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層流—乱流遷移境界層渦のクラスタ解析

Clustering analysis of vortices appearing in the process of laminar-turbulent transition

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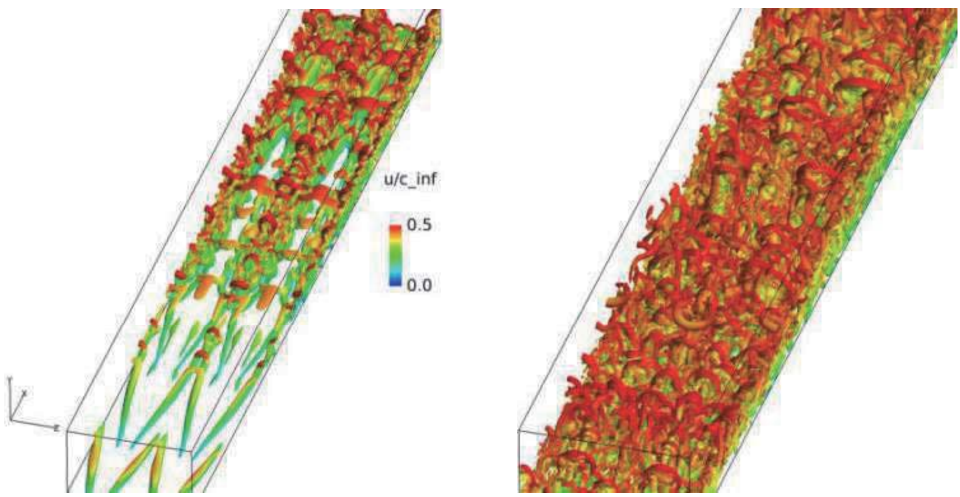
1 Introduction

Various vortices appear in the process of laminar-turbulent transition in boundary layers as shown in Fig. 1. The shapes and dynamics strongly depend on the environment of the shear layers and also the route of the process. In this study, a new data mining method which reduces the degree of freedom of vortices, and can do list management of them by putting cluster IDs on the vortices is proposed. The proposed method is applied to vortices appearing in the late stage of natural K-type transition.

2 Data Mining Method

The proposed method consists of three steps. The first step is to visualize vortex tubes by iso-surfaces such as the iso-surfaces of the second invariance of the velocity gradient tensor (SIVGT), i.e., Q or λ_2. The second step is to find out interior points within the vortex tubes, i.e., particle representation. The third step is to cluster the interior points.



(a) Re_x=3.00×10^5-5.60×10^5　　(b) Re_x=5.60×10^5-7.17×10^5

Fig. 1 Vortical structures appearing in the late stage of K-type transition, which are visualized by the iso-surfaces of SIVGT. The color shows a streamwise velocity divided by a sound speed, i.e., Mach number.

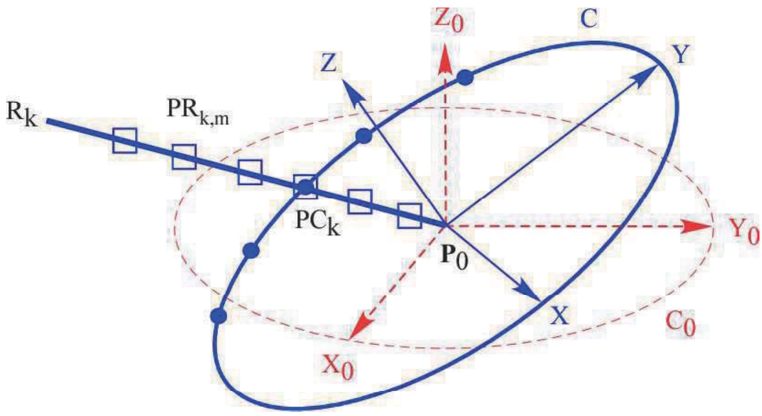


Fig. 2 Coordinate system for judging interior points of vortices enclosed by an iso-surface

2.1 Extraction of interior points (particle representation)

Interior region of a vortex tube is expressed by particles extracted by the newly devised algorithm [1]. This algorithm extracts mesh points enclosed by the iso-surface of a function $f(x,y,z)$=const. Although arbitrary function can be candidates for $f(x,y,z)$, $f(x,y,z)$ is SIVGT here.

