Economic Relationship Between Critical Minerals and Fossil Energies*

Hiroyuki Okawa†

Abstracts

This paper focuses on major fossil fuels (U.S. crude oil, European gas, and Australian coal) and critical minerals (nickel, lead, and tin), and attempts to analyze their return generation processes and dynamic conditional correlations. Using monthly data for about 20 years from April 2004 to December 2023, it is conducted an empirical analysis using the diagonal-BEKK model, one of the multivariate GARCH models. The simultaneous estimation results show that the monthly returns of nickel, lead, and tin are affected by investment in renewable energy. The dynamic conditional correlations between each fossil fuel and critical minerals are estimated, and it is found that the dynamic conditional correlations also changed during periods of economic fluctuations and increases or decreases in renewable energy investment. In addition, it is observed that the dynamic conditional correlations differed significantly between Europe, which is active in renewable energy, and the U.S. and Australia, both of which are resource-exporting countries. However, the empirical model used in this paper does not reveal the mechanism by which these dynamic conditional correlations increase or decrease, nor the markets that cause the fluctuations, which is a shortcoming of this paper and should be addressed in the future.

Keywords

*Critical Mineral, diagonal-BEKK model, Fossil Energie*s

---

* This work was supported by the Research Institute for Mathematical Sciences, an International Joint Usage/Research Center located in Kyoto University.

† Graduate School of Economics, Kobe University

1. Introduction

Since the 2000s, there has been economic activity aimed at promoting and realizing a decarbonized society and renewable energy, especially in developed countries. Along with this activity, managers and politicians have taken a critical view of fossil fuels such as crude oil and natural gas. As a result, fossil fuels became less important and valued compared to the expectations for renewable energy. In recent years, however, the intensifying discord has prompted many economically advanced countries to rethink their energy policies, most notably by reconsidering their social position on fossil fuels. In the first place, the argument for efficient operation of renewable energies was based on the premise that sufficient power supply and transmission efficiency were guaranteed. Nevertheless, it seems that the focus on climate change issues, including global warming, has tended to neglect these premises. In other words, in some cases, the lack of consideration for energy security has led to a scenario in which problems with the stable supply of electricity could arise, which could hinder the operation of renewable energy sources. Therefore, it is worthwhile to study fossil fuels in light of the current situation.

In addition, there is a need to reevaluate the relationship between fossil fuels and critical minerals, which are essential to industry and economic activity. A global trend is the introduction of technologies such as renewable energy and electrification, especially batteries and converters, which have a direct impact on electricity itself. Many of these elements are produced from minerals called nickel and lithium. In other words, nickel and lithium can be defined as one of the critical minerals. It is obvious that the market value of nickel and lithium will be high in a society where electricity is essential for smart phones and AI learning as the population grows, and institutional investors as well as actual demand will pay attention to nickel and lithium. And as energy policies are reviewed, the stability that fossil fuels offer is expected to have a reciprocal impact on critical minerals in terms of production activity and investment. In addition, analyzing the impact between the market return of critical minerals and the market return of fossil fuels under the current circumstances is expected to be a meaningful endeavor for many market participants as well as for actual demand and policy makers.

In this study, an empirical analysis of the market returns of crude oil, natural gas, and coal, which are representative fossil fuels, and nickel, tin, and lead, which are representative critical minerals related to renewable energy, will be conducted. In particular, our research will address the question of how each fossil fuel correlates with each of the three critical minerals at the market return level, and how these correlations have changed since the 2000s. Thus, the dynamic correlations between ① U.S. crude oil and nickel, tin, and lead, ② European gas and nickel, tin, and lead, and ③ Australian coal and nickel, tin, and lead are estimated using the diagonal-BEKK model of Engle and Kroner (1995), and the frequency and trends of changes in the correlations are discussed.

Empirical analysis using a sample of monthly data over a period of about 20 years, from 2004 to 2023, shows that the dynamic correlations between the monthly returns of the crude oil market and the monthly returns of nickel, tin, and lead are, on average, positive and statistically significant, respectively. Similar results are obtained for the monthly returns of the coal market, each of which is found to be positively correlated with the other. In contrast, the monthly returns of the gas market are found to be negatively correlated, particularly with the monthly returns of the nickel market, which shows a clear downward trend. Considering these facts, the empirical results suggest that regional differences are having an impact. In particular, for resource-exporting countries such as the U.S. and Australia, there is a strong tendency for a positive correlation between their own resources and minerals related to batteries, suggesting that synergistic effects can be expected to continue even in a scenario in which society reverts to fossil fuels. On the other hand, in Europe, which actively encourages renewable energy, the results suggest that investing in the natural gas market is actually a good way to hedge risk in the event that nickel market needs increase in the future.

However, the framework of the diagonal-BEKK model is limited in terms of explanatory variables, which is a drawback of this study and an area for improvement in the future. In addition, this study has not yet clarified the factors that cause variation in the dynamic correlations estimated in this study, such as why they change, and what kind of correlations exist for other minerals related to renewable energy. This is a theme for future research.

Finally, the subsequent structure of this paper is presented. In the next Section 2, it will review earlier studies that conduct empirical analysis related to the theme of this paper; in Section 3, the empirical model used in the analysis is explained and the return generating process for each market return is derived; in Section 4, the characteristics of the sample data treated and the existence of a unit root process are discussed; in Section 5, the estimation results obtained by the empirical model are discussed; and finally, in Section 6, this paper is summarized.

