[[Nontrivial modifications of the original text (i.e., for the purpose of protecting certain confidential information) are denoted via "[[…]]".]]

From: 望月新一 <shinichimochizuki1@gmail.com>
Date: 2022 年 6 月 30 日(木) 12:47
Subject: Invitation to reopen dialogue concerning IUT
To: Jakob Stix <stix@math.uni-frankfurt.de>
Cc: 望月新一 motizuki@kurims.kyoto-u.ac.jp

Dear Professor Stix,

I hope that this e-mail finds you in healthy and productive circumstances.

I apologize for the length of this e-mail, but I believe that its content is of substantial importance, so I hope that you will take the time to read it through to the end and to reflect thoughtfully on its content. I understand that you must be very busy with numerous other commitments (since I myself am also very busy with numerous other commitments!), but I hope that you will be able to provide a meaningful response, say,

within three months

or so (although you should certainly not hesitate to inform me if you need more time, e.g., six months).

Of course, if possible, a more prompt response would be most welcome. I would also like to emphasize that I am always available (modulo scheduling issues arising from numerous other commitments) for discussions concerning the content of this e-mail via e-mail or Zoom ([[xxx@xxx]]). Moreover, you are always most welcome (modulo possible scheduling or financial support issues) to visit RIMS (i.e., which is now possible as a result of the elimination, a few months ago, of various pandemic-related restrictions on entry into Japan) to discuss mathematical topics of mutual interest, if you are interested.

Already four years have elapsed since our discussions in March 2018 and the subsequent release of your 10pp. manuscript in May/August 2018. During this time, many important developments concerning IUT have transpired:

- the publication of the four IUT papers and of a sequel [ExpEst] on numerical estimates;

- the special year (including four workshops) on IUT at RIMS in 2021;

- the release of a new exposition [EssLgc] of the essential logical structure of IUT;

- the development of **exciting new applications** (for which the corresponding papers are currently in the process of being written) of upgraded versions of IUT to the Section Conjecture and to various properties of certain types of L-functions.

On the other hand, as you are presumably fully aware, your 10pp. manuscript continues to be a source of substantial confusion concerning IUT in certain sectors of the mathematical community. This confusion has resulted in quite substantial disruptions of what I refer to as the **operational normalcy** of the field of mathematics. Such disruptions have had and continue to have a **profoundly negative effect** on the careers and professional activities of numerous mathematicians, including both senior (not only myself) as well as somewhat less senior mathematicians. For various reasons, I am not at liberty to disclose the details of these negative effects on other mathematicians.

In our final e-mail correspondence in August 2018, I took the position that I do not wish to exert pressure on you (or Peter Scholze) to continue discussions concerning IUT if you are not interested in doing so. On the other hand, the fact that your 10pp. manuscript does not contain detailed, rigorously formulated statements and proofs of the key assertions cannot be ignored and constitutes a **serious violation** of article (6.)

> "Mathematicians should not make public claims of potential new theorems or the resolution of particular mathematical problems unless they are able to provide full details in a timely manner."

of the subsection entitled "Responsibilities of authors" of the EMSCOP (i.e., the Code of Practice of the European Mathematical Society).

That is to say, the very serious consequences for the development of the field of mathematics that ensued from your 10pp. manuscript just serve to highlight the fact that, in accordance with this passage of the EMSCOP, you have a **fundamental, ethical duty** as a professional

mathematician working under the auspices of the EMS either to

"provide full details in a timely manner"

or to

publicly withdraw your 10pp. manuscript.

Surely, as a professional mathematician in a position of responsibility, e.g., as a journal editor, you must be fully aware of the fundamental importance of such ethical issues from the point of view of administrative aspects of the field of mathematics. Indeed, I personally am familiar with these details of the EMSCOP precisely as a result of my work as editor in chief of PRIMS, which is published by the EMS.

