (\sigma_t)^{-1} (b_t - r_t \mathbf{1}_d) = \hat{X}_t (\sigma_t^*)^{-1} l_0(t). \\ \text{Here } Y_t^0 &= E_{\mathbb{P}}[\frac{d\mathbb{P}^0}{d\mathbb{P}} / \mathcal{F}_t] \text{ where } \mathbb{P}^0 \text{ is risk neutral probability measure,} \\ dY_t^0 &= -Y_t^0 \sigma_t^{-1} (b_t - r_t \mathbf{1}_d) dW_t, \text{ and finally} \\ dW_t^0 &= dW_t + \sigma_t^{-1} (b_t - r_t \mathbf{1}_d) dt \text{ is a } (\mathcal{F}, \mathbb{P}^0) - \text{Brownian motion.} \end{aligned}$$

Remark that since σ is invertible, $\mathbb{P}^0$ is the unique risk neutral probability measure, so the market is both complete and viable.

In case of side information, π is no more $\mathcal{F}-$predictable and thus we need first to make sense to wealth equation (1), then to exhibit risk neutral probability measure $\hat{Q} = Y\mathbb{P}$ in each case.

The sketch of the proof is as following: for $\hat{Q}$ risk neutral probability measure a necessary and sufficient condition for admissibility is: $E_{\hat{Q}}[B_T^{-1} X_T^\pi / \mathcal{G}_0] \leq X_0$. Thus, to optimize $\pi \mapsto E_{\mathbb{P}}[\ln X_T^\pi / \mathcal{G}_0]$, we introduce the Lagrangian function

$$\mathcal{L}(X_T^\pi, \lambda) = E_{\mathbb{P}}[\ln X_T^\pi - \lambda Y_T (B_T^{-1} X_T^\pi - X_0) / \mathcal{G}_0],$$

and this yields the result.

2 THREE TYPES of SIDE INFORMATION

2.1 Initial strong information

In this case the trader's information is $L = P^1_{T+\varepsilon}$, for instance, from the beginning, we say that he has a strong information. The tool is the initial enlargement of filtration, the filtration $\mathcal{F}$ is enlarged with the knowledge of L, so we get the "strong" filtration as following

$$\mathcal{G}^S_t := \cap_{s>t}(\mathcal{F}_s \vee \sigma(L)), \quad 0 \leq t \leq T.$$

To make the wealth equation meaningful, we suppose:

$$(H^S) \quad \mathbb{P}\{L \in ./\mathcal{F}_t\} \sim \mathbb{P}\{L \in .\}, \ \forall t \in [0,T], \ a.s.$$

So let be p the density of probability: $\mathbb{P}\{L \in dx/\mathcal{F}_t\} = p(t,x)\mathbb{P}\{L \in dx\}$. Using Jacod's results [19], we get the predictable representation of this martingale as following: $dp(t,x) = p(t,x)\beta(t,x)dW_t$, we define $\rho^S_t := \beta(t,L)$, thus

$$dW^S_t = dW_t - \rho^S_t dt \text{ is a } (\mathcal{G}^S, \mathbb{P}) \text{ Brownian motion.}$$

Let the Doléans exponential

$$d(Z^S)^{-1}_t = -(Z^S_t)^{-1}\rho^S_t dW^S_t.$$

Thus we get a risk neutral probability measure for the insider's point of view:

$$\hat{Q}^S := Y^0(Z^S)^{-1}\mathbb{P} = Y^S\mathbb{P}.$$

As announced, a necessary and sufficient condition for admissibility is $E_{\hat{Q}^S}[B_T^{-1}X^\pi_T/\mathcal{G}^S_0] \leq X_0$. So we get a Lagrangian function to modelize the optimization problem under constraint:

$$\mathcal{L}(X^\pi_T, \lambda) = E_{\mathbb{P}}[\ln X^\pi_T - \lambda Y^S_T(B_T^{-1}X^\pi_T - X_0)/\mathcal{G}^S_0].$$

2.2 Progressive strong information

Some agents have an additional information more and more precise as time evolves. They know at time t the random variable $L_t = f(L, \varepsilon_t)$, with $Var(\varepsilon_t)$ decreasing as t goes to T, ε is a noise. The tool is the progressive enlargement of filtration as Corcuera et al. [6] do it, not as in [23]. So we get the "progressive enlarged" filtration as following

$$\mathcal{G}^P_t := \cap_{s>t}(\mathcal{F}_s \vee \sigma(L_u, u \leq s)), \quad 0 \leq t \leq T.$$