2. Literature Review

Fossil fuels, also known as dirty energy, are relatively easy to analyze because they are traded in the market (futures, spot, options, or ETFs). Crude oil, in particular, has become a popular and large market from the perspective of hedging strategies that take advantage of the fact that it is a physical commodity. For this reason, empirical analysis of the crude oil market is widely conducted in the field of finance, and price-based analysis is possible because crude oil is a real commodity despite being an energy product. On the other hand, it is difficult to answer the question of whether there is a similar market for renewable energy at present. Although a market for carbon emissions trading has recently been launched, there is currently no open-source market for renewable energy itself.

Since the 2000s, however, some researchers have been interested in the impact of renewable energy and have studied it using alternative approaches. Recent studies include, for example, Tiwari et al. (2023) and Dawar et al. (2021). They conducted an empirical analysis of the correlation with fossil fuels and other factors using an index called the S&P Global Clean Energy Index (S&P Clean).

As an overview, S&P Clean is an index of the top 100 companies that have a corporate policy to reduce CO2 emissions or are making efforts to become carbon neutral. It is important to note that the index may include companies that are not involved in renewable energy, since they are only striving to reduce emissions. Therefore, it should be pointed out that it is not a pure renewable energy index, but rather a measure of the performance of characteristic companies and does not deal purely with renewable energy. However, it is an interesting research effort and Tiwari et al. (2023) analyzed the long-term relationship between West Texas Intermediate crude oil (WTI), Oman and Dubai crude oil, and SP Clean. The estimated results indicated co-movements between each crude oil and SP Clean (similar to a cointegration relationship), but not stable. Dawar et al. (2021) noted that WTI returns are positively correlated with SP Clean returns regardless of whether the WTI market is in a bullish or bearish state. Based on these two previous studies, despite the problems with the S&P Clean index mentioned earlier, from the perspective of economic activity and energy demand, we would expect a positive correlation trend between the index for renewable energy and the returns on fossil fuels.

Another study using a different but similar index to S&P Clean is Dias et al. (2023). Dias et al. (2023) analyzes how changes in a sustainable energy index called The Wilder Hill Clean Energy Index affect fossil fuels such as Brent crude oil and coal. The empirical analysis indicates that changes in the index affect[1] fossil fuels in both of the two time periods[2]. In both studies, the indexes related to renewable energy and fossil fuels are analyzed, but considering the origin of each index, the question arises whether they tend to be positively correlated with the economy. In other words, if the economy is favorable, business performance improves due to factors other than renewable energy, and fossil fuels (especially crude oil and natural gas) perform well in terms of actual demand, the analysis would suggest the possibility of a positive correlation. Therefore, the conclusion is that it is dangerous to easily use indices related to renewable energy.

Second, the empirical analysis of critical minerals and fossil fuels, which is the subject of this paper's analysis, has already been addressed in previous studies. A representative study is that of Baffes (2007), who analyzed the correlation between oil prices and minerals. Baffes (2007) conducted an empirical analysis by dividing each commodity price by one of the deflator indices called the MUV index. This is a highly commendable method that allows for a pure correlation analysis without the effects of inflation by returning each commodity price to the real price as a variable. What this previous study shows is that both of the commodities analyzed are commodities, and that the effects of the economy or inflation must be taken into account when analyzing commodities. However, one drawback of the Baffes (2007) study is that the MUV index required to exclude inflation is limited to

---

[1] Interpreted as probably positive impact since it is stated in the conclusion that it increases evolution.

[2] From May 2018 to December 2019, defined as a period of economic stability, and from January 2020 to April 2023, defined as a period of economic stress.

annual data and cannot be treated on a monthly or daily basis.

On the other hand, some studies are critical of inflation concerns. Kanamura (2022) analyzed the correlation between S&P Clean, The Wilder Hill Clean Energy Index, S&P 500 Index, WTI, and Henry Hub natural gas, and found a positive correlation between renewable energy-related indices and fossil fuels. So far, this is similar to previous studies, but in the discussion, the authors state that considering that electricity spot prices increase in tandem with energy prices, it is reasonable to assume that the value of renewable energy projects that sell electricity in the spot market will increase as energy prices rise. In other words, the empirical results suggest that the economy is not the only factor that is positively correlated. Thus, it can be interpreted as a conclusion that the impact of the economy cannot be said to cause significant errors in all the results of the analysis.

Based on these previous studies, the empirical analysis in this paper will incorporate the Wilder Hill New Energy Global Innovation Index (WH NEX) variable into the return generation process, in addition to the variables controlling for the impact of the economy. WH NEX differs from the renewable energy-related indexes covered in earlier studies in that it is composed of approximately 80-90 companies from around the world with innovative technologies focused on clean energy, renewable energy, decarbonization, etc., and is also indexed based on a specific calculation method. The most important feature of the index is that it is limited to only those companies with technology, so that changes in the index can be interpreted as a surrogate index of trends related to investment in renewable energy. In other words, by quantifying the trend in renewable energy investments, the index is expected to measure the influence of renewable energy trends in the return generation process of critical minerals. These details will be discussed in the model structure in Section 3.

3. Empirical Modeling

This section explains the diagonal-BEKK model by deriving the monthly return generation process for each commodity market used in the cleaning estimation in the diagonal-BEKK model.

