Financial risk transfer of Catastrophe *

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Abstract

Serious disasters had occurred in Japan as Kobe earthquake, East-Japan Tsunami and Covid-19 and Noto earthquake of this year. The economic burden to recover from disasters is expanding government deficit further. Reinsurance and securitization such as catastrophe bond are analyzed for application to overcome future financial burden of disasters .

1 Introduction

Japan is located the place where 4 tectonic plates go down. The earthquakes of magnitude over 6.0 in Japan have been more than 20 % of the world in the recent decade. In 1995 the earthquake had attacked the one of major city Kobe and the port. The scale of earthquake was M.7.3 and 4,571 casualties and amount of insurance and reinsurance were 78 billion yen. The total 5 years government special budget account was 9.2 trillion yen. In 2011 the East Japan earthquake had been M.9.0 and Tsunami and Nuclear Reactors were in catastrophe. The Government special budget account for the earthquake is total 32.9 trillion yen for 10 years. The recent earthquakes in Japan and the insurance amounts payed are seen in Table 1. Government total debts and the increment rates caused by the catastrophe are seen in the last column

In the following section, firstly we review the Government policies for residential earthquake insurance where private insurance companies and government cooperate to help the victims of earthquake and tsunami. As the recent repeated earthquakes the insurance for buildings of residential uses might be well covered by the corporation with insurers and government. However earthquake insurance for commercial property is not well covered by insurance nor reinsurance in Japan. By the theoretical approach for the fund raising model of [2] we compare the insurance with reinsurance and the securitization of risky asset such as catastrophe bonds for insurance companies.

2 Reinsurance scheme for earthquakes

Residential earthquake insurance has started since 1966. All non-life insurance companies take contract with housing owners with fire insurance. Private company that is named as Japan Earthquake Reinsurance Co. Ltd shares the half of insurance money by the reinsurance contract. The Reinsurance Co. is owned by all the non-life insurance companies. Furthermore the government has the Earthquake reinsurance special account in the National budget system.

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As seen in Table 1 insurance money were payed increasingly after 2011. The scheme of reinsurance has been revised for many times before the recent 2023.4.1. version as seen in Table 2. For the damage that is less than 153.3 billion yen the insurance money ought to be pay the half from the insurance company and the rest from the Reinsurance Co., The claim from 153.3 billion to 216.3 billion yen is payed by 50% by the private insurers and the rest is pay by the Government account. The claim more than 216.3 billion yen to 1.20 trillion yen is payed 99.6% by the government and the rest is from the reinsurance. For the large claim private insurers will 228,7 bill. yen including Japan Earthquake reinsurance and Government will pay 11,771.3 bill. yen and the total equals to 12,000 bill. yen, which is 99.63% of 12 trillion yen of maximum of the reinsurance system. The historical allocation to Government vs reinsurance company are compared in Figure 1.

Table 1: Earthquakes in Japan (100M yen)

year	place	scale	Insur	Reins(Gov)	Gov. Debt(trill.yen)
1995	Kobe	M7.3	783	62+*	452(+12%)
2000	Tottori	M7.3	28		
2001	Hiroshima	M6.7	169		
2004	Niigata	M6.8	148		
2005	Fukuoka	M7.0	169		
2007	NiigataOki	M6.8	82		
2011	EastJapan	M9.0	12000	5872 +**	993(+7%)
2016	Kumamoto	M7.3	3620	1378	
2018	Osaka	M6.1	1030	182	
2021-22	FukusimaOki	M7.4	4580	2002	
2024	NotoPen	M7.6	900		

The decrease of private insurance is caused by the problem of decreasing reserves as seen in Figure 1. The year of Kobe EQ 1995 has started to decreasing and after the EastJapan EQ the reserve for private reinsurance had lost most of reserves. In 2023 the contribution to Reserve by Government vs insurance companies was changed from 8 : 2 to 3 : 7 before the reserve of the companies reaches to 1000 bill. yen. Although exists the huge government debt, the contribution by government has been expanded.

Table 2: Reinsurance Scheme from 2023.4.1.

billion yen	Private insures	Government
Up to 153.3	100%	0
Up to 216.3	50%	50%
Up to 12,000	0.37%	99.63%

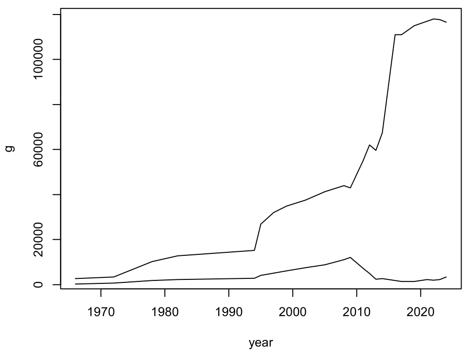

Figure 1: Reinsurance Scheme (Government vs Private)

3 Reinsurance vs Securitization

The insurance contracts for residential buildings is increasing because of the experience of huge earthquakes as Kobe and EastJapan. But the non residential insurance seems to stay in low level. It is reported as annual expected natural catastrophe property damage losses by Swiss Re [6] as Table 3.