Failure to comply with this passage from the EMSCOP, i.e., to "provide full details in a timely manner", has the effect of creating a sort of

blackhole relative to the issue of
mathematical accountability

that can lead to significantly harmful effects on the development of mathematics for decades or even centuries (cf. the discussion of [EssLgc], ¥S 1.5, ¥S 1.10, ¥S 1.12). Of course, one famous example is the case of Fermat's asserted proof (for which he never provided "full details in a timely manner"!) of "Fermat's Last Theorem". Another (though somewhat less famous) example is the assertion, on the part of Deligne, of some sort of logical relationship between the SectionConjecture and the Mordell Conjecture. Although I am not familiar with all of the details (please feel free to correct me if I am misunderstanding something!), I believe that you yourself played an important role in trying to pin down the "full details" (i.e., of a proof that some form of the Section Conjecture implies the Mordell Conjecture) and eventually in convincing Deligne to publicly withdraw his assertion. In particular, the important role that you played in this situation involving Deligne's assertion suggests all the more strongly that you should be fully aware of the fundamental importance of compliance with this "full details" passage from the EMSCOP.

If you are no longer interested in being involved with IUT or writing out the "full details" of the statements and proofs of key assertions of your 10pp. manuscript, then I would like to emphasize that the simplest course of action for you to take would be to simply

publicly withdraw your 10pp. manuscript,

i.e., to publicly make clear that

you do **not** have any **rigorous mathematical reason**
not to acknowledge the mathematical validity of IUT.

On the other hand, if you do not wish to publicly withdraw your 10pp. manuscript, then I wish to emphasize once again that you have a **fundamental, ethical duty** as a professional mathematician working under the auspices of the EMS to

"provide full details in a timely manner".

As I discussed at the beginning of this e-mail, if you are interested in discussing the mathematical content of IUT either from the point of view "providing full details" or independently just to deepen your understanding of IUT, I would like to emphasize once again that I am always available (modulo scheduling issues arising from numerous other commitments) for discussions via e-mail or Zoom.

Moreover, with regard to making various public announcements, I am always prepared to cooperate so that such announcements can be made in a coordinated and harmonious fashion and attract a minimal amount of sensationalist media attention(since I think that this is in our common best interest, as well as in the best interest of the field of mathematics!). For instance, we could try to coordinate a public announcement that could be made through some mathematician in a position of social/political authority in such a way that the announcement would provide a clear, definitive statement with regard to **mathematical content**, but in such a way that it is engineered so as to attract a minimal amount of attention to you personally.

Next, I would like to turn to a discussion of mathematical content. The key starting point of the argument in Section 2.2 of your 10pp. manuscript

(KP1) "In order for a meaningful inequality to be concluded,
one must consistently identify all of these."

(l. -2 of p. 9 of the August 2018 version) --- i.e., a certain **identification** of various copies of R

(the real numbers) --- is an assertion that appears **nowhere** in IUT and is given without any mathematical justification in your 10pp. manuscript. This is the first **key point** (KP1) that requires a rigorous argument with "full details", in accordance with the EMSCOP. The second **key point** (KP2) that requires a rigorous argument with "full details", in accordance with the EMSCOP, is your assertion in footnote 12 (of the August 2018 version of your 10pp. manuscript)

> (KP2) "with the simplifications outlined above, such as identifying identical copies of objects along the identity, the critical [IUTT-3, Theorem 3.11] does not become false, but trivial."