First, in the fossil fuel group, the variables used in the empirical model are based on monthly data transformed into log-returns variables, which is expressed as in equation (1) below.

$$y_{i,t} = \log\left(\frac{price_{i,t}}{price_{i,t-1}}\right), i = oil, gas, coal,\ GPR,\ BCIUS,\ BCIEU,\ BCIAUS \tag{1}$$

where, GPR represents monthly data for the geopolitical risk index. BCIUS, EU, and AUS represent monthly data for the U.S., European, and Australian business sentiment indices provided by FRED.

Using these variables, the monthly return generation process for the three fossil fuels can be expressed as equations (2) through (4) below.

$$y_{oil,t} = \omega_{oil} + \alpha_1 y_{oil,t-1} + \alpha_2 y_{GPR,t-1} + \alpha_3 y_{BCIUS,t} + \varepsilon_{oil} \tag{2}$$
$$y_{gas,t} = \omega_{gas} + \beta_1 y_{gas,t-1} + \beta_2 y_{GPR,t-1} + \beta_3 y_{BCIEU,t} + \varepsilon_{gas} \tag{3}$$
$$y_{coal,t} = \omega_{coal} + \gamma_1 y_{coal,t-1} + \gamma_2 y_{GPR,t-1} + \gamma_3 y_{BCIAUS,t} + \varepsilon_{coal} \tag{4}$$

where, ω in each monthly return generation process refers to the drift term, and each ε is assumed to follow a standard normal distribution. The reason for including the log returns of the business sentiment index as an explanatory variable in equations (2) through (4) is that it is assumed that there is an effect of the business cycle on monthly returns at time t, as described in Section 2. In addition, monthly returns of the geopolitical risk index are included because it is also assumed that changes in geopolitical risk in the past affect the returns of fossil fuels based on studies by Khan et al. (2021) and others. Thus, the process of generating monthly returns on fossil fuels can be expressed as an equation consisting of an autoregressive term one period ago, GPR monthly returns one period ago, BCI monthly returns at the same point in time, and an error term (or information from the market at time t).

The next step is to derive the monthly return generation process for the three critical minerals. As with the monthly return generation process for fossil fuels, the variables transformed into log returns expressed as in equation (5) below are treated as variables in the empirical model.

$$y_{j,t} = \log\left(\frac{price_{j,t}}{price_{j,t-1}}\right), j = nickel,\ lead,\ tin,\ WH\ NEX, BCIUK \tag{5}$$

where, because monthly data for each critical mineral is derived from the British stock exchange, the business confidence index is also treated in the same way, i.e., as the British one (BCIUK).

Using these variables, the monthly return generation process for the three critical minerals can be expressed as equations (6) to (8) below.

$$y_{nickel,t} = \omega_{nickel} + \delta_1 y_{nickel,t-1} + \delta_2 y_{WH\ NEX,t} + \delta_3 y_{BCIUK,t} + \varepsilon_{nickel} \tag{6}$$
$$y_{lead,t} = \omega_{lead} + \phi_1 y_{lead,t-1} + \phi_2 y_{WH\ NEX,t} + \phi_3 y_{BCIUK,t} + \varepsilon_{lead} \tag{7}$$
$$y_{tin,t} = \omega_{tin} + \psi_1 y_{tin,t-1} + \psi_2 y_{WH\ NEX,t} + \psi_3 y_{BCIUK,t} + \varepsilon_{tin} \tag{8}$$

where, ω and ε for each monthly return generation process is treated as in equations (2) through (4). In addition, $y_{WH\ NEX,t}$ is the monthly returns of WH NEX at time t, and $y_{BCIUK,t}$ is the monthly returns of the UK business confidence index at time t. Thus, the critical mineral monthly return generation process can be expressed as an equation consisting of an autoregressive term one period ago, WH NEX monthly returns at the same time, BCI monthly returns at the same time, and an error term (or information from the market at time t).

The following combinations will be used for peer estimation using the diagonal-BEKK model based on these monthly return generation processes.

Model ①
Monthly return generation process for crude oil market: Equation (2)
Monthly return generation process for three types of critical minerals: Equations (6), (7), and (8)

Model ②
Monthly return generation process for natural gas market: Equation (3)
Monthly return generation process for three types of critical minerals: Equations (6), (7), and (8)

Model ③
Monthly return generation process for coal market: Equation (3)
Monthly return generation process for three types of critical minerals: Equations (6), (7), and (8)

The aim of the simultaneous estimation of the above combinations is to analyze how nickel, tin, and lead, which are important for batteries, the core of renewable energy, correlate with each fossil fuel, and how the correlations evolve over time.

Finally, the diagonal-BEKK model is explained. Since this paper is an empirical analysis as a test standpoint, the basic model will be used first. Therefore, the Engle and Kroner (1995) model is used as is, and the variance-covariance equation is expressed as follows

$$H_t = C + A_k(\varepsilon_{t-1}\varepsilon'_{t-1})A'_k + B_k(H_{t-1})B'_k, k = 1,2,3 \tag{9}$$

where, H_t denotes the 4 × 4 conditional variance-covariance matrix, and matrices A and B are restricted to be diagonal matrices. C is an indeterminate matrix, and ε_{t-1} is a 4 × 1 error term vector. The subscript k of matrix A and matrix B in equation (9) means the combination of simultaneous estimation, where k=1 means model ①, k=2 means model ②, and k=3 means model ③.

