Estimates for oscillatory integrals with damping factors

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This talk concerns estimates for the Fourier transform of surface-carried measures, which is a fundamental subject in harmonic analysis. Such estimates have a wide range of applications, including to dispersive equations.

Let σ be the surface measure on a smooth hypersurface $\mathcal{H}$ in $\mathbb{R}^{d+1}$. When $\mathcal{H}$ is non-degenerate, the stationary phase method provides the optimal estimate for $\widehat{\sigma}$. However, when the surface is degenerate, satisfactory results are only known for some special cases, such as convex finite-type surfaces.

In this talk, we consider the oscillatory integral estimate

$$|(\kappa^{1/2}\sigma)^\wedge(\xi)| \le C|\xi|^{-d/2}$$

with the damping factor $\kappa^{1/2}$, where κ is the Gaussian curvature on $\mathcal{H}$. The role of $\kappa^{1/2}$ is to mitigate the poor behavior of the oscillatory integral near the degeneracy, thereby recovering the optimal decay rate. In view of the stationary phase expansion, $\kappa^{1/2}$ arises as the natural and optimal damping factor. Unfortunately, as shown by Cowling–Disney–Mauceri–Müller, such decay estimates generally fail for $d \ge 5$, even for convex finite-type surfaces.

Under the additional assumption that the surface $\mathcal{H}$ is analytic, we obtain an essentially complete result. That is to say, we prove the estimate for $d = 2, 3$, and with an additional logarithmic factor for $d = 4$. Our approach is inspired by the recent stationary set method introduced by Basu–Guo–Zhang–Zorin-Kranich. We also discuss applications of these estimates to convolution, maximal, and adjoint restriction operators associated with $\mathcal{H}$, incorporating mitigating factors of optimal order.