The coordinate system for explaining this algorithm is shown in Fig. 2. First, a mesh point $\mathbf{P}_0$ is judged as an interior point or not. Around $\mathbf{P}_0$, a unit circle C is considered in $\mathbf{R}^3$. On the circle, equispaced points PC_k, k=1,…,M_1 are generated. Here, the circle C around $\mathbf{P}_0$ is generated by rotating in $\mathbf{R}^3$ a unit circle C_0, $(x,y,z)=(\cos\Phi, \sin\Phi, 0)^T$. Using Euler angle, this transformation is expressed as follow [2]:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = T(\alpha,\beta,\gamma)\begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix},$$
$$T(\alpha,\beta,\gamma)= \begin{pmatrix} \cos\gamma\cos\beta\cos\alpha - \sin\gamma\sin\alpha & \cos\gamma\cos\beta\sin\alpha + \sin\gamma\cos\alpha & -\cos\gamma\sin\beta \\ -\sin\gamma\cos\beta\cos\alpha - \cos\gamma\sin\alpha & -\sin\gamma\cos\beta\sin\alpha + \cos\gamma\cos\alpha & \sin\gamma\sin\beta \\ \sin\beta\cos\alpha & \sin\beta\sin\alpha & \cos\beta \end{pmatrix} \quad (1)$$

From the center $\mathbf{P}_0$ to each PC_k, a line segment R_k is drawn. On the line segments, equispaced points $PR_{k,m}$, m=1,…, M_2 are generated. From an ordered set of the values of $f(PR_{k,m})$ obtained by linear interpolation, the existence of the iso-surface of $f(x,y,z)=\varepsilon$ on the line segment, i.e., the cutting of the line segment by the iso-surface, is judged. Here, ε is a threshold value used for visualizing vortices. If iso-surface of $f(x,y,z)=\varepsilon$ exists on all line segments $PR_{k,m}$, k=1,…,N on a circle C which can be obtained by rotating C_0 at P_0, i.e., if such a circle C can be found, then point P_0 is judged as an interior point. The length of R_k is taken as δ_{in} in this study. Here, δ_{in} is the displacement thickness of a boundary layer.

2.2 Clustering

From the previous step, a set of particle coordinates $\{\boldsymbol{x}_i \in \boldsymbol{R}^3, i = 1, \cdots, N\}$ is obtained. These points are divided into K clusters C_0, …,C_K. Three candidate clustering methods are investigated in this study.

2.2.1 Successive incorporation clustering

First, initial seed points belonging to different clusters $\boldsymbol{p}_0 \in C_0$, …,$\boldsymbol{p}_K \in C_K$ are assumed. For a pair $(\boldsymbol{x}_i, \boldsymbol{x}_j)$, a *logical*-type relationship of "connection", i.e., $con(\boldsymbol{x}_{i,} \boldsymbol{x}_j)$, is considered. If $\|\boldsymbol{x}_i - \boldsymbol{x}_j\| < \varepsilon$, and also there is no variation going through the selected Q-criterion along the segment $\overline{\boldsymbol{x}_i \boldsymbol{x}_j}$, $con(\boldsymbol{x}_{i,} \boldsymbol{x}_j) = TRUE$, and $con(\boldsymbol{x}_{i,} \boldsymbol{x}_j) = FALSE$ in other cases. When a cluster C_m is computed, connected points in terms of $con(\boldsymbol{x}_{i,} \boldsymbol{x}_j) = TRUE$ are successively incorporated from $\boldsymbol{p}_m$.

2.2.2 K-means clustering [3,4]

Here, we represent $\boldsymbol{x}_i = (x_{i1}, x_{i2}, x_{i3}), i = 1, \cdots, N$, and try to allocate each point to one of k clusters so as to minimize the within-cluster sum of squares:

$$\sum_{m=1}^{K} \sum_{i \in S_m} \sum_{j=1}^{3} (x_{ij} - \overline{x}_{mj})^2 \quad (2)$$

Where S_m is the set of points in the m-th cluster and $\overline{x}_{mj}$ is the mean for the variable j over cluster m. In addition, a K by 3 matrix giving the initial cluster centers for the K clusters is required. The points are the initially allocated to the cluster with the nearest cluster mean. The procedure is then to iteratively search for the K-partition with locally optimal within-cluster sum of squares by moving points from one cluster to another. In order to conduct the above clustering, NAG library routine *g03eff* was used [5].

2.2.3 Spectral clustering [6]

Normalized spectral clustering based on a fully connected graph is employed. Here, all points are simply connected with positive similarity with each other, and weights w_{ij} between points $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ are evaluated by

$$w_{ij} = \exp(-\|\boldsymbol{x}_i - \boldsymbol{x}_j\|^2 / (2\sigma^2)) \quad (3)$$

where the parameter σ controls the width of the neighborhoods. The weighted adjacency matrix of the graph is defined as $W = (w_{ij})_{i,j=1,\cdots,N}$. The degree of a vertex is defined as $d_i = \sum_{j=1}^{N} w_{ij}$, and the degree matrix D is defined as the diagonal matrix with the degrees $(d_i)_{i=1,\cdots,N}$ on the diagonal. The normalized Laplacian L_{rw} is computed as

$$L_{rw} = I - D^{-1}W \tag{4}$$

The first k generalized eigenvectors $u_1, \ldots, u_k$ of the generalized eigenvalue problem

$$Lu = \lambda Du \tag{5}$$

are computed. A matrix $U \in R^{n\times k}$ containing the vectors $u_1,\ldots,u_k$ as columns is constructed. For i=1,…,N, $y_i \in \boldsymbol{R}^k$ be the vector corresponding to the i-th row of U. The points $(\boldsymbol{y}_i)_{i=1,\cdots,N}$ in $\mathbf{R}^k$ are clustered into clusters $C_1,\ldots,C_K$ with the k-means algorithm. Finally, clusters $A_1, \ldots, A_k$ with $A_i = \{ j \mid \boldsymbol{y}_j \in C_i \}$.

