プレート沈込み型地震に対する半特異面数学モデルの構築

A Mathematical Model with a semi-Singular Plane for the Earthquake of Plate-Subduction Type

筑波大学 名誉教授　佐々木 建昭 [*1]
Tateaki Sasaki
Professor emeritus, University of Tsukuba

筑波大学医学医療系　讃岐 勝 [*2]
Masaru Sanuki
Faculty of Medicine, University of Tsukuba

日本数学検定協会　稲葉 大樹 [*3]
Daiju Inaba
The Mathematics Certification Institute of Japan

Abstract

(`This paper assumes mathematicians as main readers`)

In 1750, great mathematician Leonhard Euler developed a comprehensive theory of the motion of a point mass, a massive rigid body, a massive **elastic body**, and massive fluid (with no viscosity), under a given force [2, 3]. It is amazing that, even now, Euler's system for the earthquake waves requires no change, although it is pretty complicated. Unfortunately, Euler's system has no mechanism of causing the earthquake; the **plate tectonics** was not known in Euler's era. This paper clarifies the limitations of Euler's system and searches for the origin of big earthquakes.

The earthquake of **plate-subduction** type occurs between a "continental plate" and a "ocean plate", where the latter subducts the former. We image that the ocean plate is the Pacific plate and the continental plate is the North American plate which covers the north half of Japan. In this case, the ocean plate approaches the continental plate at a speed of about 8 cm/year. In the subduction, both plates touch each other and they slide, without breaking others largely; see Figure 1 in the text. Hence, two plates facing each other have a thin "**fault**" (**断層** in Japanese) between them.

First, we express the above situation mathematically. As the 0-th approximation, we assume that the fault width is zero in subduction zone. Then, the plate boundary, let it be $B(x, y, z)$, is settled uniquely. The plates above and below $B(x, y, z)$ are of different speeds, hence $B(x, y, z)$ is a **singular plane**. Actually, both plates are apart from each other a little. The gap between two plates was measured to be about 8 cm at a point near the Japan trench; see the text around Figure 2. So, we introduce a **gap-function** $G(x, y, z)$ which simulates the gap, and we replace $B(x, y, z)$ by $G(x, y, z)$.

The second point of this paper is the "**asperity**" proposed by Theme Lay and Hiroo Kanamori in 1980. Asperities are big bumps on the surface of ocean plate. Lay and Kanamori insist that the asperities make the ocean and continental plates "**locked**"(**固着した** in Japanese), which is the main reason of big earthquakes. The surface of Pacific plate became clear after 1990s by development of the method of investigating structure of the underground of sea, see Figure 2 in the text. Setting the surface of ocean plate by imitating Figure 2, we propose *a realistic model of the plate-locking*.

Finally, we must say that Euler's system will be refined again and again in future, because we have so far considered neither thermodynamics nor big destruction of the plate itself.

[*1] 〒 305-8571 茨城県つくば市天王台 1-1-1　E-mail: `sasaki@math.tsukuba.ac.jp`
[*2] 〒 305-8576 茨城県つくば市天王台 1-1-1　E-mail: `sanuki@md.tsukuba.ac.jp`
[*3] 〒 110-0005 東京都台東区上野 5-1-1　E-mail: `d.inaba@su-gaku.net`

1 A brief survey of Euler's theory

We review (simplified) Euler's equations of motion of the elastic body (we call them *Euler's system*), and trace how the equations for earthquake waves were derived from Euler's system.

Definition 1: Elastic body (弾性体 in Japanese) and **Strain** (歪み in Japanese). The elastic body can be deformed by the external force, and returns to the original form as soon as the external force is removed. The strain of the elastic body is the change of relative positions between two arbitrary points of the body, except for the rotation and the parallel movement with no deformation of the body. □

By $[x_1, x_2, x_3]$ we denote the rectangular coordinate system and let $\boldsymbol{x} = (x_1, x_2, x_3)$ be a point of the elastic body. Let $\boldsymbol{u}(\boldsymbol{x}) = (u_1(\boldsymbol{x}), u_2(\boldsymbol{x}), u_3(\boldsymbol{x}))$ be the **strain vector** of the elastic body at $\boldsymbol{x}$. The $\boldsymbol{u}(\boldsymbol{x})$ is often complicated. So, we do not treat $\boldsymbol{u}(\boldsymbol{x})$ directly but transform it into a system of differential equations as follows, by assuming that $\boldsymbol{u}(\boldsymbol{x})$ is *continuous* w.r.t. each x_j (below, "$\stackrel{\text{def}}{=}$" means "define").

