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In my previous report I mentioned a new invariant in connection with the theory of local uniformizations and expressed the hope that this invariant would help to prove a crucial improvement of this theory, which I also described in that report. This invariant did what it was supposed to do, but ultimately it failed to prove the desired result; instead it showed that the well-known algorithms for uniformizations or resolutions of singularities are inadequate for this purpose. I then had to resort to the computation of some examples in order to decide if there is a possibility to improve the algorithms or if my conjecture might perhaps be wrong. This took some time, but I hazard now to say that probably the former is the case and since much of my older work, including a possible proof of the Fujita Conjecture, heavily depends on this somewhat technical point, it is of considerable importance to me [1–4].

Ever since Oscar Zarisky proved the existence of local uniformizations in arbitrary dimensions over a field of characteristic zero in 1940, the algorithms always relied on an induction on the dimension. Here, one has to find an appropriate hyperplane and an ideal or function on this hyperplane, which has to be uniformized or resolved first (which is conveniently done by the induction hypothesis) before one can proceed to uniformize or resolve the original function or ideal. The hyperplane became eventually a 'maximal contact hyperplane' and the ideal on that hyperplane the 'coefficient ideal'. This is all very canonical and convenient, but as my examples indicate, the coefficient ideal is too large for my purpose and indeed, it is in general not necessary to uniformize all the coefficients. Instead, I have now an algorithm, which does not rely on induction, but rather carefully blows-up only what seems to be absolutely necessary; on the other hand I have developed a refined method to establish the required multiplicity bound and while there is certainly still some more research necessary, I believe those two ingredients will eventually prove my conjecture.

But the theory turns out to be much more interwoven than originally expected and my uniformization theory [6] depends on parts of the theory of filtrations I developed in [5], while on the other hand the latter parts of the filtration theory depend on the uniformization theory, so that the division in those two parts is no longer adequate. I will instead divide this research now in four parts: Filtrations, Uniformizations, Deformations and Applications, or put them all together.

[1] S. Helmke, *On Fujita's conjecture,* Duke Math. J. 88 (1997), 201–216.

[2] S. Helmke, *On global generation of adjoint linear systems,* Math. Ann. 313 (1999), 635–652.

[3] S. Helmke, *The base point free theorem and the Fujita conjecture,* Vanishing theorems and effective results in algebraic geometry, ICTP Lecture Notes 6, Trieste, 2001, 215–248.

[4] S. Helmke, *Multiplier ideals and basepoint freeness,* Oberwolfach reports 1, 2004, 1137–1139.

[5] S. Helmke, *New Combinatorial Methods in Algebraic Geometry,* in preparation.

[6] S. Helmke, *On local uniformizations,* in preparation.