In addition, the dynamic conditional correlation (DCC_t), which is the purpose of this paper, can be expressed as in the following equation (10) using the covariance obtained in equation (9) and standard deviation of each log returns.

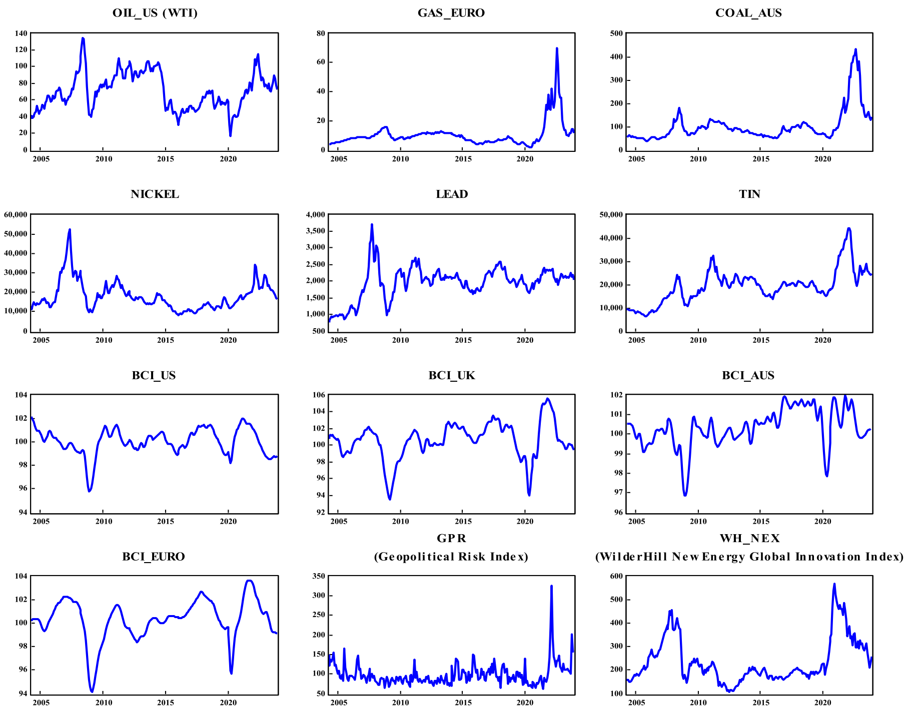
$$DCC_t = \frac{Cov(y_{i,t}, y_{j,t})}{Std.Dev._{y_i} Std.Dev._{y_j}}$$
$$, i = oil, gas, coal \quad j = nickel, lead, tin \tag{10}$$

4. Data Properties

Section 4 describes the distributional characteristics of the sample data used in this paper and examines the presence of a unit root process. The period for all sample data is April 2004 through December 2023 and is monthly data. The crude oil market is West Texas Intermediate crude oil, the benchmark for the U.S. crude oil market. The natural gas market is based on European gas, and the coal market is based on Australian coal. Critical minerals are those traded on the British stock exchange. These commodity market data are spot prices and are obtained from the World Bank's Commodity Prices “Pink Sheet”. The BCI for each country or area is obtained from FRED, the GPR from Caldara and Iacoviello (2022) and their published web page, and the WH NEX from Investing.com.

Figure 1 plots these sample data in time-series order. From Figure 1, it appears that each commodity market price changes at the same time that the geopolitical risk index and the business sentiment index are fluctuating significantly. In particular, the prices of European gas and Australian coal have shown almost the same price movements as the geopolitical risk index. On the other hand, U.S. crude oil prices seem to lag behind changes in the U.S. business sentiment index, suggesting that the U.S. may have a more distinctive relationship with the U.S. With regard to critical minerals, the relationship is broadly similar to that of the WH NEX, but also seems to be linked to BCI of each country. Thus, as pointed out in studies such as Baffes (2007), on a level basis, commodity markets are expected to be affected by the business climate.

Figure 1. Monthly data of commodity markets and indices



Next, a description of the distributional properties and the presence or absence of a unit root process for each sample level and log-returns variable transformed data will be examined. Tables 1 and 2 summarize the mean, standard deviation, skewness and kurtosis, the Jarque-Berra test for the shape of the normal distribution, and the Augmented Dickey-Fuller test (ADF test) for the existence of a unit root process. On a level basis, the results are slightly worrisome because all sample data ADF tests could be rejected except for WH NEX, but the results are rejected at the 10% European gas and tin significance level. On the other hand, in the log returns, all samples could be rejected by the ADF test. Therefore, it is appropriate to perform variable transformations to avoid sham regressions, which can be expected to provide some statistical security for the estimation results in this empirical model.