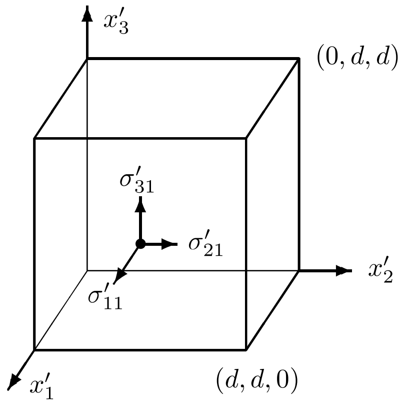
$$\frac{\partial u_i}{\partial x_j} \stackrel{\text{def}}{=} \lim_{\delta x_j \to 0} \frac{u_i(\cdots, x_j + \delta x_j, \cdots) - u_i(\cdots, x_j, \cdots)}{\delta x_j} \qquad (1 \le i, j \le 3), \tag{1}$$

$$\tau_{ij} \stackrel{\text{def}}{=} \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big), \quad \theta_{ij} \stackrel{\text{def}}{=} \frac{1}{2}\Big(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\Big) \qquad (1 \le i, j \le 3). \tag{2}$$

The τ_{ij} will be used a little later. The θ_{ij} represents the rotation and not used in this paper.

Definition 2 (Euler?) **:** **Stress** (応力 in Japanese). Every point of the elastic body is influenced by the external force such as the gravity, and any neighboring parts of the body, separated by a plane $\overline{\mathcal{P}}$, influence each other. <u>The latter force is the stress</u>. Let p be a small square of area d^2 ($\ll 1$) located at $\boldsymbol{x}$, where $p \in \overline{\mathcal{P}}$. Let $\boldsymbol{s}$ be the whole stress acting on p, then $\boldsymbol{S}(\boldsymbol{x}) \stackrel{\text{def}}{=} \boldsymbol{s}/d^2$ is the **normalized stress** at $\boldsymbol{x}$ of the body. The $\boldsymbol{S}(\boldsymbol{x})$ can be expressed by mutually orthogonal three components. □

We derive a key formula on $\boldsymbol{S}(\boldsymbol{x})$. Let $[x'_1, x'_2, x'_3]$ be a rectangular coordinate system, where its origin is at $\boldsymbol{x}$ and its main axes are in parallel to those of $[x_1, x_2, x_3]$. Consider a small cube of volume d^3 located at $\boldsymbol{0}' = (0', 0', 0')$. We have 6 planes through which the stress influences the cube. We consider the stress acting on the front surface: a plane specified as ($x'_1 = d,\ 0 \le x'_2, x'_3 \le d$); see the right figure. Let x'_1-, x'_2-, x'_3-components of the stress be $\sigma'_{11}, \sigma'_{21}, \sigma'_{31}$, resp. Next, consider the stress acting on the back surface ($x'_1 = 0$). Let $\sigma''_{11}, \sigma''_{21}, \sigma''_{31}$ be x'_1-, x'_2-, x'_3-components of the stress, resp. Finally, computing $\lim_{d\to 0}\big(\sigma'_{11} - \sigma''_{11}, \sigma'_{21} - \sigma''_{21}, \sigma'_{31} - \sigma''_{31}\big)/d$, we readily obtain $(\partial/\partial x_1)\big((\sigma_{11}(\boldsymbol{x}), \sigma_{21}(\boldsymbol{x}), \sigma_{31}(\boldsymbol{x})\big)$. Since we have similar formulas for $(\partial/\partial x_2)$ and $(\partial/\partial x_3)$, we find that the stress becomes the first sum of the r.h.s. of formula in (3) below.



Euler's system is obtained by adding the stress term to Newton's equations of the motion.

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sum_j \frac{\partial \sigma_{ij}}{\partial x_j} + f_i\,(\text{external force}) \qquad (i = 1, 2, 3). \tag{3}$$

Here, ρ is the density of the body, t is the time, and σ_{ij} is the (i, j)-component of the $\boldsymbol{S}(\boldsymbol{x})$. Note that ρ is a function in x_1, x_2, x_3 and t. See also the comments at the tail of this section.