Table 3: Annual expected natural catastrophe property damage loses

Japan	EQ	Wind	flood	Total
insured	600	360	120	1080
uninsured	2760	480	120	3360
unins. %	82	57	50	76
total	3360	840	240	4440

billion yen

The estimates numbers are in annual base and including non-residential facilities. Surprisingly high is the uninsured amount for earthquake damage. In this section we discuss the possibility of alternative extension for insurance which are reinsurance and securitization. The insurance model for catastrophe loss proposed by Subramanian and Wang [2] is considered. In the model the insurers can choose insurance retention or reinsurance or securitization. The insurers are insurance companies or insures who have the specific risk which are distributed as a probability function. The model is followed from Subramanian and Wang.

3.1 Insurance company model

The insurance company has the capital W and a risky portfolio of insurable risks. He receives the premium A with probability $1-p$ in the good state. Or he pays the insurance money B net with the loss probability p in bad state. If $B > W$, the insurance money payed is more than the company's capital W, then the company is bankrupt and it costs C. Let $F(p)$ be investors prior believe distribution on the insurance company loss probability p which is also the label for risk type of insurance company in the continuous framework.

3.2 Reinsurance

The insurance company of the risk type p has the reinsurance contract $(A_r(p), B_r(p))$. Let $A_r(p), B_r(p)$ be respectively the reinsurance premium and reinsurance money. The reinsurance requires the minimum premium rate δ for the service charge.

The shortage of capital is $B - W$ is covered by the reinsurance whith $B_r(p)$. Let define $\bar{B} := B - W$. For the reinsurance is not too expensive, we assume $\delta < \frac{C}{\bar{B}}$.

The optimal reinsurance contract for the insurance company of p solves

$$\max_{A_r(p),B_r(p)} \quad (W + A - A_r(p))(1-p) + (W - B + B_r(p))p$$

$$s.t.$$

$$A_r(p)(1-p) \geq (1+\delta)pB_r(p),$$
$$W + A - A_r(p) \geq 0,$$
$$W - B + B_r(p) \geq 0$$

Theorem 1 (Reinsurance Contract) *The probability for reinsurance contract for insurance company p:*

$$p_r := \frac{W}{B} - \frac{\delta}{1+\delta} \tag{3.1}$$

if $p < p_r$ then the insurance company takes reinsurance
if $p \geq p_r$ then no reinsurance, retention or securitization.

The optimal reinsurance premium is

$$A_r^*(p) = \frac{\bar{B}p(1+\delta)}{1 - p(1+\delta)}$$

where $p < 1/(1+\delta)$.
Cost of Reinsurance is the sum of premium and insurance money;

$$B_r^*(p) = A_r^*(p) + \bar{B} = \frac{\bar{B}}{1 - p(1+\delta)}.$$

The retention cost is $\bar{B}p$.

The example of reinsurance vs retention is seen in Figure 2.

3.3 Securitization

The insurer risk type p has the securitization contract $(A_s(p), B_s(p))$. Let denote $A_s(p)$ as dividend payed to the investors if no incident occurs. $B_s(p)$ is the payment received from the investors in the incident. The insurer of risk type p chooses pooling securitization if $p \in [p_s, 1]$ or retention if $p \in [0, p_s)$. Let $F(p)$ be investors prior belief distribution of insurer's type p and

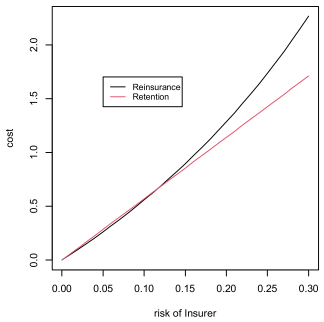

Figure 2: Reinsurance vs Retention

$\mu_s(p)$ be the investors probability function to choose the pooling securitization of trigger p_s such as

$$d\mu_s(p) = \frac{dF(p)}{1 - F(p_s)}$$

The securitization contract satisfies the following theorem of Subramanian & Wang [2].

Theorem 2 (Securitization contract) *Suppose there is a unique p_s s.t.*

$$Cp_s = \bar{B}R(p_s) \tag{3.2}$$

where subsidization ratio $R(p)$ of insurance money $\bar{B}$.

$$R(p_s) := \frac{\int_{p_s}^1 t d\mu_s(t) - p_s}{1 - \int_{p_s}^1 t d\mu_s(t)} =: \frac{X}{Y}, X + Y = 1 - p_s$$

(i) Insurers with the risk type $\forall p \in [p_s, 1]$ choose pooling securitization (A_s^*, B_s^*)
The loss of the high premium group Y is shared as

$$A_s^* = \frac{\bar{B}}{1 - \int_{p_s}^1 t d\mu_s(t)} \int_{p_s}^1 t d\mu_s(t), B_s^* = \bar{B} + A_s^*$$

(ii) Insurers with the risk type $\forall p \in [0, p_s)$ choose self-insurance.