Table 1. Distributional properties of monthly data on commodity markets and indices

	Commodities					
	Oil	Gas	Coal	Nickel	Lead	Tin
Mean	70.497	10.293	100.765	17966.020	1959.408	19290.121
Std. Dev	22.107	8.630	69.222	7351.155	496.979	7210.579
Skewness	0.308	3.665	2.861	1.646	-0.306	0.723
Kurtosis	2.456	19.708	11.853	6.802	3.850	4.345
Jarque-Berra	6.674**	3286.986***	1097.256***	249.852***	10.832***	38.534***
ADF	-3.402**a	-2.851*a	-2.942**a	-3.112**a	-3.323**a	-2.854*a
	Indices					
	BCI_US	BCI_UK	BCI_AUS	BCI_EU	GPR	WH_NEX
Mean	100.032	100.578	100.316	100.309	98.703	236.612
Std. Dev	1.114	2.175	0.962	1.742	28.425	94.958
Skewness	-0.959	-0.723	-0.829	-1.074	3.076	1.2885
Kurtosis	4.991	4.415	4.592	5.064	20.477	4.0035
Jarque-Berra	75.458***	40.457***	51.959***	87.642***	3389.969***	75.419***
ADF	-3.871***a	-5.330***b	-3.714**b	-3.997***a	-6.742***a	-0.625c

Note: The sample period of monthly observation runs from April 2004 to December 2023. Significance at the 1, 5 and 10 % level is denoted by asterisks ***, **, *, respectively. The ADF test uses three different types of constraints, with trend term, with trend term and intercept and with none of them are indicated by a, b, and c next to the estimated value. Jarque–Bera statistics and ADF tests are followed as $\chi 2$ distribution.

Table 2. Distributional properties of monthly log returns on commodity markets and indices

	Commodities (log returns)					
	Oil	Gas	Coal	Nickel	Lead	Tin
Mean	0.003	0.005	0.004	0.001	0.003	0.004
Std. Dev	0.109	0.126	0.094	0.088	0.072	0.068
Skewness	-0.937	-0.016	-0.247	-0.024	-0.713	-0.268
Kurtosis	10.902	6.943	6.966	4.712	5.447	3.952
Jarque-Berra	651.271***	153.502***	157.754***	28.950***	79.223***	11.784***
ADF	-11.590***c	-11.154***c	-11.595***c	-11.139***c	-12.335***c	-11.006***c
	Indices (log changes)					
	BCI_US	BCI_UK	BCI_AUS	BCI_EU	GPR	WH_NEX
Mean	-0.0001	-5.123e-05	-1.197e-05	-3.517e-05	0.001	0.002
Std. Dev	0.002	0.005	0.003	0.003	0.193	0.082
Skewness	0.150	-0.021	0.022	-1.166	0.547	-0.979
Kurtosis	5.935	6.779	4.889	10.179	4.839	6.842
Jarque-Berra	85.938***	141.039***	35.099***	562.696***	45.215***	183.589***
ADF	-6.169***c	-6.667***c	-5.206***c	-4.878***c	-12.236***c	-13.024***c

Note: The sample period of monthly observation runs from April 2004 to December 2023. Significance at the 1 % level is denoted by asterisks ***. The ADF test uses three different types of constraints with none of trend term and intercept is indicated by c next to the estimated value. Jarque–Bera statistics and ADF tests are followed as $\chi 2$ distribution.

5. Results

5 −1. Estimation results with diagonal-BEKK model

This subsection describes and discusses the results of the simultaneous estimation of the grouped models ①, ② and ③ and the results of the estimation of the variance-covariance equation by equation (9).

Table 3 summarizes the results of the simultaneous estimation of models ①, ② and ③. In Table 3, the asterisk mark (*) next to the estimated values indicates the significance level, meaning *** for the 1% level, ** for the 5% level, and * for the 10% level, respectively. Also, ω_i is the drift term of the monthly return generation process for each fossil fuel, for example it is denoted as ω_{oil} for crude oil. AR (1) refers to the autoregressive term of the monthly return generation process for each fossil fuel at time t-1.

Let us begin by looking at the monthly returns for fossil fuels, and it is clear that the results are distinctive. Crude oil returns are significantly positively correlated with GPR returns a period earlier and with BCIUS over the same period. This relationship suggests that monthly returns in the crude oil market are sensitive to changes in geopolitical risk and economic fluctuations. The empirical results support the findings of Goldstein and Yang (2022), who indicated that financialized commodity markets are linked to the real economy. In contrast, only the autoregressive terms for gas and coal returns were significantly positively correlated. In other words, the gas and coal markets are more independent than the crude oil market, suggesting that they still have a strong random walk nature even though the financialization of commodity markets is said to be progressing. This contrasting result raises the further question of whether the geographical background of these markets is a factor.

Table 3. Estimated results of monthly returns generating process at three types of models

	Model ① (Oil)	Model ② (Gas)	Model ③ (Coal)
Variables	coefficients	coefficients	coefficients
ω_i	0.009*	0.004	-0.005
$AR\ (1)$	0.073	0.271***	0.297***
$y_{GPR,t-1}$	0.036*	0.030	0.022
$y_{BCIUS,t}$	12.806***	0.616	2.038
ω_{nickel}	-0.003	-0.002	-0.002
$y_{nickel,t-1}$	0.256***	0.274***	0.256***
$y_{Eco,t}$	0.187***	0.176***	0.243***
$y_{BCIUK,t}$	0.561	1.595	1.042
ω_{lead}	0.001	-0.002	0.001
$y_{lead,t-1}$	0.033	0.086	0.090
$y_{Eco,t}$	0.122**	0.156***	0.191***
$y_{BCIUK,t}$	1.725*	1.979**	1.817*
ω_{tin}	-0.0004	0.0002	0.001
$y_{tin,t-1}$	0.245***	0.258***	0.232***
$y_{Eco,t}$	0.153***	0.173***	0.178***
$y_{BCIUK,t}$	3.013***	3.675***	3.347***

Notes: The sample period of monthly observation runs from April 2004 to December 2023. Significance at 1, 5 and 10% level is denoted by ***, ** and *, respectively.