In order to compute the earthquake waves, we must express the stress in terms of the strain. We know empirically that the extension and the shrinkage of an elastic bar is proportional to the force added to the bar. On the other hand, the destruction of a rock by a strong force depends on the structure of the rock; if the rock consists of special crystals then the rock will be destroyed along several peculiar directions specified by the crystals. This phenomenon is called the *anisotropy* (非等方性 in Japanese) of the rock. In discussing the earthquake waves generally, it is common to restrict the elastic body to be *linear*, i.e., there is a linear relationship between σ_{ij} and τ_{kl}, as shown by the left equality in (4) below, and that the elastic body shows no anisotropy, which plays a critical role in deriving the r.h.s. relation in (4).

$$\sigma_{ij} = \sum_{k,l} C_{ijkl}\,\tau_{kl}, \quad \text{where} \quad C_{ijkl} \in \mathbb{R} \qquad \Longrightarrow \qquad \sigma_{ij} = \lambda\,(\tau_{11}+\tau_{22}+\tau_{33})\delta_{ij} + 2\mu\,\tau_{ij}. \tag{4}$$

Here, $\delta_{ij}=1$ if $i=j$ else 0 (Kronecker's delta), and λ and μ are called Lamè's constants.

The C_{ijkl} are called *elastic constants*, and we have $3^4 = 81$ constants. Since the stress is not affected by the external force, we have $\sigma_{ij} = \sigma_{ji}$ due to the balance of moment, and the same is true for τ_{ij}. These symmetries decrease the number of constants to 36. Furthermore, considering the strain energy function which is not given in this paper, the number is decreased to 21. Finally, assuming that the elastic body is isotropic, which introduces many symmetries, the number of free elastic constants reduces to 2, as in the r.h.s. of (4). (`Derivation of the r.h.s. expression in (4) is pretty complicated. The reader had better refer to a book or the Web on` *`Solid Mechanics`*`.`)

We note that the definition of τ_{ij} allows us to rewrite the r.h.s. expression in (4) as follows.

$$\tau_{11}+\tau_{22}+\tau_{33} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \;\Longrightarrow\; \sigma_{ij} = \lambda\Big(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\Big)\delta_{ij} + \mu\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big)$$

Using the above expression for σ_{ij}, we can rewrite the stress-term in (3) as follows.

$$\sum_j \frac{\partial \sigma_{ij}}{\partial x_j} \;\Longrightarrow\; \frac{\partial \sigma_{ij}}{\partial x_j} = \lambda\,\frac{\partial}{\partial x_j}\Big(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\Big)\delta_{ij} + \mu\,\frac{\partial}{\partial x_j}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big)$$
$$\Longrightarrow\; \sum_j [\text{above last term}] = \mu \sum_j \frac{\partial^2 u_i}{\partial^2 x_j} + \mu\,\frac{\partial}{\partial x_i}\Big(\sum_j \frac{\partial u_j}{\partial x_j}\Big)$$

Substituting the above expressions for the stress-term of (3), we obtain well-known Navier's equations.

$$\rho\,\frac{\partial^2 u_i}{\partial t^2} = (\lambda+\mu)\frac{\partial}{\partial x_i}\Big(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\Big) + \mu\Big(\frac{\partial^2 u_i}{\partial x_1^2} + \frac{\partial^2 u_i}{\partial x_2^2} + \frac{\partial^2 u_i}{\partial x_3^2}\Big) + f_i \qquad (i=1,2,3). \tag{5}$$

This system was solved very beautifully, giving the P- and S-waves; see the Book[1], for example.

Euler's original system is more complicated in that it contains the law of mass conservation (質量保存則 in Japanese). This law is unnecessary for the rigid body and the fluid. As for the elastic body, the law is necessary in general, and we must treat ρ to be a function in $\boldsymbol{x}$ and t. However, the change of ρ is almost negligible even during the earthquake. So, it is common to ignore the mass conservation law.