A simple version of proof for discrete case is given in Appendix A.

The securitization condition in the theorem 2 is $Cp_s = \bar{B}R(p_s)$ where $Cp_s$is the expected bankrupt cost with retention and $\bar{B}R(p_s)$ is subsiding cost for pooling securitization. Insurers with type greater than p pool together by offering a single contract where the premium reflects the average of pool as

$$a(p) = \int_p^1 t d\mu_s(t)$$

$R(p)$ is the measures of degree of subsidization for p as

$$R(p) = \frac{\int_p^1 t d\mu_s(t) - p}{1 - \int_p^1 t d\mu_s(t)} = \frac{a(p) - p}{1 - a(p)}$$

Insurers of securitization for $(0, p_s)$ will be in retention, and for $(p_s, 1)$ the insures take the pooling securitization, where investors $p \in (p_s, a(p))$ subsidy to investors $p \in (a(p), 1)$.

3.4 A model for numerical example

For investors density to securitization for incident $d\mu(p) = \frac{dF(p)}{1-F(p_s)}$, let assume $F(p) = p^h$ for $0 \le h \le 1$, then

$$d\mu(p) = \frac{hp^{h-1}}{(1 - F(p_s))}$$

Let define $p_m(h)$ be the average probability which is greater than p_s with parameter h

$$p_m(h) = \int_p^1 t\mu(t)dt = \frac{h}{(h+1)(1-p_s^h)}(1 - p^{h+1})$$

4 An example of reinsurance vs securitization

As the numerical example we set the following numbers from a balance sheet data of a company [11]. The insurance company has the capital $W = 1.4$. It receives the premium $A = 0.4$ and pays the net insurance money $B = 5.7$ in the case of earthquake. If the insurance money exceeds the capital as $B > W$ then it will be the financial distress which costs $C = 0.8$.

The minimum premium ratio for reinsurance company is assumed as $\delta = 0.15$. The insurers of $0 < p \le p_r^* = 0.115$ are in retention, where $p_r^* = \frac{W}{B} - \frac{\delta}{1+\delta}$. The cost of reinsurance is $A_r^*(p) = \frac{\bar{B}p(1+\delta)}{1-p(1+\delta)}$ and the cost of retention is Bp as Figure 2.

4.1 Investors for securitization

Let investors prior distribution of insurer's type p be $F(p) = \sqrt{p}$ for the case $h = 0.5$ and the density function $d\mu(t)$ to securitization of trigger p_s is

$$d\mu(t) = \frac{1}{2(1-\sqrt{p_s})\sqrt{t}}.$$

The average probability of pooling securitization of $h = 0.5$ is

$$p_m(0.5) = \int_p^1 t\mu(t)dt = \frac{1}{3(1-\sqrt{p_s})}(1 - p\sqrt{p}).$$

Then the ratio of subsidization to high-risk companies is

$$R(p) = \frac{p_m(0.5) - p}{1 - p_m(0.5)}.$$

4.2 Three cases of Investors belief distribution functions

For the risk parameter of insurance company in the case of $h = 0.3$, the investor distribution is considered more concentrate in the middle as $F(p) = p^{0.3}$. Insures which are $p = (0.21, 0.30)$ pay the premium as the pooling securitization and they will be in retention $p = (0.115, 0.21)$ as the first row in Table 4.

In the case of $h = 0.5$, $F(p) = \sqrt{p}$. Insures which are $p = (0.29, 0.39)$ pay the premium as the pooling securitization and they will be in retention $p = (0.115, 0.29)$ seen in the the second row in Table 4.

In the case of $h = 0.7$, which is the diverse market of insurers. Insures which are $p = (0.33, 0.45)$ pay the premium as the pooling securitization and they will be in retention $p = (0.115, 0.33)$ as the last row in Table 4.

The density functions of $h = 0.3, 0.5, 0.7$ for insurers risk are seen in Figure 3.

