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Ground States for $2D$ Spin Glasses

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Edwards-Anderson (EA) Spin Glass

For $G_N = (V_N, E_N)$, graph with N vertices, let

$$H_N(\sigma) = -\sum_{(x,y)\in E_N} J_{xy}\sigma_x\sigma_y \ .$$

with $J := \{J_{xy}\}$ indep. mean zero Gaussian (or other).

- For SK model [SK], G_N is the complete graph.
- For EA model [EA], $G_N \subset \mathbb{Z}^d$.
- Spin glasses exhibit many, very different, states of low energy that lead to complex behavior.
- How many ground states as $N \to \infty$?
- How different is EA than SK? For what d?

Gibbs States and Ground States

- The Gibbs measure is concentrated on σ's with small H_N.

$$\mathcal{G}_{\beta,N}(\sigma) = \frac{\exp -\beta H_N(\sigma)}{Z_N(\beta)}$$

- $\mathcal{G}_{\beta,N}$ gives information on σ's close to the minimum.

Take $G_N = (V_N, E_N)$, a box in $\mathbb{Z}^d$ (of volume N).

- Take periodic b.c. (so $H_N(\sigma) = H_N(-\sigma)$).
- Write $\alpha := \{\sigma, -\sigma\}$, a **Ground State Pair (GSP)**.
- Study the measure on the unique GSP $\alpha_N(J)$: $\delta_{\alpha_N(J)}$.
- Equivalent to studying Gibbs state with

$$\beta \to \infty, \text{ then } N \to \infty .$$

Metastate [AW, NS1]: a measure on GSP's

- $\alpha_{N,J}$ could change drastically between N and N'.
- Consider the joint distribution of $(\alpha_{N,J}, J)$,

$$K_N := \delta_{\alpha_{N,J}} \; \nu_N(dJ) \; .$$

- Take a (subsequence) limit of K_N as $N \to \infty$.
- Express the limit K as $K_J \; \nu(dJ)$.

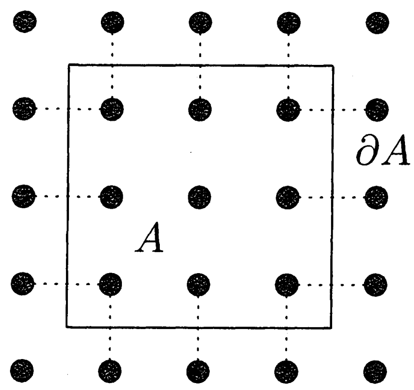
- $K_J \; \nu(dJ)$ is **translation invariant**. (Periodic b.c.)
- K_J is a measure on GSP's on $\mathbb{Z}^d$ for given J.

Ground States in Infinite Volume

Definition (Ground State Property)

$\alpha = \{\sigma, -\sigma\}$ *is a GSP on* $\mathbb{Z}^d$ *for* J *if for any finite* $A \subset \mathbb{Z}^d$

$$- \sum_{(x,y)\in\partial A} J_{xy}\sigma_x\sigma_y < 0 \ .$$



Ground States in Low Dimension

Conjecture (Uniqueness of Ground States)

For low d ($d < d_c = 6?, = 8?, = \infty?$),

- *the limit $K = K_J\,\nu(dJ)$ exists (no subseq. needed);*
- *K_J is supported on a single GSP.*

($d = 2$ numerics of Palassini-Young [PY], Middleton [M])

Strategy:

1. Let α and α' be replica GSP's.
2. Study the **interface**:

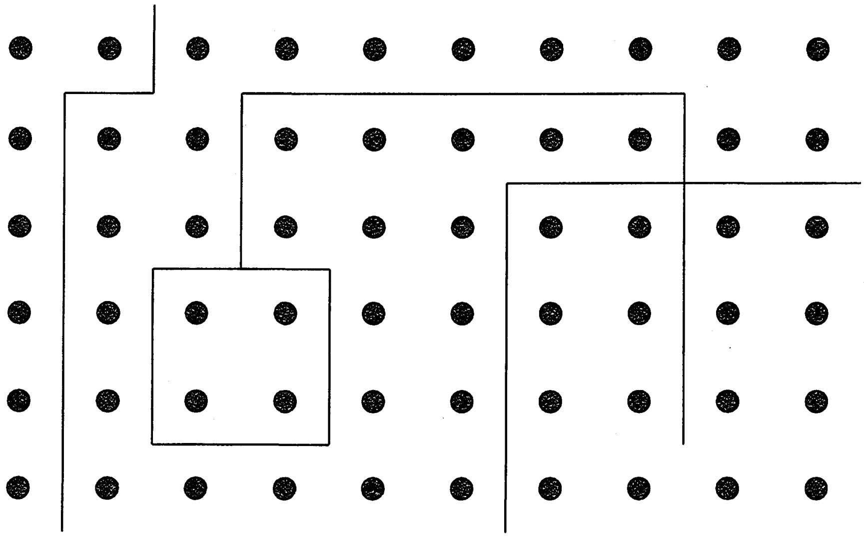
$$\alpha\Delta\alpha' := \{(x,y) : \alpha_{xy} \neq \alpha'_{xy}\} \,,$$

   where $\alpha_{xy} := \sigma_x\sigma_y$.