To make the wealth equation meaningful, we suppose:

(H^P) f is measurable, L is $\mathcal{F}_T$−measurable, $(\varepsilon_t, \ t \leq T)$ and $\mathcal{F}_T$ are independent,

$$\mathbb{P}\{L \in ./\mathcal{F}_t\} << \mathbb{P}\{L \in .\},\ \forall t \in [0,T],\ a.s..$$

Once again, let us define the density of probability: $\mathbb{P}\{L \in dx/\mathcal{F}_t\} = p(t,x)\mathbb{P}\{L \in dx\}$. Using Jacod's results [19], we get the predictable representation of this martingale as following:
$dp(t,x) = p(t,x)\beta(t,x)dW_t$, we define $\rho_t^P := E_{\mathbb{P}}[\beta(t,L)/\mathcal{G}_t^P]$, thus

$$dW_t^P = dW_t - \rho_t^P dt \text{ is a } (\mathcal{G}^P, \mathbb{P}) \text{ Brownian motion.}$$

We get a risk neutral probability measure with $d(Z_t^P)^{-1} = -(Z_t^P)^{-1}\rho_t^P dW_t^P$,

$$\hat{Q}^P := Y^0(Z^P)^{-1}\mathbb{P} = Y^P\mathbb{P}.$$

A (only) necessary condition (a budget constraint) for admissibility for π is

$$E_{\hat{Q}^P}[B_T^{-1}X_T^\pi/\mathcal{G}_0^P] \leq X_0.$$

It is not a characteristic property; nevertheless it is usefull to define the Lagrangian function and to exhibit an optimal strategy which will be, a posteriori, admissible:

$$\mathcal{L}(X_T^\pi, \lambda) = E_{\mathbb{P}}[\ln X_T^\pi - \lambda Y_T^P(B_T^{-1}X_T^\pi - X_0)/\mathcal{G}_0^P].$$

2.3 Weak information

Finally some agents have a weak information, in this case the trader's information is only the density of law of L: he/she knows ν, the law of L under $\mathbb{P}$. A weak informed agent doesn't know probability measure $\mathbb{P}$, but he /she knows the risk neutral probability measure $\mathbb{P}^0$ and bets the law of L under $\mathbb{P}$.

In such a case we don't need enlargement of filtration, but the common fact with the two previous cases is the equivalent change of measure with respect to the basic risk neutral measure $\mathbb{P}^0$. Following Fabrice Baudoin [5], we suppose that the law ν is equivalent to the law of L under $\mathbb{P}^0$ and that:

$$(H^W)\quad \mathbb{P}^0\{L \in ./\mathcal{F}_t\} \sim \mathbb{P}^0\{L \in .\},\ \forall t \in [0,T],\ a.s.$$

Under such an hypothesis Fabrice Baudoin shown that there exists a unique probability measure $\mathbb{P}^\nu \sim \mathbb{P}^0$ such that $\forall X$ bounded,

$$E_{\mathbb{P}^\nu}[X/L] = E_{\mathbb{P}^0}[X/L] \text{ and the law of } L \text{ under } \mathbb{P}^\nu \text{ is } \nu.$$

Then Caroline defines process $\zeta_t = E_{\mathbb{P}^0}[\frac{d\mathbb{P}^\nu}{d\mathbb{P}^0}/\mathcal{F}_t]$. This process ζ is a $(\mathcal{F}, \mathbb{P}^0)$−martingale and using the predictable representation property, there exists process ρ^W such that $d\zeta_t = \zeta_t\rho_t^W dW_t^0$. In this case we define $Y^W := (\zeta)^{-1}$ and we get a link between the probability measures:

$$\mathbb{P}^0 = Y^W\mathbb{P}^\nu.$$

Once again, a necessary and sufficient condition (a budget constraint) for admissibility is

$$E_{\mathbb{P}^0}[B_T^{-1}X_T^\pi] \leq X_0.$$

The weak informed agent wants to optimize $\ln X_T^\pi$ under his own probability measure $\mathbb{P}^\nu$, so we get the Lagrangian function:

$$\mathcal{L}(X_T^\pi, \lambda) = E_{\mathbb{P}^\nu}[\ln X_T^\pi - \lambda Y_T^W (B_T^{-1}X_T^\pi - X_0)].$$