2 Plate tectonics and various questions

Nowadays, the *plate tectonics* is a common knowledge in earth science. However, for us, non-expert beginners, there are many questions on the plate tectonics. First, we list important features of the plate tectonics concisely for mathematicians:

F0) The surface of the earth is covered by many (12 or so) plate-like rocks named *plate*s.
F1) Except for big *continental plate*s, many plates are moving at a speed of several cm/year.
F2) The front of *ocean plate* either collides other ocean plate or subducts a continental plate.
F3) The collision and subduction cause big earthquakes and "tsunami"s (津波 in Japanese).
F4) The continental plate is ∼150 km in depth, and the ocean plate is ∼80 km in thickness.
F5) The ocean plate is heavy and hard, while the continental plate is light and not hard.
F6) The ocean plate consists of two layers, named *lithosphere* and *asthenosphere*.
F7) The upper lithosphere behaves as an elastic body within at least 10^5 years.
F8) The lower asthenosphere becomes a fluid-like matter after several 10 years at most.
F9) The mud and sand will become the rock if compressed strongly during many years.

In this paper, we image that the ocean plate is the Pacific plate and the continental plate is the North American plate which covers the north half of Japan. In this case, the Pacific plate approaches to the North-American plate at a speed of ∼8 cm/year and the former subducts the latter.

The depth distribution of earthquakes around Japan

This rectangle shows a plane along the latitude ∼ 38°N;
width: (horizontal: 650km, vertical: 250km.)
Before bending, many earthquakes occur below the Japan trench
not only in lithosphere but also in asthenosphere,
in deep area s.t. 20 km ≤ depth ≤ 100 km.
Pacific plate (80km thickness) begins to bend at Japan trench
⇒ subducts ∼180 km ⇒ ∼40°-bending.
After the bending, Pacific plate subducts straightly.
Earthquakes on Pacific plate: very many on the surface, inside
lithosphere: definite number, till depth 150km,
Earthquakes in N.A.plate: very many within depth ≤30km.
just above the Pacific plate: vary many,
in particular, within depth ≤ 75km.

Figure 1: This figure is taken from Hasegawa et. al. 1978 [8],

Figure 1 shows that the Pacific plate bends at the Japan trench then subducts the North American plate in upper left-side of the figure. Each dot shows the "epicenter"(震源 in Japanese) of an earthquake.

As for the origin of big earthquakes, T. Lay and H. Kanamori [9] proposed a concept of "asperity" in 1980, to be objects which cause strong "lock" (固着 in Japanese) of the ocean and continental plates in the subduction zone. They suggested that the asperity is nothing but **bumpy** (凸凹 in Japanese) surfaces of the plates. They drew many figures as asperities by hand, each figure is different from plate to plate, so we are not easy to image the asperities.

As non-experts in earth science, the present authors have many questions on the behaviors of plates.

Q1) How is the surface of the ocean plate? In other words, what are the actual "asperities"?
Q2) How the hard and thick ocean plate bends when it begins to subduct the continental plate?
Q3) Let R be the radius of earth. The area of surface of the earth at the depth $R-d$ decreases by $(1-d/R)^2$. However, the ocean plate cannot shrink. How is this discrepancy resolved?
Q4) What role does the asthenosphere play during the subduction, in particular, in the bending?

3 Remarkable investigations on the Pacific plate

As for investigating the structure of earth and bottom of sea deeply and widely, very effective method named "seismic reflection survey" (反射法地震探査 in Japanese) was developed as early as 1980s. In Japan, we have now a private company which performs the seismic reflection survey.

In addition, after the Hanshin (Kobe) Earthquake in 1995, Japan equipped with "Hi-Net" (High Sensitivity Seismograph Network) and "GEONET" (GNSS Earth Observation Network). Furthermore, a special ship named "Chikyu" ("Earth" in English) was constructed so as to investigate the bottom of sea. Its main job is to drill the bottom of sea until the plate boundary. The tip of drill is a pipe of diameter 16 cm, so as to get the matter existing at the plate boundary. However, on March 11 in 2011, Japan was attacked by the extremely strong earthquake "2011-Tohoku EarthQuake(EQ)".

One year later, JAMSTEC (Japan Agency for Marine Earth Science and Technology) executed the **Japan Trench Fast Drilling Project**[7]. The drilling point was chosen near the cross-point of the Japan Trench and the latitude of the epicenter of 2011-Tohoku-EQ. Figure 2 below shows a vertical structure of the bottom of sea (depth $6\sim9$ km from the sea surface, horizontal width ~ 20 km along the latitude 38.26° N, and the Japan trench is about 2km right of the figure center). The red line shows how Chikyu made a boring: it excavated the bottom of sea about 850 m and reached at the boundary of the Pacific and the North American plates. We can see both ocean and continental plates clearly.