As I have stated many times, I **completely agree** that once one imposes the condition of consistently identifying the various copies of R (i.e., (KP1)), one obtains a contradiction/meaningless situation. The point is that imposing this condition of consistently identifying the various copies of R yields a theory with a fundamentally different, **logically unrelated** **logical structure** from that of IUT. This is what is explained in detail in [EssLgc] (cf., e.g., the discussion of [EssLgc], Example 2.4.8). In a word, identification of these copies of R as in (KP1) --- or the closely related stronger version of identification of Hodge theaters, as in (KP2) --- on opposite sides of the theta-link has the effect of **invalidating** the crucial **logical AND** relation --- i.e., to the effect that

> the ("abstract", in your terminology) prime-strip that appears in the theta-link is **simultaneously** the ("concrete", in your terminology) theta-pilot prime-strip in the domain of the theta-link **AND** the ("concrete", in your terminology) q-pilot prime-strip in the codomain of the theta-link

--- that forms the **logical centerpiece** of the logical structure of IUT. It is this crucial logical AND relation that is invalidated by **deleting the distinct labels** attached to distinct copies of R (or distinct Hodge theaters) on opposite sides of the theta-link.

It seems that a misunderstanding of the crucial nature of this crucial logical AND relation is the **fundamental cause** of the misunderstandings that underlie the argument in Section 2.2 of your 10pp. manuscript. Indeed, do you remember that you emphasized during our March 2018 discussions that

> the abstract prime-strip that appears in the theta-link

> is **either** the ("concrete") theta-pilot prime-strip in
> the domain of the theta-link **OR** the ("concrete") q-pilot
> prime-strip in the codomain of the theta-link, but **not**
> **both simultaneously**?

I remember very vividly the image of you emphasizing this **OR assertion** at a **key point** in our March 2018 discussions, since this assertion struck me as being a sort of **epicenter** of the misunderstandings that underlie the argument in Section 2.2 of your 10pp. manuscript.

One phenomenon that has come to my attention in recent months and which I personally found somewhat **shocking** is the phenomenon that it seems that many mathematicians in a position of substantial political authority in the European mathematical community seriously believe that the central RC (i.e., "redundant copies") assertions (that is to say, concerning identification of copies of R, as in (KP1), or of distinctly labeled Hodge theaters, as in (KP2)) of your 10pp. manuscript are not in fact your assertions at all, but rather assertions that were fabricated by me! I found this phenomenon to be so shocking not only because of the explicit mention of (KP1), (KP2) in your 10pp. manuscript, but also because I remember so vividly how ardent you (and Peter Scholze) were in making these assertions orally during our March 2018 discussions. Moreover, I remember that already during the December 2015 workshop at Oxford (as well as during the subsequent months), Brian Conrad often made assertions concerning RC, and that you yourself made assertions concerning the possibility of achieving a substantial simplification of IUT by eliminating such RC. Also, a few years subsequent to this, I heard reports to the effect that Matthew Morrow was also making assertions concerning RC. At any rate, my treatment of this issue in substantial detail in my recent manuscript [EssLgc] is based entirely on what I continue to understand as assertions concerning RC by **other mathematicians** (i.e., you yourself, Scholze, Conrad, Morrow, and most probably several other mathematicians) which these mathematicians seem to regard as the centerpiece, relative to mathematical content, of their negative position concerning IUT.

Another closely related phenomenon surrounding these assertions (KP1), (KP2) is the lack of a **readily accessible infrastructural apparatus** that would allow the mathematical community to digest the mathematical content of these assertions. That is to say, typically, when a substantial mathematical result is publicly asserted, various **related activities** occur:

- the release of a properly written mathematical paper that gives rigorously formulated statements and proofs with full details (in accordance with the EMSCOP), and which is then submitted to a

mathematical journal for peer review;

- talks, workshops, and seminars concerning the result where the result is discussed and analyzed in substantial detail;

- the development of a community of mathematicians who are thoroughly familiar with the mathematical content of the result and prepared to discuss this mathematical content with other mathematicians;

- expository texts by other mathematicians concerning the result;

- further research (typically leading to new mathematical papers) that builds on the mathematical content of the result.