3 OPTIMIZATION PROBLEM

In these three cases the optimization problem is formally the same. We only deal with an equivalent change of probability measure and thanks to the kind form of the utility function (logarithm) we can exhibit the optimal portfolio. In any case, the agents have to optimize

$$\pi \to E_{\mathbb{P}}\left[\ln(X_T^\pi)/\mathcal{G}_0\right],$$

respectively concerning a weak insider agent

$$\pi \to E_{\mathbb{P}^\nu}\left[\ln(X_T^\pi)\right].$$

under the constraint that π is G_T-predictable admissible, meaning the budget constraint $E_{\hat{Q}}[B_T^{-1}X_T^\pi/\mathcal{G}_0] \leq X_0$. So we get a Lagrangian function and we obtain the optimal policy:

$$\forall t \in [0,T], \quad \left\{ \begin{array}{rl} B_t^{-1}\widehat{X}(t) & = X_0 \frac{1}{Y}(t) \\ \widehat{\pi}(t) & = (\sigma^*(t))^{-1}\widehat{X_t}(l_0(t) + \rho_t), \end{array} \right\}$$

where $l_0(t) = \sigma_t^{-1}(b_t - r_t)$ and ρ is the "drift information" (cf. [18], [4]) respectively: ρ^S, ρ^P, ρ^W. $W - \int \rho$ is a $\mathbb{P}$-Brownian motion for the enlarged filtration (but in case of weak information $W^0 - \int \rho^W$ is a $\mathbb{P}^\nu$-Brownian motion for the natural filtration) and Y is respectively: Y^S, Y^P, Y^W.

In conclusion, the comparison of agents'strategies relies on the comparison of processes Y in the three cases of side information; more precisely, the optimal portfolios are based upon the drift information.

4 COMPARISON, SIMULATIONS

The key of the simulations is the generation of process Y in the three cases.

4.1 Analytic comparison, based on processes Y.

An analytic comparison of insider's optimal portfolio with respect to a non insider one is available. For instance, let us suppose that the insider knows $L = \mathbf{1}_{[a,b]}(P^i_{T+\varepsilon})$ for some i.

Let be t not a time jump, then $\frac{\widehat{\pi}}{\widehat{X}}(t) = (\sigma^*(t))^{-1}(l_0(t) + \rho_t)$. We denote as $\rho_i, i = 1, 2$, the components of ρ corresponding to the Brownian part, respectively the Poisson part.

$$\begin{aligned} \frac{\widehat{\pi}}{\widehat{X}}(t) - \frac{\widehat{\pi_0}}{\widehat{X_0}}(t) &= (\sigma^*(t))^{-1}(l_t - l_0(t)) = (\sigma^*(t))^{-1}\left((\rho_1)^*, (\frac{\rho_2 - \mathbb{I}_2}{q})^*\right)^*(t) \\ &= (\sigma^*(t))^{-1}\left(\sigma_{iW}(t), \frac{\ln(1 + \sigma_{iN}(t))}{q}\right)^* a(t). \end{aligned}$$

In a purely diffusive market

$$\frac{\widehat{\pi}}{\widehat{X}}(t) - \frac{\widehat{\pi_0}}{\widehat{X_0}}(t) = (\sigma^*(t))^{-1}\sigma_i^*(t)a(t) = (0, \cdots, 0, \overbrace{a(t)}^{\text{component } i}, \cdots, 0)^*.$$

Another example is when the insider knows $L = \ln(P^1_{T+\varepsilon}) - \ln(P^2_{T+\varepsilon})$.

Let be t not a jump time, then

$$\frac{\widehat{\pi_k}}{\widehat{X_k}}(t) - \frac{\widehat{\pi_0}}{\widehat{X_0}}(t) = (\sigma^*(t))^{-1}\left((\sigma_{1W} - \sigma_{2W})(t), \frac{1}{q}\ln\left(\frac{1 + \sigma_{1N}}{1 + \sigma_{2N}}\right)(t)\right)^* a(t)$$

In a purely diffusive market,

$$(\sigma^*(t))^{-1}(\sigma_{1W} - \sigma_{2W})^*(t) = (1, -1, \cdots, 0, \cdots, 0)^*.$$

In both cases, a(t) is a scalar function depending on the side information.

4.2 Simulations

Here we present the simulation of portfolio parts on bond and known asset for a non informed agent (interrupted line) and an informed agent (continuous line) in the figures 4.2 and 2. This is in a continuous case (no jumps), the information is $L = \mathbf{1}_{[0.85\ ;\ 1.2]}(P_T^1)$ and P_T^1 is supposed to be 0.5096. First are the bond component of the portfolios.