The structure of the underground of sea bottom
`obtained by the seismic reflection survey method`

This rectangle shows a plane along the latitude $\sim 38°$;
size: (horizontal $\sim$25km, vertical 4000m).

The upper white area shows the sea: depth $6000\sim7000$.

The middle [edge:fat, else:thin] area shows N.A.plate.

The lower [edge:thin, else:fat] area shows Pacific plate.

We see several hills of height $300\sim500$m on Pacific plate.
Note that edges of each hill are sharp.

There is a fourth thin layer over the Pacific plate.
This layer will be composed of breccia.

Figure 2: This Figure was pictured by Weiren Lin (Univ. Kyoto.)
(The reader can see Figure 2 at Web: <No.3> of [6])

The JAMSTEC project clarified the following three important facts.

- **Fact-1** The surface of ocean plate is very **bumpy** (凸凹 in Japanese); there are many big hills of $300\sim500$ m high and several km long. It must be a real figure of the "asperity".
- **Fact-2** The gap between mutually facing plates is very narrow; the width is as thin as **8 cm**.
- **Fact-3** The materials in the gap are water(20 %) and **very fine clay**(粘土 in Japanese).

Here, we give comments on the sea bottom. The surface of sea bottom is covered by mud, sand, shells, bones of fishes, etc. At the beginning of subduction, these covers are pushed to the continental plate, and they finally change to rocks. By the ocean plate, we mean the plate without these covers.

4 Let us introduce two kinds of plates into Euler's system

In this section, we will rewrite the Euler's system so that it consists of the continental plate and the ocean plate, in such a way that these two plates are separated by a very thin gap.

We start our rewriting from Euler's system (3) given in Sec.1, where we assumed that ρ is t-independent by neglecting the law of mass conservation. That is, we assumed ρ to be static as a whole, and we discussed only the local oscillation (振動 in Japanese) of ρ. From now on, however, we consider that ρ is t-dependent. Note that we still ignore the law of mass conservation. In the subduction, the ocean plate moves dynamically, hence we must consider that ρ is a function in $\boldsymbol{x}$ and t.

We explain important points in this section, one by one.

● **We separate ρ into $\rho^{(+)}$ and $\rho^{(-)}$, for continental plate and ocean plate, respectively.** Similarly, we separate σ_{ij} into $\sigma_{ij}^{(+)}$ and $\sigma_{ij}^{(-)}$. We have $\rho^{(+)} \cup \rho^{(+)} = \emptyset$ for any point $\boldsymbol{x}$. Note that we can define $\sigma_{ij}^{(+)}$ and $\sigma_{ij}^{(-)}$ only for such $\boldsymbol{x}$ that $\rho^{(+)}(\boldsymbol{x}, t) \neq 0$ and $\rho^{(-)}(\boldsymbol{x}, t) \neq 0$, respectively.

● **The above Euler's system has a shortage mathematically.** The ρ contains a fault between the continental plate and the ocean plate, hence ρ is **not analytic** at the the fault. Since the $\rho\, \partial^2 u_i / \partial t^2$ is the main term of Euler's system, this shortage is serious. Our study of the earthquake was motivated by this shortage. This shortage is removed as follows. We have separated the continental and the ocean plates by expressing them by $\rho^{(+)}$ and $\rho^{(-)}$, respectively, just above, and will express ρ as $\rho^{(+)} G\, \rho^{(-)}$, just in the following two paragraphs, where G is not singular but an ordinary function.

● **Introduction of Boundary function $B(\boldsymbol{x}, t)$.** We have seen in the previous section that the gap between the Pacific plate and the North American plate in the subduction zone is very narrow (see **Fact-2**). Hence, as the 0-th approximation, we assume that the gap width is 0. Then, the plate boundary in the subduction zone is fixed uniquely. We denote the fixed boundary by $B(\boldsymbol{x}, t)$. The plates just above and just below $B(\boldsymbol{x}, t)$ move at different speeds (the difference is about 8 cm/year). Therefore, $B(\boldsymbol{x}, t)$ is a singular plane mathematically. We note that the continental and the ocean plates are different not only in their rocks but also in the mathematical interpretation of their movements.