For instance, in the case of IUT, numerous such related activities have occurred over the past decade, and there is now a quite substantial community of mathematicians involved with IUT who continue to engage in such related activities, such as the development of new versions and applications of IUT. On the other hand, to my knowledge, **none** of these related activities has occurred with regard to the **key points** (KP1), (KP2) of your 10pp. manuscript, and indeed I continue to search for even a single mathematician who feels that he/she understands the mathematical content of (KP1), (KP2) and is prepared to discuss this mathematical content with me (or other mathematicians in the IUT community). It is precisely this sort of situation that gives rise to the creation of a sort of **blackhole** relative to the issue of **mathematical accountability** (i.e., as discussed above).

Put another way, in short, I think that the lack of any accessible infrastructural apparatus for processing these key points (KP1), (KP2) is one of the main causes of the (quite frankly, somewhat absurd!) confusion/ignorance, on the part of many mathematicians in a position of political authority in the European mathematical community, mentioned above concerning the mathematical content of your 10pp. manuscript, despite the fact these mathematicians tend to cite this 10pp. manuscript so frequently and with such intense conviction as a justification of their negative position concerning IUT!

Let me return to the discussion of mathematical content. I often refer to the well-known example of the gluing construction of the **projective line** (cf. [EssLgc], Example 2.4.7, for more details) --- i.e., by gluing together two copies of the affine line (which are ring schemes!) along the

(multiplicative group scheme) G_m in a fashion that is **not** compatible with the respective ring scheme structures of the two copies of the affine line --- as an elementary and efficient way of explaining the AND/OR misunderstandings that underlie the argument in Section 2.2 of your 10pp. manuscript (cf. also [EssLgc], Example 2.4.8, for another elementary example that discusses the essential issues that occur in Section 2.2 of your 10pp. manuscript).

At a more technical level, if one denotes the two copies of the affine line by U_0 and U_1 and the respective standard coordinates on these two copies by t_0 and t_1, then the gluing construction of the projective line involves an identification t_0=t_1^{-1}, i.e., the creation of a new rational function on the glued object that is **simultaneously** equal to t_0 on U_0 **AND** equal to t_1^{-1} on U_1. Of course, if one arbitrarily deletes the distinct labels "0" and "1", then one obtains a contradiction "t=t^{-1}", but this contradiction does not imply the existence of some intrinsic flaw in the gluing construction of the projective line! Rather, it simply means that arbitrary deletion of the distinct labels yields a mathematically non-isomorphic object to the object (i.e., the projective line) obtained by maintaining the distinct labels "0" and "1". Here, we note that

- the two copies of the affine line correspond, in IUT, to the two Hodge theaters on opposite sides of the theta-link;

- G_m corresponds to the ("abstract") prime-strip that occurs in the theta-link;

- the copies of Z (the rational integers) that contain the exponents "1" or "-1" that occur over the various coordinate functions t_0, t_1, t correspond to the copies of R in Section 2.2 of your 10pp. manuscript;

- the contradiction "t=t^{-1}" (i.e., at the level of exponents in copies of Z, "1=-1") corresponds to the contradiction arising from the assertion (KP1) in Section 2.2 of your 10pp. manuscript.

If one specializes the base field of the discussion of the preceding paragraph to C (the complex numbers), then the multiradial representation of IUT (i.e., [IUTchIII], Theorem 3.11) and its use in deriving [IUTchIII], Corollary 3.12, may be understood as corresponding to the well-known **geodesic metric geometry** of the **sphere**, regarded as the result of gluing the northern and southern hemispheres along the equator (cf. [EssLgc], Example 2.4.7, (v)). That is to say, this geodesic metric geometry allows one to "compute" the southern hemisphere from the point of view of the northern hemisphere --- **not** via an **arbitrary/naive identification, or crushing

together,** of the two hemispheres (!), but rather --- by applying the geodesic flow, which allows one to represent the southern hemisphere as

> "the northern hemisphere, shifted by a certain
> explicitly bounded deviation/displacement".