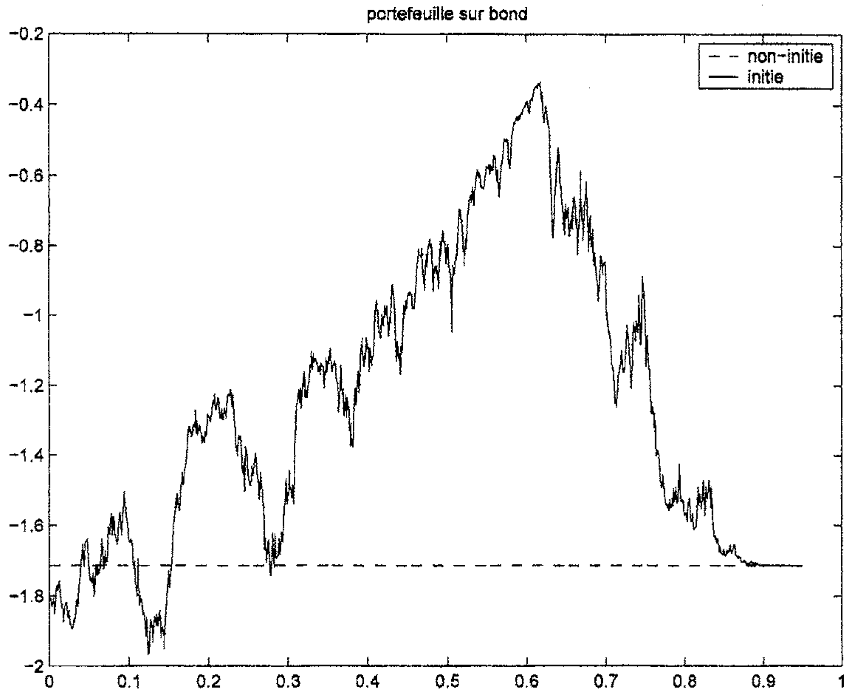

Figure 1: Portfolio on bond.

Then are the asset 1 component of the portfolios.

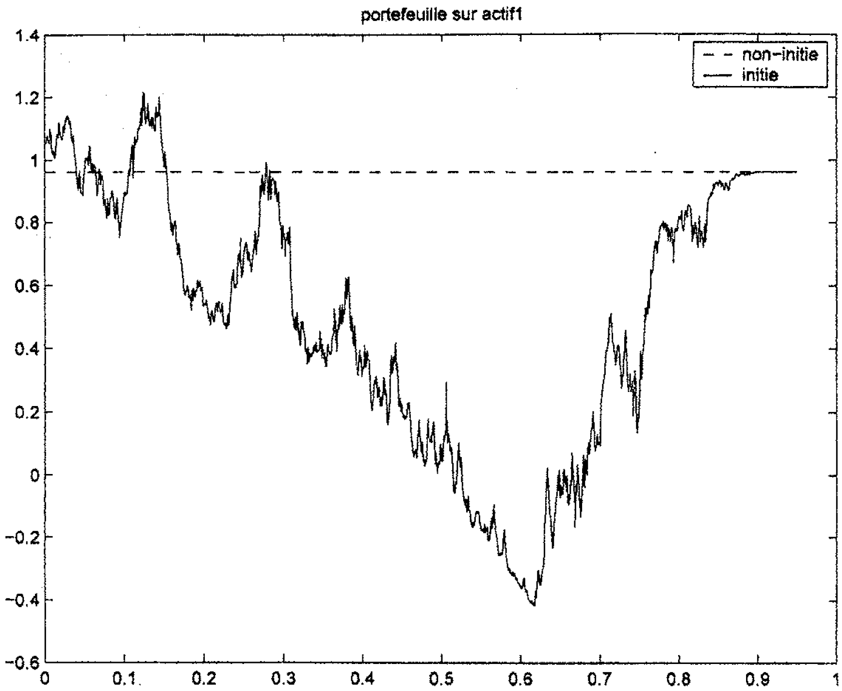

Figure 2: Portfolio on known asset.

Now, in case of mixed Brownian-Poisson process, here is the simulation of processes Y^{-1} and Y_0^{-1}; indeed, in any case, the insider's optimal wealth is $X_0 B_t Y^{-1}$ and the non insider one is $X_0 B_t Y_0^{-1}$. Here it is clear that the insider's wealth is more increasing than the non insider one. In this simulation and in the two next ones, $a = 0.7$, $b = 1.1$ and the information is $P_1(T) = 1.5422$. The first Poisson process jumps are on times 0.7039 and 0.9695, the second on times 0.8280 and 0.9129.