3 Computational Cases

The above method is applied to the boundary-layer transition of K-regime without free-stream turbulence. In this scenario, disturbances comprising of a two-dimensional Tollmien-Schlichting wave and a pair of oblique waves are superimposed on the Blasius solution. The governing equations are the unsteady three-dimensional fully compressible Navier-Stokes equations written in general coordinates for body-fitted mesh geometries. The system of equations is closed by the perfect gas law. A constant Prandtl number of Pr=0.72 is assumed. The equations are solved a sixth-order finite-difference method. Time-dependent solutions to the governing equations are obtained using the third-order explicit Runge-Kutta scheme. The numerical details are explained in [7].

4 Results and Discussion

Figure 3 shows the results. Part (a) shows the vortical structures represented by the iso-surfaces of SIVGT, and also interior points enclosed by the iso-surface. Part (b) shows a cluster corresponding to an unstable hairpin leg extracted by the successive incorporation clustering algorithm. These results show that the present algorithm works successfully, and the present clustering method can selectively pick up a connected vortex structure (a leg part in this example), which is located close to other longitudinal vortices. The results of clustering by K-means and spectral clustering algorithms are also shown in Fig. 4. Some clusters are distributed over separate vortex structures, and thus erroneous results are obtained.

5 Conclusions

The new data mining method which reduces the degree of freedom of vortices, and can do list management of them by putting cluster IDs on the vortices is proposed. The proposed method is applied to natural transition. It is found that the present method with successive incorporation clustering can successfully extract unstable hairpin leg.

Acknowledgements

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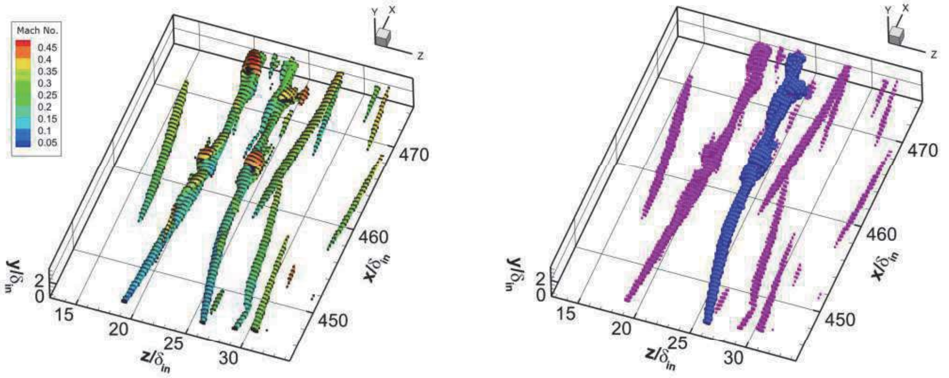
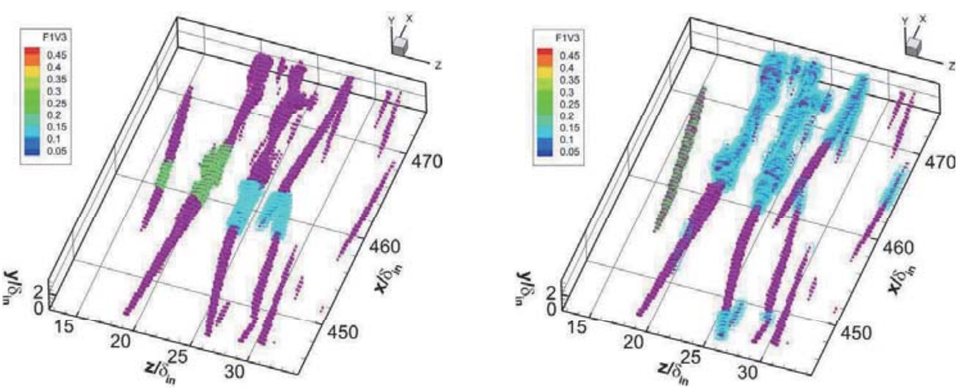


(a) Vortical structure visualized by SIVGT and its representation by particles (black points) enclosed by the iso-surfaces, The color in the legend is Mach No.

(b) Extracted cluster corresponding to a unstable hairpin leg by successive incorporation clustering (blue points)

Fig. 3 Instantaneous vortex structures appearing in the laminar-turbulent transition, its particle representation, and an example of an extracted cluster



(a) K-means, light green: cluster 2, cyan: cluster 3

(b) Spectral clustering, light green: cluster 1, cyan: cluster 10

Fig. 4 Clustering of interior points by k-means clustering and spectral clustering

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