● **Introduction of Gap function $G(\boldsymbol{x}, t)$ and Gap matter $\gamma(\boldsymbol{x}, t)$.** However, the gap width is not zero actually, though it is very small. Hence, we introduce a **Gap function** $G(\boldsymbol{x}, t)$ which simulates the actual gap, hence its width is greater than 0, and we replace $B(\boldsymbol{x}, t)$ by $G(\boldsymbol{x}, t)$. By this, the singular plane is removed from the Euler's system. We will discuss $G(\boldsymbol{x}, t)$ and $\gamma(\boldsymbol{x}, t)$ in the next section.

● **How we treat the "Asperity".** Just blow Figure 2 in Sec. 3, we wrote that the 凸凹 must be a real figure of the "asperity". Here, we strengthen this statement much as "the asperity is nothing but the 凸凹 s of the surface of ocean plate"; the asperity is very clear in our subduction model.

● **Finally, we rewrite the Euler's system.** Let $\rho^{(+)}$, $\sigma_{ij}^{(+)}$ and $u_i^{(+)}$ be for the continental plate and $\rho^{(-)}$, $\sigma_{ij}^{(-)}$ and $u_i^{(-)}$ be for the ocean plate, respectively. Then, the above 1-st point leads us to relations in (6). We can compute $u^{(+)}$ and $u^{(-)}$ independently, hence we obtain equations in (7).

$$\rho = \rho^{(+)} G\, \rho^{(-)}, \qquad \sigma_{ij} = \sigma_{ij}^{(+)} G\, \sigma_{ij}^{(-)}, \tag{6}$$

$$\rho\, \frac{\partial^2 u_i^{(+)}}{\partial t^2} = \sum_j \frac{\partial(\sigma_{ij})}{\partial x_j} + f_i\,(\text{e. f.}), \qquad \rho\, \frac{\partial^2 u_i^{(-)}}{\partial t^2} = \sum_j \frac{\partial(\sigma_{ij})}{\partial x_j} + f_i\,(\text{e. f.}). \tag{7}$$

If the gap matter $\gamma(\boldsymbol{x}, t)$ is slippery, such as **Fact-3** in Sec. 3, then the stress $\sigma_{ij}^{(+)}$ cannot affect $u_i^{(+)}$ much, and $\sigma_{ij}^{(-)}$ cannot affect $u_i^{(-)}$ much, hence *Euler's system is unable to cause big earthquakes.*

5 A mechanism of the plate-locking

The plate-locking[1)] is a phenomenon which locks the ocean plate and its partner continental plate each other. The locking occurs at the boundary of both plates. It was said that the 2011-Tohoku-EQ was caused by this phenomenon. An earthquake glossary explains the plate-locking as follows: *A shallow part of the plate-boundary is locked usually by "static friction" (静摩擦 in Japanese), and it slips all at once by the earthquake.* The present authors doubt this explanation because it relies on dynamics of small event *static friction*, although they know that the plate-locking is supported by the "crust" (地殻 in Japanese) deformation found by Suwa et al. [10] in 2004. The plate-locking is mysterious for the beginners.

As for the earthquake, the readers can get detailed and comprehensive information from Proceedings of "Coordinating Committee for Earthquake Prediction" (地震予知連絡会 in Japanese)[4]. The Proceedings, however, did not give us any reasonable definition nor any clear mechanism of the plate-locking. Although many authors mentioned about the relation between the asperity and the big earthquakes, see [6], they gave no mention on the mechanism of plate-locking. In this section, we propose a clear and reasonable mechanism (probably new) of the plate-locking. We must note, however, that *our proposal is now in a conceptual level* and not in any mathematical formula.