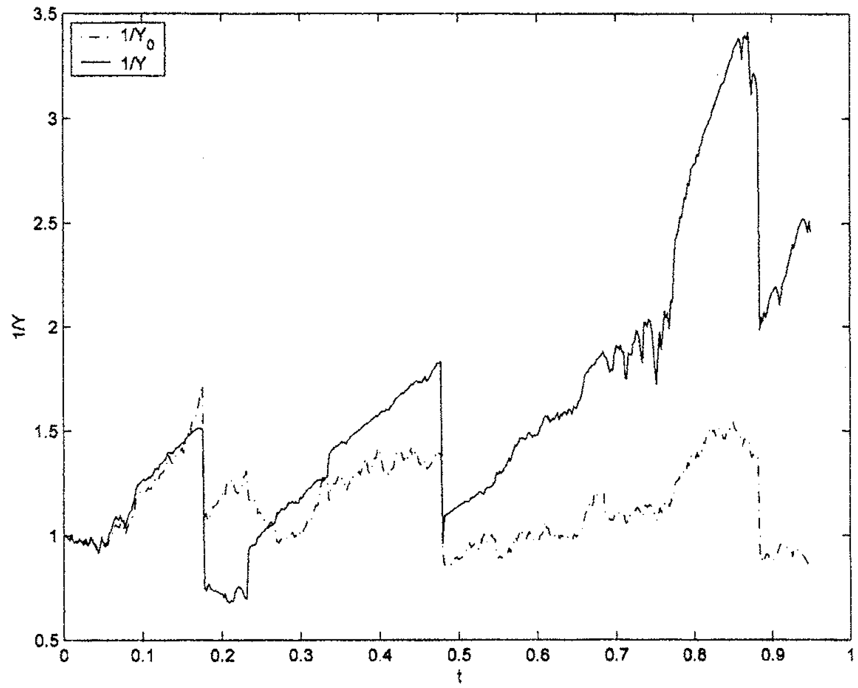

Figure 3: Simulation of processes Y^{-1}.

Finally, in case of mixed Brownian-Poisson process, we present the simulation of portfolio parts on bond and known asset for a non informed agent (interrupted line) and an informed agent (continuous line) in the figures 4 and 5.

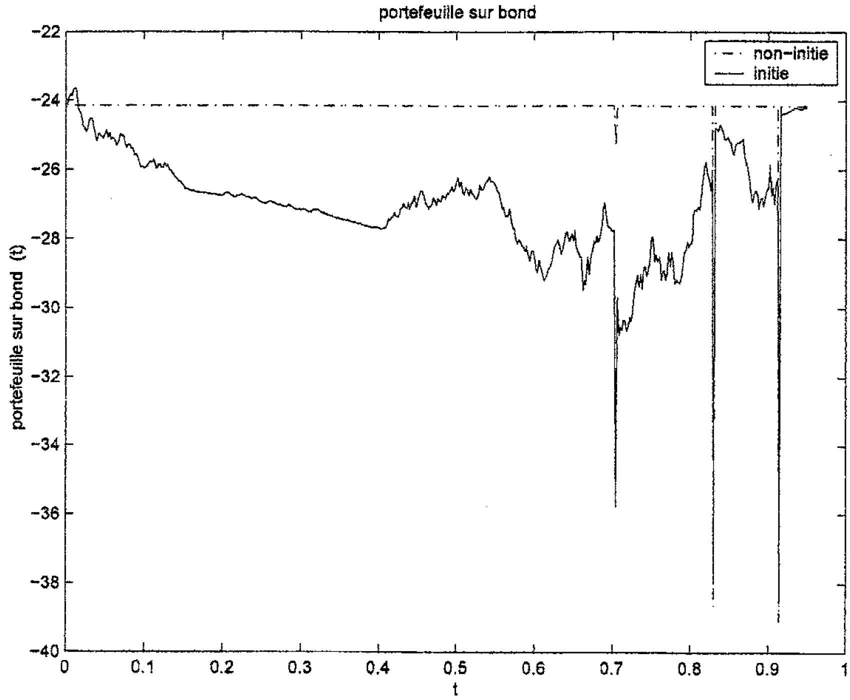

Figure 4: Portfolio on bond.

Then are the asset 1 component of the portfolios.