Next, the monthly returns for the critical mineral will be discussed. One feature of any model is that the estimated results for the explanatory variables are consistent. The monthly returns for nickel are significantly positively correlated with the past autoregressive terms and the monthly returns for WH NEX, while the monthly returns for lead are significantly positively correlated with the monthly returns for BCIUK and WH NEX. It is also found that the monthly returns of tin have significant positive correlations with all explanatory variables. The first thing that can be deciphered from these estimation results is that all of the critical minerals are affected by the trend in renewable energy investment. In addition, under the increase in battery demand associated with economic expansion, the results are consistent with our intuition that the monthly returns of lead and tin are positively correlated with changes in the economy due to their relationship as catalysts for batteries. On the other hand, it is quite interesting to note that the estimated results show that only the autoregressive term and the trend in renewable energy investment are significantly related to the monthly returns of nickel, although it was expected that the monthly returns of nickel would be affected by changes in the business cycle from the standpoint of batteries. The relationship between the nickel market, which is sensitive to the impact of investment in renewable energy but not to fluctuations in the business cycle, may be an important factor in considering the future development of the battery industry and its coexistence with renewable energy.

The last part of this subsection discusses the estimation results for the variance-covariance equation. Table 4 summarizes the estimation results for matrix A, the coefficient of the square of the error term, and matrix B, the coefficient of H_{t-1}, for models ①, ② and ③, respectively. The asterisk (*) in the table indicates the significance level, with 1% meaning *** and 10% meaning *. In model ①, the estimation results for matrices A and B are all statistically significant. On the other hand, several expected results were not obtained in models ② and ③. In model ②, matrix A of the variance-covariance equation with monthly returns of nickel is not significant. Also, in model ③, the matrix A of the variance-covariance equation due to monthly returns of tin is significant at the 10% level, but the result is nonnegative. However, since the non-significant vectors in the estimation results of models ② and ③ are both error term vectors, it can be concluded that they do not pose a significant problem for DCC, which is the objective of this paper. Therefore, it can be concluded that these parts are problems of the model specification, and future work is needed to address them.

Table 4. Estimated results of variance-covariance equations at three types of models

Variance Equation Coefficients with diagonal-BEKK					
Model ① (Oil)		Model ② (Gas)		Model ③ (Coal)	
	Coefficients		Coefficients		Coefficients
A1(1,1)	0.633***	A2(1,1)	0.432***	A3(1,1)	0.380***
A1 (2,2)	0.309***	A2 (2,2)	0.029	A3 (2,2)	0.342***
A1 (3,3)	0.268***	A2 (3,3)	0.219***	A3 (3,3)	0.239***
A1 (4,4)	0.180***	A2 (4,4)	0.261***	A3 (4,4)	-0.119*
B1 (1,1)	0.417***	B2 (1,1)	0.912***	B3 (1,1)	0.908***
B1 (2,2)	0.863***	B2 (2,2)	0.983***	B3 (2,2)	0.692***
B1 (3,3)	0.947***	B2 (3,3)	0.959***	B3 (3,3)	0.950***
B1 (4,4)	0.947***	B2 (4,4)	0.900***	B3 (4,4)	0.888***

Notes: The sample period of monthly observation runs from April 2004 to December 2023. Significance at 1 and 10% level is denoted by *** and *, respectively.

5 −2. Analysis of dynamic conditional correlations

The DCCs calculated according to Equation (10) using the covariances of each model ①, ②, and ③ obtained in Equation (9) will be discussed in this subsection.

Table 5 summarizes only mean values of the basic statistics for the 3x3 dynamic conditional correlations (DCC_t)over a sample period of about 20 years, from 2004 to 2023. Reviewing from model ①, it is found that DCC_t with the monthly returns of nickel, tin, and lead are positive in the value of Mean for the monthly returns of crude oil, respectively. The same is also positive for the monthly returns of coal. In contrast, the DCC_t with monthly returns of gas and monthly returns of nickel, tin, and lead are all negative values. It is interesting to note that these results show a clear difference between Europe, which encourages renewable energy, and the United States and Australia, both of which are fossil fuel resource exporters. Based on these results, it is analyzed how DCC_t of each model is changing.

Table 5. Distribution (Mean) of estimated DCC_t

	Sample periods: April 2004 ~ December 2023		
	Nickel returns	Lead returns	Tin returns
Oil returns (US)	0.187	0.130	0.191
Gas returns (EU)	-0.036	-0.032	-0.051
Coal returns (AUS)	0.172	0.196	0.170

Figure 2 plots in time-series order the changes in DCC_t and the related indices (BCI, GPR, and WH NEX) for model ①, i.e., the monthly returns of the crude oil market and the critical minerals. It can be read from Figure 2 that the three DCC_t in model ① are not stable fluctuations, but rather frequent wild fluctuations. It can also be seen that DCC_t tends to fluctuate significantly during periods when the U.S. BCI fluctuates significantly and WH NEX rises sharply. Comparing the period before and after 2015 after the Paris Agreement, it cannot be claimed that the transition of DCC_t has not changed extremely.