Table 4: Insurers type p and the policies

h	Reinsurance	Retention	Securitization
0.3	(0,0.115)	(0.115,0.21)	(0.21,0.30)
0.5	(0,0.115)	(0.115,0.29)	(0.29,0.39)
0.7	(0,0.115)	(0.115,0.33)	(0.33,0.45)

h : Insurers risk distribution parameters

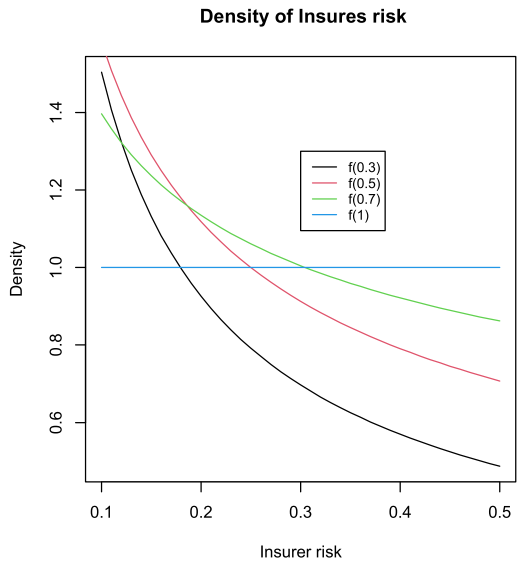

Figure 3: The Securitization in the distribution of insurers

5 Concluding remark

The main theme of the paper is how we can manage the catastrophic Earth Quake in Japan. The common measures agaist catastrophe risk in Japan are "Self-help, Mutual-help and Government-help". However, catastrophic earth quakes of Kobe and East-Japan were mainly supported by Government budget, which had worsened significantly the government deficit as seen in the last column of Table 1. The theme of the paper is to promote the Mutual-help principle utilizing the Catastrophe bonds.

Residential risk of Earth Quake seems to be covered by reinsurance of the Governmental support in Japan, however non-residential risk above the certain level of Earth Quake is not covered by reinsurance from the theoretical viewpoint of SW[2] and in acutual statistics. For the risk over the certain level of Catastrophe risk, the securitization could be pooling and sharing damage among insurers from the theoretical point of view.

From the simple assumption of insurers risk distribution the diversion expands the retention decision area in the insurers risk. Less diverse risk distribution promotes securitization. For the risk above the certain level as Catastrophe risk, the securitization should be pooling and sharing damage among insurers.

Insurance companies get collaborate to common CAT Bond which serves dividend to the owners and in the catastrophe the value of Bond serves the aid to victims. It will be Mutual-help for Catastrophe risk.

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Appendix A Proof of Pooling Bayesian Equilibrium in Discrete model

(1) Let p^h the probability of risk for insurance company h and let p^l be the probability of risk for insurance company l and we assume

$$p^h > p^l.$$

(2) Risk management of insurance companies by securitization
Let $v, 1-v$ be the portfolio to the company h and l, respectively
The pooling securitization is assumed to be better than the separating securitization.

$$A_s^* = \frac{B_s^*(vp^h + (1-v)p^l)}{\{v(1-p^h) + (1-v)(1-p^l)\}}$$

$$B_s^* = B - W$$

where C is the bankrupt cost and is assumed to satisfies,

$$Cp^l > \frac{B_s^* v(p^h - p^l)}{\{v(1-p^h) + (1-v)(1-p^l)\}}. \tag{A.1}$$

$$v(1-p^h) + (1-v)(1-p^l) = 1 - (vp^h + (1-v)p^l)$$

Define the average rate of investment as $Ev = vp^h + (1-v)p^l$, then

$$A_s^* = B_s^* \frac{Ev}{1-Ev}$$

is equivalent as (2) of theorem 2. The cost of default C satisfies

$$Cp^l > B_s^* \frac{Ev - p^l}{1-Ev}$$

which is also the same as (4) in theorem 2.

Proof.

- Separating equilibrium of securitization
  Let (A_s^h, B_s^h) be the securitization for the high risk, and (A_s^l, B_s^l) be for the low risk company
  If $B - W \leq B_s^h$ then the high risk insurance company will not be bankrupt. From the budget constraints for low risk insurance company

$$(1-p^l)A_s^l = p^l(B-W).$$

  The expected payoff of low risk insurer for separating securitization is

$$EU_s^{sep} = (1-p^l)(W+A) + p^l(W-B) - p^lC.$$

- Pooling equilibrium of securitization
  In the pooling (A_s^*, B_s^*) satisfies,

$$[v(1-p^h) + (1-v)(1-p^l)]A_s^* = [vp^h + (1-v)p^l](B_s^* - W).$$

  Then the xpected payoff of low risk insurer for pooling satisfies

$$\begin{aligned} EU_s^{pool} &= (1-p^l)(W+A) + p^l(W-B) - ((1-p^l)A_s^* - p^lB_s^*) \\ &= (1-p^l)(W+A) + p^l(W-B) - \tfrac{(B-W)(vp^h+(1-v)p^l)}{v(1-p^h)+(1-v)(1-p^l)} \end{aligned}$$

  Then we conclude from the condition of C of (A.1)

$$EU_s^{pool} > EU_s^{sep}$$

QED