● **Plate-locking must be based on clear facts.** We are confident of that the subduction of the ocean plate is a global or semi-global event and never a local one. Similarly, since the plate-locking is necessary for causing huge earthquakes, we are sure that the locking is based on clear evidences on the plates. Since the plate-locking has been said to occur at the plate boundary, the locking will become clear if we will be able to see the plate boundary. Fortunately, we can see a small part of the boundary of Pacific and North American (N.A. in short) plates by Figure 2 in Sec. 3. In Figure 2, **we see three hills** on the Pacific plate. The size of each hill is as high as 300∼500 m and as long as several km. Although the hills are tiny compared with the thick Pacific plate, they are considerably large compared with the thickness of continental plate covering only the epicenter area of 2011-Tohoku-EQ. Figure 2 also suggests us that the hills are distributed randomly and considerably widely over the surface of Pacific plate. The hills are parts of Pacific plate composed of hard rocks, hence not easy to destroy hills. On the other hand, Japan islands are composed of soft rocks and "gauge" (古くて丸い破砕物質 in Japanese), because most rocks of Japan islands are circulating between low mountains and shallow sea.

● **Proposal of a mechanism of plate-locking based on a clear fact: the existence of hill.** If there is no hill on the surface of Pacific plate then the plate will subduct the N.A.plate smoothly. Hence, it must be sure that the plate-locking is caused by the hills. Then, how the hills cause the locking of the Pacific plate and the N.A.plate ? In order to find how the hills do so, we trace the subduction of the Pacific plate in our brain and watch a hill on the plate surface (⇐ brain-experiment).

We assume that **average hill is considerably high**, say several 100 m, **hard enough** so that they will not be destroyed during the subduction. In addition, we assume that **the hills are distributed widely and rather densely** over the surface of the Pacific plate. This means that the existence of hills is based on the history of Pacific plate itself.

Starting from the Japan trench and moving to the west at the same speed of the Pacific plate, of course, the hill first pushes the surrounding gauge aside and also the rocks of the low tail of N.A.plate aside, where the height of tail is less than that of hill. The hill next meets the tail of N.A.plate which is taller than it. Then, the hill pushes the rocks at the bottom of the N.A.plate, mostly to the west and

[1)] Some researchers call the plate-locking as *inter-plate coupling.*

partly upward. Note that the hill is powerful, because it is a part of the Pacific plate. So, the rocks being pushed by the hill will be fractured; the fractured rocks are angular and called "breccia" (角礫 in Japanese), and they will go behind the hill. Many tiny earthquakes will help the breccia to move. If the N.A.plate is thicker then the Pacific plate pushes the N.A.plate more strongly. We call this process **hill-pushing**. During the hill-pushing, the N.A.plate is pushed to the west by the Pacific plate through the hill. Therefore, **the hill-pushing is nothing but the plate-locking**.

● **Increasing the strain in the underground**. At inside-JapanTrench sea, the N.A.plate is pushed by the Pacific plate to the west, from the east edge first to the west part of the plate in turn. Therefore, the strain is such that the N.A.plate is pulled to the west, from the Japan trench toward the coast of Tohoku. When a huge earthquake, including aftershocks (余震 in Japanese), happens then the strain having been accumulated is released almost fully. However, the fractured rocks are never reset but remain behind hills. This means that, after many big earthquakes, the composition of the Pacific plate will be changed: *the surface of the Pacific plate will become hills and breccia.*

6 Details of the plate-locking based on hill-pushing

By "slip" (滑り in Japanese) in the earthquake, we mean the relative sudden motion of the rocks of each side of the fault by the earthquake. The amount of slip is $5m \sim 10m$ in a large earthquake.

We already know that the 2011-Tohoku-EQ was not only very large in the magnitude but also very wide in the "rapture size" (地震破壊規模：一つの地震で滑りが起きた部分の広さ in Japanese). If the hill-pushing is an actual mechanism of the plate-locking then we can understand these two features of the 2011-Tohoku-EQ; see the next paragraph for the reason.

● **The hill-pushing is a very effective mechanism of plate-locking**. Suppose that the plate-locking originates at a local area and propagates to surrounding, then the resulting earthquake will be not so strong and the resulting area of the plate-locking will be never so wide as the rapture size of the 2011-Tohoku-EQ. On the other hand, many hill-pushings start from a north-to-south line of the Pacific plate and advance to the west almost simultaneously at the speed of 8cm/sec. Therefore, the plate-locking due to hill-pushing will create the strain as widely as possible. Furthermore, the plate-locking due to hill-pushing is **mild** in that *a whole big job is distributed widely among quite many nearly equal-sized small jobs.* As a result, the hill-pushing will create the strain as much as possible.