Figure 3 plots the transition of DCC_t and related indices (BCI, GPR, WH NEX) in time-series order in Model ②, i.e., the monthly returns of the natural gas market and the DCC_t with critical minerals. Compared to the transition of DCC_t in model ①, it can be seen from Figure 3 that the transition is significantly different. In particular, the DCC_t of gas and nickel were characterized by a steady decline until a certain period. It was always in a declining trend until 2008, when the Lehman Shock occurred, but it can be seen that the correlation has begun to exceed 0.0 since 2015. However, the DCC_t of gas-lead and gas-tin has been observed to increase in correlation at several timings, and it is not possible to read from this graph whether it is changing due to the economy, geopolitical risk, or renewable energy investment. Rather, one cannot rule out the possibility that other market or macro factors are related to the DCC_t of gas-lead and gas-tin. Lastly, Figure 4 plots model ③, i.e., the evolution of DCC_t with monthly returns and critical minerals in the coal market and the related indices (BCI, GPR, and WH NEX) in chronological order. It can be seen from Figure 4 that several periods are characterized by large declines in DCC_t, which occurred around 2008, when the Lehman Shock occurred, and around 2015, when the Paris Agreement was implemented.

These things can be summarized in the fact that there is no clear evidence at this time as to what is the factor that increases or decreases the variation in DCC_t of any of the models. Certainly, at first glance, it seems that the U.S. may be affected by economic fluctuations and trends in renewable energy investment, but it may be necessary to change one's mind if one asks whether the same thing affects Australia, which also uses the same fossil fuels. It is also undeniable that the correlation with critical minerals may be different in Europe, which is aggressive in expanding renewable energy, than in the U.S. because of its reluctance to invest in the gas market in the first place. Therefore, it is our future task to verify what is behind the characteristic DCC_t obtained in this empirical analysis.

