**助教 Stefan Helmke (Algebraic Geometry)**

In recent years my research concentrated on an improvement of the uniformization theory for algebraic functions over a field of characteristic zero. My previous research on the Fujita Conjecture [1–4] eventually failed to prove this conjecture, but led to a new approach [5] which I briefly outlined in my last report to which I refer here. Unfortunately this new approach depends on a strong version of uniformizations which was not yet known and which cannot be directly derived from existing results about uniformizations or resolutions of singularities. As with resolutions, the difficulty is to find good invariants and ultimately a statement which can be proved by induction over the dimension. This was indeed so frustrating that I had to give up this problem for a while and do something else, of which I will report at some other time. But when I restarted this research, after making the same mistakes as before as usual, I suddenly found apparently the exactly right invariants to make the induction work and this will finally become my long prospected paper [6].

Here, now, I will briefly explain the result. One begins with a regular local ring $R$ and a valuation $v$ which we will assume for simplicity to be of rank 1, so that its valuation group can be embedded into the real numbers, and which has a non-trivial center on $R$. Then, for any given real number $\epsilon > 0$ there exists a regular local ring $S \supset R$ which is obtained from $R$ by a sequence of blow-ups in admissible centers and localizations in the corresponding centers of $v$ with the following properties. There is a regular system of parameters $(x_1, \ldots, x_d)$ of $S$, such that the exceptional locus is given by $x_1 \cdots x_r = 0$ and $v(x_1), \ldots, v(x_r)$ form a basis of the valuation group over the rational numbers. Then, we put $\deg x_i = v(x_i)$ for $i = 1, \ldots, r$ and $\deg x_i = 0$ for $i > r$ and extend this degree to a valuation of $S$. The smallest degree term in a power series expansion of an element $f \in S$ can then be considered as a power series in the variables $x_{r+1}, \ldots, x_d$ and we denote its order by $\operatorname{ord} f$. The main condition is now that for any $f \in R$ one has

$$\operatorname{ord} f \;\leq\; \epsilon \cdot \deg f.$$

So, for example, if for a given $f \in R$ one chooses $\epsilon < 1/v(f)$, then, since $\deg f \leq v(f)$, the right-hand side of the inequality is less than 1 and hence $f$ will be uniformized by a simple toric modification of $S$ in the usual sense. Thus ordinary uniformizations are a special case, but it is this generalization which makes the methods I had previously developed in [5], though not yet published, really powerful and reversely, some of the more basic techniques in there are now crucial in the proof of this result.


[1] S. Helmke, *On Fujita's conjecture,* Duke Math. J. 88 (1997), 201–216.

[2] S. Helmke, *On global generation of adjoint linear systems,* Math. Ann. 313 (1999), 635–652.

[3] S. Helmke, *The base point free theorem and the Fujita conjecture,* Vanishing theorems and effective results in algebraic geometry, ICTP Lecture Notes 6, Trieste, 2001, 215–248.

[4] S. Helmke, *Multiplier ideals and basepoint freeness,* Oberwolfach reports 1, 2004, 1137–1139.

[5] S. Helmke, *New Combinatorial Methods in Algebraic Geometry,* in preparation.

[6] S. Helmke, *On local uniformizations,* in preparation.