This is the same geodesic geometry as the geometry that is used to create rectangular, cartesian maps (in the cartographic sense!) of the (non-Euclidean!) spherical globe (i.e., the surface of the planet earth), i.e., by allowing for a relatively mild, bounded deviation/distortion/displacement. Here, we note that the gluing "t_0=t_1^{-1}" that gives rise to the projective line may be thought of, in the current metrized situation, as corresponding to the following (at first glance, self-contradictory!) phenomenon:

> an oriented flow along the equator --- which may be thought of
> physically as a sort of wind current --- that flows from east
> to west appears **simultaneously** to be flowing, from the point
> of view of the northern hemisphere, in the clockwise direction
> **AND**, from the point of view of the southern hemisphere, in
> the counterclockwise direction.

Again, this qualitative model (involving the sphere/projective line) of the argument in IUT --- i.e., of embedding the q-pilot and theta-pilot objects in a **common container**, in a fashion compatible with the gluing constituted by the theta-link, by introducing suitable distortions/indeterminacies, which allow one to remove the distinct labels --- consists of entirely elementary and well-known mathematics. In particular, I continue to fail to see why it seems to be so difficult for you to digest the general flow of the argument in IUT. That is to say, in the final few lines of your 10pp. manuscript, you state, in effect, that you "do not understand" the use of indeterminacies in IUT in order to remove the distinct labels without violating the crucial logical AND relation. This "inability to understand" the general flow of the argument in IUT appears to be the main (entirely non-rigorous!) justification for the RC identification assertions (KP1), (KP2). But in the example above of the geometry of the sphere, it is not difficult to see that embedding the northern and southern hemispheres in the common container provided by the entire sphere, in a fashion compatible with the gluing along the equator, yields a situation in which one can use the intrinsic geodesic metric geometry of the sphere to qualitatively estimate the southern hemisphere as a slightly distorted/displaced version of the northern hemisphere **without** ever crushing the two hemispheres into a single hemisphere!

Incidentally, with regard to the topic of indeterminacies, one remark that I should make is the following: Although it seems that during our March 2018 discussions and also in your 10pp. manuscript you were mainly concerned with the indeterminacies (Ind1) and (Ind2), it should be understood that in fact the indeterminacy **(Ind3)** is the most important/fundamental indeterminacy in IUT. For more details, I refer to the discussion of [EssLgc], ¥S 3.5, ¥S 3.11.

At any rate, I would like to emphasize that not only I myself, but also many of our colleagues --- such as Yuichiro Hoshi, Go Yamashita, Arata Minamide, Akio Tamagawa, Ivan Fesenko, Mohamed Saidi, Emmanuel Lepage, Benjamin Collas, etc., etc., many of whom have played a quite nonnegligible (to say the least!) role in your own career up till now as a professional mathematician! --- have a **keen interest** in obtaining detailed information concerning the precise mathematical content that underlies your perceived difficulties in understanding the central argument in IUT. Moreover, I would like to emphasize once again that sharing these "full details" of the precise mathematical content underlying your assertions (KP1) and (KP2), in accordance with the EMSCOP, is not only in the best interest of those mathematicians who are substantially involved with IUT, but also in the best interest of the entire field of mathematics!

In the context of the situation involving our colleagues, I would like to explain in a bit more detail the situation surrounding Emmanuel Lepage, since this situation may be of interest to you. My discussions with him concerning IUT took place, for the most part, between the summer of 2017 and the fall of 2021. They were conducted partly in person (prior to the pandemic) and partly via e-mail/Zoom after the pandemic began. Although (as you may see from the content of the exposition) many of the earlier portions of [EssLgc] were based on our March 2018 discussions and your 10pp. manuscript, the latter portions of [EssLgc] were in fact based, to a substantial extent, on discussions with Lepage.