● **The hills will be buried gradually with the breccia**. This phenomenon will decrease the pushing power of the hills. However, the hill is not easy to push its front side, because there is almost no empty space in the underground. (At shallow places, the hill may raise local parts of the N.A.plate easily). Hence, the hill-burying will be slow, so the hill will push the N.A.plate very steadily. Consequently, *the plate-locking will be performed over very wide area of off the coast of Tohoku.*

● **The above-mentioned mechanism is applicable widely**. The mechanism mentioned above is applicable to any area between the ocean plate and the continental plate so long as the topographic conditions (地形学的条件 in Japanese) are nearly the same. In fact, the 2011-Tohoku-EQ ranged to off coasts of Fukushima and Ibaraki.

● **What happens if there is a big hill**. Compared with the plate-collision, the plate-locking due to hills on the surface of ocean plate is not so strong. However, if there is a big hill (or big hills) such as of height 1000 m, then the effect of hills is considerable. The present authors are sure that the big hill creates some remarkable phenomenon.

7 On $G(\boldsymbol{x}, t)$– Toward a system of plate-subduction with plate-locking

The gap-function $G(\boldsymbol{x}, t)$ was originally introduced to bury the difference between the real plate-boundary and the mathematical boundary $B(\boldsymbol{x}, t)$ the width of which is 0. As mentioned in the abstract, we replace the $B(\boldsymbol{x}, t)$ by the gap-function $G(\boldsymbol{x}, t)$ of width > 0. However, in clarifying the mechanism of plate-locking, we were urged to handle such gap functions that we had never thought before; rocks of N.A.plate are fractured to breccia and carried to the gap between two plates. Below, we list the characteristic features of such gap functions, one by one, and explain each feature in details.[2)]

1) `How to specify the ‘‘gap’’ when hills are fracturing rocks of the N.A.plate`?
In our mechanism of plate-locking, rocks at the bottom of N.A.plate are fractured by hills and resulting breccia are carried to the surface of Pacific plate. Hence, we specify the gap to be *the space between the bottom of N.A.plate and the surface of Pacific plate, <u>where the hills are not included</u>*.

2) `How to treat the fracturing of rocks by hills at the bottom of N.A.plate`?
The present authors have few knowledge about the fracturing. They are planning to learn from book [1]. Anyway, the fractured rocks must be angular and small, hence easy to accumulate on the surface of Pacific plate almost evenly. As the subduction proceeds, the gap which contains only hills initially will be filled with breccia, and the fracturing power of hills will decrease gradually.

3) `How to treat theoretically the very wide area where the Plate-locking occurs`?
As explained in Secs. 5 and 6, the 2011-Tohoku-EQ was so huge because huge amount of strain was accumulated over very wide area. So, we had better treat the very wide subduction area honestly in the computer simulation of hill-pushing. Once we obtained a convincing image of plate-locking after many simulations, then we may convert the image to a mathematical expression.

4) `How to treat soft rocks of continental plate and hard rocks of ocean plate`?
Existence of two kind of rocks, soft and hard, is crucial in our model of plate-locking. Both rocks must be elastic, because the strain is accumulated even to the soft rocks. Hard rocks of the ocean plate fracture the soft rocks of the continental plate, with not much damage to the hard ones.

5) `How to express theoretically the whole system of plate-subduction with earthquake`?
In the whole system, the law of mass conservation does not hold between the continental and ocean plates. We can express the mass conservation by differential equations. However, the resulting equations will be useless, because the mass of N.A.plate is transported only through the bottom of N.A.plate which is very wide, and we must distinguish each hill from others. As we have explained in **3)**, we had much better treat the wide bottom of N.A.plate as a whole.

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[2)] At the oral presentation of this research in “Symposium on Computer Algebra (Dec. 18-20, 2023)”, Res. Inst. Math. Sci., Kyoto Univ.”, the old definition of the gap-function $G(\boldsymbol{x}, t)$ was very complicated. The old one was based on the extended-Hensel construction which the present authors have invented, and the authors were very proud of it. However, they noticed later that the old gap-function is bad. This is obvious by considering how an important role the gap-function plays in the mechanism of plate-locking. Furthermore, the old gap-function was too mathematical, hence most seismologists will disgust at the function. In this manuscript, we have defined the gap function to be acceptable by seismologists.

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