特定助教 Wojciech Porowski (Anabelian geometry)

My research is based in anabelian geometry over global and local fields. As a brief summary of my previous work: in [1] it is shown how one can reconstruct group theoretically the notion of the local height of a rational point on an elliptic curve over a local field from the group theoretic data consisting of the étale fundamental group of a once-punctured elliptic curve and a given section, whereas [2] presents a group theoretic criterion for determining the reduction type of an elliptic curve over a p-adic local field from the data of the geometrically maximal pro-p quotient of the fundamental group of a once-punctured elliptic curve.

My current projects concern the notion of a Galois section, namely a splitting of the fundamental exact sequence associated to a geometrically connected variety X over a number field K. Every K-rational point of X gives rise to such a splitting and there is a well-known conjecture (Grothendieck Section Conjecture) which describes precisely the remaining sections in the case of a hyperbolic curve X as a set of so-called cuspidal sections. In this generality, this conjecture is widely open. Therefore one may restrict the problem to some subsets of all sections with more geometric properties, for example we may consider locally geometric sections. These sections are assumed to come locally from rational points over the completion K_v of K at every nonarchimedean place v.

Then, the two main problems that I currently consider are: (a) suppose that two local sections s and t are locally conjugate at all places v, does it imply that they are globally conjugate?, (b) suppose instead that s and t are locally conjugate for all places v from some 'large' subset S of all places, can we deduce that they are locally conjugate for all places?. Assuming the Section Conjecture, both these questions should have affirmative answers for hyperbolic curves.

I have managed to obtain some positive results for both these question. Regarding (a), one can obtain affirmative answer when X is a hyperbolic curve and one section is assumed to be locally geometric on a set of positive density. The main input of the proof is a modification of a certain argument of Stoll and the vanishing of centralizers of sections for hyperbolic curves. Currently I am trying to strengthen this method so that we could remove the assumption on the local geometricity.

Regarding question (b), one can obtain a positive result at the cost of a stronger assumption. We assume that both sections are adelic (local points together determine an adelic point of X) and that X admits a finite étale cover Y which is also a finite étale cover of a hyperbolic curve of genus zero. In this setup, we can prove that for subsets S of all valuations of K which are strongly of positive density (i.e., their lifts to all finite field extensions are of positive density) the answer to (b) is positive (which also implies global conjugacy by the affirmative case of (a)). Here the main technique of the proof is a construction of a a suitable Belyi map and certain strong multiplicity one property from the theory of ℓ-adic representations.

[1] W. Porowski, Anabelian reconstruction of the Néron-Tate local height function, *European Journal of Mathematics* **8**, 1172–1195 (2022)

[2] W. Porowski, Pro-p criterion of good reduction of punctured elliptic curves, to be published in *Publications of the Research Institute for Mathematical Sciences* of Kyoto University.