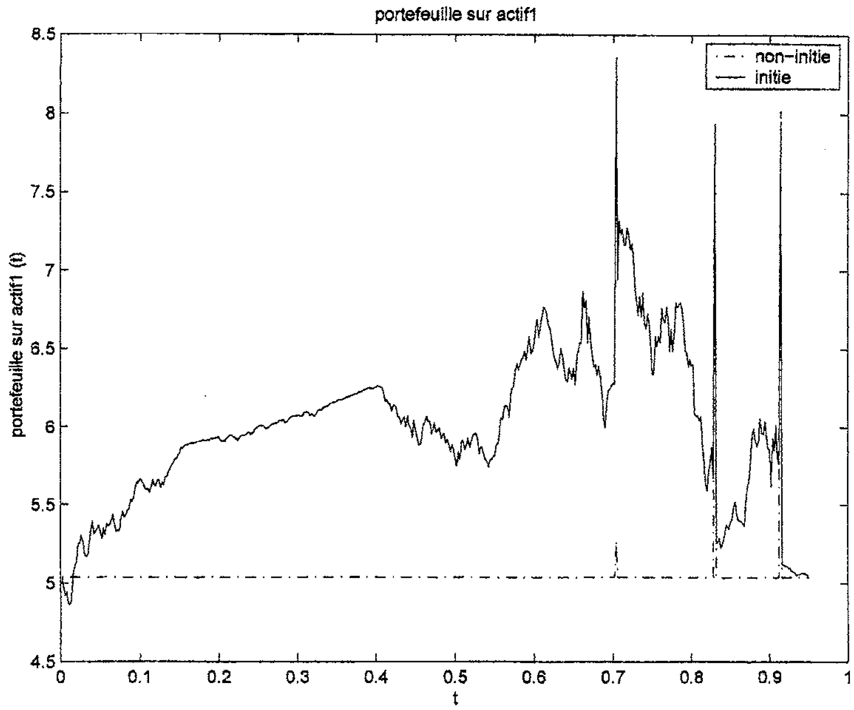

Figure 5: Portfolio on known asset.

Finally, we present the comparison of differently informed agents'wealth, clearly, the more informed they are, the greater wealth they have.....

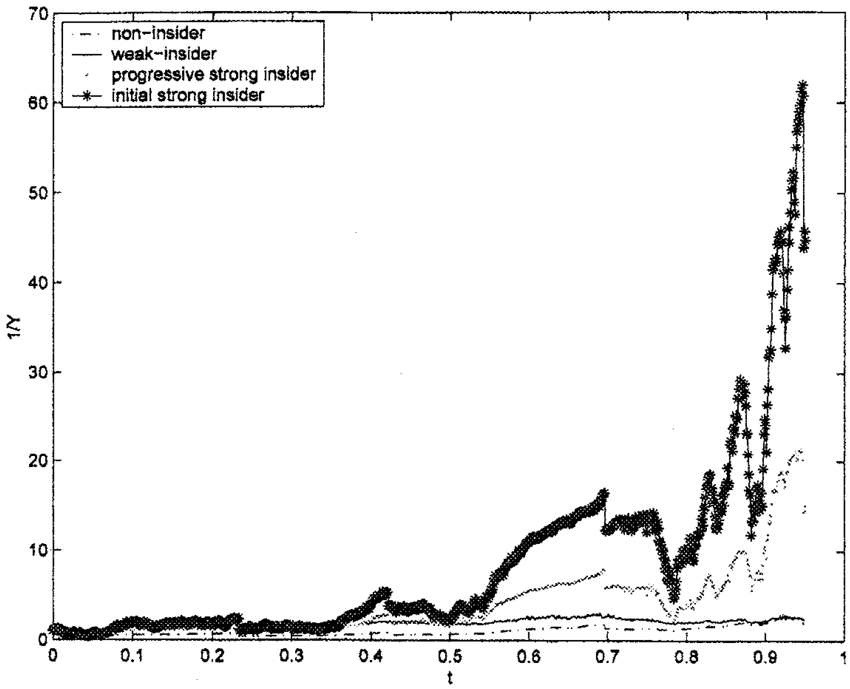

Figure 6: Differently informed agents' wealth.

5 CONCLUSION

We would like to stress an interesting point: in the case of a pure diffusive market, it is shown that the comparison between insider's portfolios and non insider's one is quite simple. Thus, if the question is to detect an insider trading, we can exploit this fact. If somebody chooses a portfolio with one or more component, ith component for instance, opposite to the ones of other traders, we can guess that he has an information on the ith asset.

This is to be connected with AMF (Autorité des marchés financiers)'s process: if somebody realizes π^i wich is over the 0.95 quantile, the AMF decides an inquiry on the trader who did it.

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