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On a characteristic property of the tent map

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As a generalization of a characteristic property of fundamental chaotic maps such as the tent map, a primitive chaos is defined by $(X,\ \{X_{\lambda},\ \lambda\in\Lambda\},\ \{f_{X_{\lambda}},\ \lambda\in\Lambda\})$ which satisfies the following property, where X is a set, $\{X_{\lambda},\ \lambda\in\Lambda\}$ is a family of subsets of X which contains no empty set, and $\{f_{X_{\lambda}},\ \lambda\in\Lambda\}$ is a family of maps $f_{X_{\lambda}}:X_{\lambda}\rightarrow X$ [1].

(P) For any infinite sequence $\omega_{0},\ \omega_{1},\ \omega_{2},\ldots$ of sets, there exists an initial point $x_{0}\in\omega_{0}$ such that $f_{\omega_{0}}(x_{0})\in\omega_{1},\ f_{\omega_{1}}(f_{\omega_{0}}(x_{0}))\in\omega_{2},\ldots$, where $\omega_{i}\in\{X_{\lambda},\ \lambda\in\Lambda\}$ for each i.

The primitive chaos is closely related to general problems about determinism, causality, free will, and irreversibility [1], which are vital problems in science (e.g., refs. [2-26]).

Then, the following propositions present sufficient conditions for guaranteeing the primitive chaos.

Proposition 1 [1]. *If X is a nondegenerate Peano continuum, for any $\varepsilon>0$, there exist finitely many nondegenerate Peano subcontinua $X_{1},\ldots,X_{n}$ covering X such that dia $X_{i}<\varepsilon,\ i=1,\ldots,n$. Then, for each i, for any positive integer n^{i}, for any n^{i} points $x_{1}^{i},\ldots,x_{n^{i}}^{i}\in X_{i}$ and $y_{1}^{i},\ldots,y_{n^{i}}^{i}\in X$, there exists a continuous surjection $f_{X_{i}}:X_{i}\rightarrow X$ such that $f_{X_{i}}(x_{1}^{i})=y_{1}^{i},\ldots,f_{X_{i}}(x_{n^{i}}^{i})=y_{n^{i}}^{i}$, and they satisfy the property* (P).

Proposition 2 [27]. *If X is a Cantor set, for any positive integer n, there exist a partition $\{X_1, \ldots, X_n\}$ of X and maps $f_{X_i} : X_i \to X$, $i = 1, \ldots, n$, and they satisfy the property (P).*

Here, a nondegenerate space means the space that consists of more than one point, a Peano continuum is a locally connected continuum, and a continuum is a nonempty connected compact metric space. A Cantor set is any space that is homeomorphic to the Cantor middle-third set, and a space is a Cantor set if and only if it is a zero-dimensional, perfect compact metric space [28, Theorem 8.1].

Proposition 1 explains the reason why we are surrounded by diverse chaotic behaviors [29], and Proposition 2 implies the possibility of the Cantor set for a new recognition of natural phenomena.

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