Figure 2. DCC estimated by model ① and monthly data of energy related indices

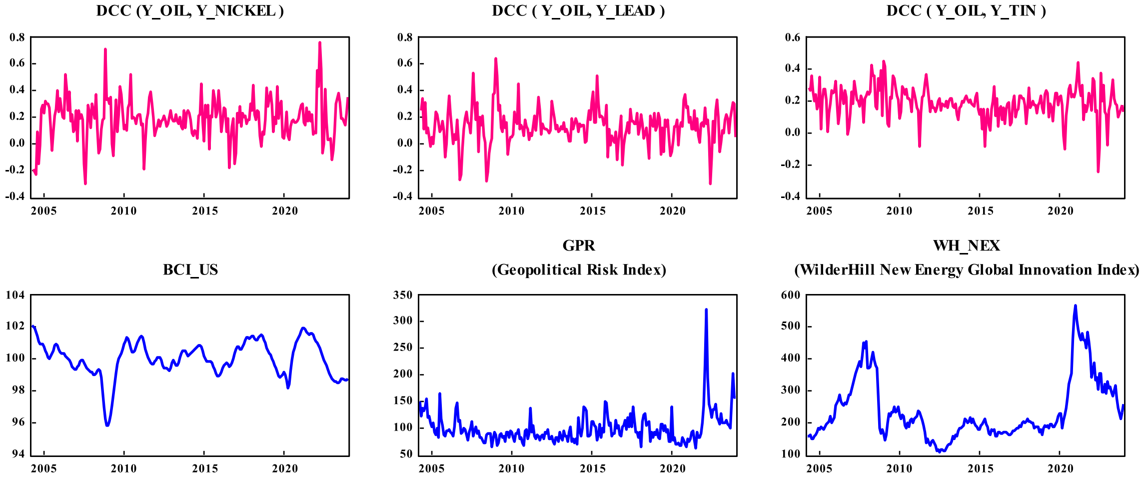


Figure 3. DCC estimated by model ② and monthly data of energy related indices

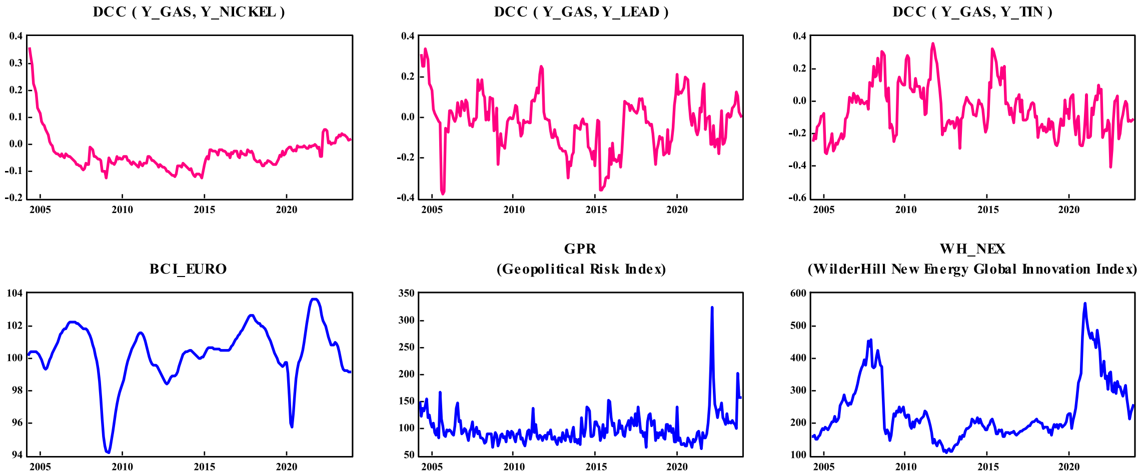
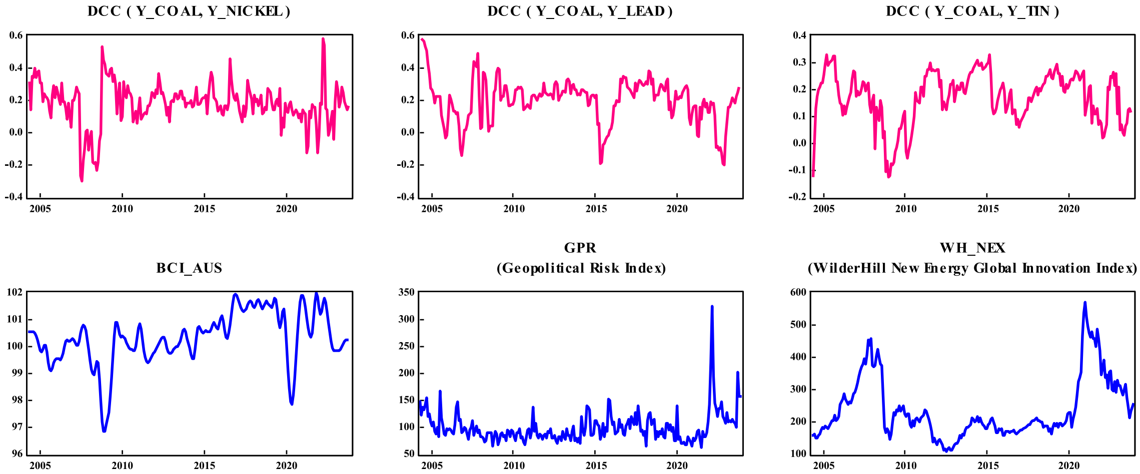


Figure 4. DCC estimated by model ③ and monthly data of energy related indices



6. Conclusion

This paper focuses on major fossil fuels (U.S. crude oil, European gas, and Australian coal) and critical minerals (nickel, lead, and tin), and attempts to analyze their return generation processes and dynamic conditional correlations. The empirical analysis was conducted using the diagonal-BEKK model with monthly data for about 20 years from April 2004 to December 2023, and several characteristic results were obtained. First, the simultaneous estimation shows that the crude oil market, unlike the gas and coal markets, is sensitive to changes in geopolitical risk and business cycle fluctuations. In addition, all three critical minerals are commonly affected by changes in renewable energy investment, suggesting that trends in renewable energy investment are relevant. The results can also be estimated and calculated for all DCC and found that U.S. crude oil and Australian coal have a positive trend with critical minerals. In the U.S. and Australia, DCC_t seems to change in accordance with the increase or decrease in the economy and the trend of renewable energy investment. On the other hand, European gases were found to have a negative trend with critical minerals, suggesting that the region of Europe may have something to do with the results. These empirical results are expected to provide evidence for further consideration of the prospects for the battery business and fossil fuels, which are likely to expand further in the future.

However, there are two notable shortcomings of this paper. first, the limitations of model selection, such as the need to carefully select explanatory variables for the convergence of simultaneous estimation due to the adoption of the diagonal BEKK model. second, the inability to clarify what factors cause changes in the estimated DCC, why such fluctuations occur. Therefore, future research should explore the most appropriate approaches, analytical methods or additional analyses.

Acknowledgements

I would like to thank the professors who provided comments and suggestions while writing this paper. And I would also like to thank Professor Tsujimura of Doshisha University for the opportunity to present this paper.

References

[1] Baffes, J. (2007). Oil spills on other commodities, *Resource policy*, 32, 126-134.

[2] Caldara, D. and Iacoviello, M. (2022). Measuring Geopolitical Risk. *American Economic Review*, 112(4), 1194–1225.

[3] Dawar, I., Dutta, A., Bouri, E. and Saeed, T. (2021). Crude oil prices and clean energy stock indices: Lagged and asymmetric effects with quantile regression, *Renewable Energy*,163, 288-299.

[4] Dias, R. T., Chambino, M. and Alexandre, P. M. (2023). Strength in Transition: Resilience of Sustainable Energy vs. Fossil Energy, *7th International Scientific Conference ITEMA 2023-Conference Proceedings-*, 157-166.

[5] Engle, R.F. and Kroner, K.F. (1995). Multivariate simultaneous generalized ARCH. *Econometric Theory*, 11 (1), 122–150.

[6] Goldstein, I. and Yang, L. (2022). Commodity Financialization and Information Transmission. *The Journal of Finance*, 77, 2613–67.

[7] Kanamura, T. (2022). A model of price correlations between clean energy indices and energy commodities, *Journal of Sustainable Finance & Investment*, 12(2), 319-359.

[8] Khan, K., Su, C–W., Tao, R. and Umar, M. (2021). How do geopolitical risks affect oil prices and freight rates? *Ocean and Coastal Management*, 215.

[9] Tiwari, A. K., Trabelsi, N., Abakah, E. J. A., Nasreen, S. and Lee, C. C. (2023). An empirical analysis of the dynamic relationship between clean and dirty energy markets, *Energy Economics*, 124, 106766.