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SEMIGROUPS PRESENTED BY FINITE CONGRUENCE CLASSES *

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In this article, we study semigroups presented by finite congruence classes.

Definition 1 (1) *Let* X *be finite alphabets and* R *a subset of* $X^{+}\times X^{+}$. *Then* R *is called a rewriting system.*

(2) *For* $u, v\in X^{+}$, $(w_{1}, w_{2})\in R$, $uw_{1}v\Rightarrow_{R}uw_{2}v$.

The congruence μ_{R} *on* X^{+} *generated by* $\Rightarrow_{R}$ *is called the Thue congruence defined by* R.

(3) *A semigroup* S *is (finitely)presented if there exists a (finite) set of* X, *there exists a surjective homomorphism* ϕ *of* X^{+} *to* S *and there exists a (finie) rewriting system* R *consisting of pairs of words over* X *such that the Thue congruence* μ_{R} *is the congruence* $\{(w_{1}, w_{2})\in X^{*}\times X^{+}\mid\phi(w_{1})=\phi(w_{2})\}$.

In this case, we say that S *has a presentaion by* X *and* R *denoted by* $S=<X:R>$.

Definition 2 *A semigroup* S *has a presentation with regular [resp. finite] congruence classes if there exists a finite set* X *and there exists a surjective homomorphism* ϕ *of* X^{+} *to* M *such that for each word* $w\in X^{+}$, $\phi^{-1}(\phi(w))$ *is a regular [resp. finite] language.*

In [3], we investigated the properties of semigroups presented by regular congruence classes. The class of such semigroups contains many important classes of semigroups, for example, Burnside semigroups ([1]), Baumslag-Solitar semigroups ([2]) and so on. Even the class of semigroups presented by finite congruence classes looks so interesting. So we give a characterization of one-relator rewriting systems all Thue congruence classes of that are finite.

*This is an absrtact and the paper will appear elsewhere.

Definition 3 *A word u over a finite alphabet X is unbordered if there exists no non-empty word v with* $u \in vX^+ \cap X^+v$.

We have

Theorem. *Let* u, w *be word over a finite alphabet* X *and* $R = \{(u, w)\}$ *a one-relator rewriting system. Assume that* u *is an unbordered and the length of* u *is shorter than one of* w. *Further, assume that* u *is not a subword of* w. *Then the rewriting system* $R = \{(u, w)\}$ *gives only finite congruence classes if and only if there are no non-empty words* $l_{i,j}, r_{i,j}$ *over* X *such that* $u = l_{s,t}r_{s,t}$ $(1 \leq s \leq 2k, 1 \leq t \leq i_s)$, $w \in r_{1,1} \cdots r_{1,i_1}X^+$, $w \in X^+l_{2,i_2} \cdots l_{2,1}l_{1,i_1}$, $\cdots$, $w \in X^+l_{2k,i_k} \cdots l_{2k,1}l_{2k-1,i_{2k-1}}$ *and* $l_{2k,i_k} = l_{1,1}$.

References

[1] V. Guba, *The word problem for the relatively free semigroup satisfying* $T^m = T^{m+n}$ *with* $m \geq 4$ *or* $m = 3, n = 1$ Internat. J. Algebra Comput. **3**(1993), no. 2, 125-140.

[2] D. Jackson, *Decision and separability problems for Baumslag-Solitar semigroups* Internat. J. Algebra Comput. **12**(2002), no. 1-2, 33-49.

[3] K. Shoji, *Finitely generated semigroups which have such a presentation that all the congruence classes are regular languages,* Math. Japonicae **69**(2008), 73-78.