I think that it is important for you to know that his initial position concerning IUT was substantially **skeptical/negative**, and that this negative position continued roughly until the spring of 2021, when his negativity started to waver, and finally reached a decisive turning point in the fall of 2021, when he stated explicitly that he now realized that he had completely misunderstood IUT, and that he no longer had any mathematical reason not to acknowledge the mathematical validity of IUT. Nevertheless, despite his initially quite skeptical/negative position, I think that it is also important for you to know that our discussions (between the summer of 2017 and the fall of 2021) were carried out in a completely **orderly**, **rational**, and **friendly* atmosphere.

In particular, I continue to be entirely unable to understand why some mathematicians seem to insist on creating such a venomous and adversarial atmosphere, e.g., in the English-language mass media and internet, regarding me and my work --- an atmosphere that has the effect of significantly hindering such orderly and rational dialogue concerning the mathematics. Put another way, I see no reason why mathematical dialogue concerning IUT cannot be conducted --- as was the case during my discussions with Emmanuel Lepage! --- in an atmosphere in which the two sides genuinely feel that they are fighting (not against one another (!), but rather) **together** for the common cause of elucidating the mathematical content under consideration in a maximally efficient fashion.

In this context, it should be noted that statements to the effect that there are **misunderstandings** should be understood as statements concerning inadequacies in the communication that was conducted between the two sides. That is to say, statements to the effect that there are **misunderstandings** should not be taken as "insults" to the other side, but rather as signals that something important does not seem (for whatever reasons) to be getting across to the other side.

Next, let me turn to a discussion of the mathematical content of Lepage's misunderstanding of IUT. First, I should state clearly that unlike you (and Scholze), he never made RC identification-type assertions to the effect that one should identify isomorphic objects, i.e., as in (KP1) and (KP2). In this sense, his initial negative position concerning IUT was not precisely the same as that of you (or Scholze). On the other hand, the fundamental confusion underlying his misunderstanding was, as far as I can see, **precisely the same** as that of you (and Scholze), i.e., the AND/OR confusion discussed above (and in more detail in [EssLgc]). In this sense, my discussions with Lepage (between the summer of 2017 and the fall of 2021) may be understood as a sort of "constructive continuation" of my (and Hoshi's) March 2018 discussions with you (and Scholze).

Perhaps you will recall that in our final e-mail exchange in August 2018, I proposed continuing our discussions via **representatives** of the two sides and suggested that you propose a representative of your side, i.e., who understands the essential mathematical content of your assertions. To my knowledge, you never responded to this proposal, but in some sense, it may be said that Lepage undertook this quite formidable task of "representing the essential mathematical content of your assertions" on his own initiative!

Subsequent to the resolution of Lepage's misunderstandings in the fall of 2021, I asked

him, repeatedly on several occasions, for his advice concerning how to expedite the process of resolving such misunderstandings, e.g., how he would explain IUT to himself if he could go back in time and communicate with the version of himself that existed a few years ago. His response (on multiple occasions) was to emphasize the importance of the content of [EssLgc], ¥S 3.10, and, in particular, the important point (cf., e.g., the discussion of [EssLgc], Example 3.10.1) of avoiding the

"trap" of thinking in terms of **diagram commutativity**

(i.e., which essentially amounts to the "OR" approach discussed above!), since this is simply not the way in which the argument proceeds in IUT.

In this context, I should state that I do not see any problem with your discussing Lepage's experience with Lepage himself, but I would like to request that you refrain from discussing Lepage's experience in any sort of public situation until you obtain his explicit permission to do so.

Finally, I would like to urge you once again to take the time to reflect on the utter **gravity** of the responsibility that you have undertaken --- i.e., with regard to influencing the course of development of the field of mathematics, as well as the careers and professional activities of numerous mathematicians involved with IUT --- by **publicly making assertions** (concretely in the form of (KP1), (KP2)) concerning the mathematical validity of IUT **without** providing "full details in a timely manner" (in accordance with the EMSCOP) or indeed any other readily accessible infrastructural apparatus for digesting the mathematical content of these assertions.

Sincerely,

Shinichi Mochizuki