3. Show $\alpha\Delta\alpha'$ is empty (hence $\alpha = \alpha'$).

Interfaces between GSP's

- $\alpha\Delta\alpha' = \{(x, y) : \alpha_{xy} \neq \alpha'_{xy}\}$.
- Put a dual edge whenever $(x, y) \in \alpha\Delta\alpha'$.



Full Plane Partial Result

If α and α' from metastate are distinct; then

- $\alpha\Delta\alpha'$ cannot have dangling ends (or 3-branching points).
- cannot contain loops.
- cannot have 4-branching points.

So $\alpha\Delta\alpha'$ is one or more doubly-infinite (self-avoiding) paths.

Theorem (Newman, Stein [NS2])

In fact, non-empty $\alpha\Delta\alpha'$ can only be a **single** *path.*

Uniqueness of GSP's in the Half-Plane

Take $G_N = [-N, N] \times [0, 2N] \cap \mathbb{Z}^2$ with horizontal periodic and vertical free b.c. and let $N \to \infty$.

Horizontal but not vertical translation invariance

Theorem (Arguin, Damron, Newman, Stein [ADNS])
In the half-plane,

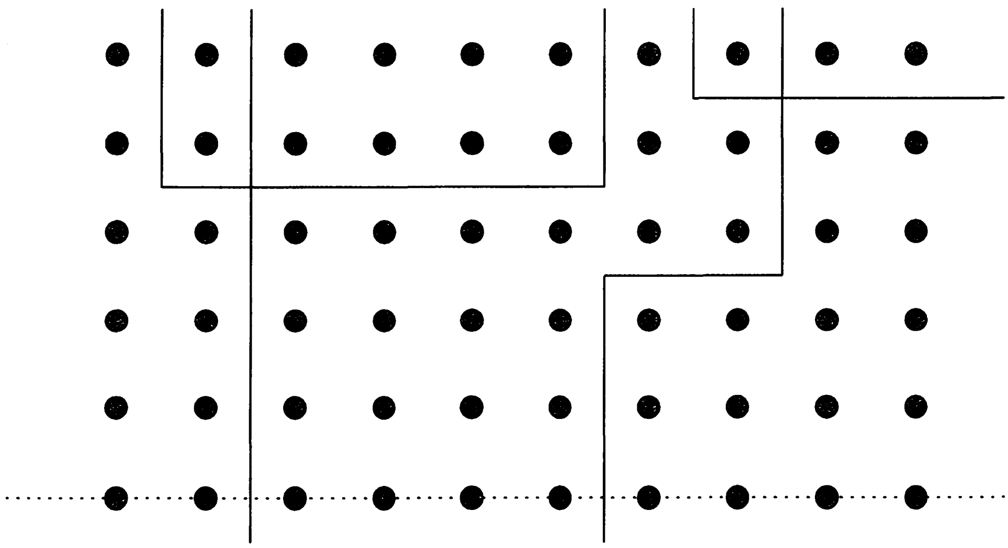
- *the limit* $K = K_J \; \nu(dJ)$ *exists (no subseq.);*
- K_J *is supported on a single GSP.*

This is the first complete result for $d > 1$.

Strategy — show interface between replicas must be empty.

Interface of GSP's in the Half-Plane

If $\alpha \Delta \alpha' \neq \emptyset$, then there are infinitely many **tethered paths**:



From Half to Full Plane

Let $\mu^* = \lim_{k\to\infty} \frac{1}{k} \sum_{l=1}^{k} T^{-l}\mu$ (along subseq.) where T is the vertical shift and μ is distrib. of (α, α', J).

- μ^* is a measure on the **full plane**,
- translation invariant in full plane by construction.

Because **we see many tethered paths**, if $\alpha\Delta\alpha' \neq \emptyset$, then $\alpha\Delta\alpha'$ is not a single path, contradicting full plane result of [NS2].

To do list for the future:

- Uniqueness of GSP's in the full plane.
- Uniqueness (or not) of GSP's for $d > 2$?
- Metastates applied to $\beta < \infty$.

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