特定助教 Ana Kontrec (Representation theory)

My main research focus is in the field of vertex algebra theory, and especially $\mathcal{W}$-algebras. In the study of conformal field theory and string theory, vertex operator algebras (VOAs) play a fundamental role as the mathematical foundation for the concept of chiral algebras. Recent developments show connection between four dimensional $N = 2$ superconformal theories and VOAs as their algebraic invariants. One of the most important families of vertex algebras are affine vertex algebras and their associated $\mathcal{W}$-algebras, which are connected to various aspects of geometry and physics, such as quantum Langlands correspondence and symmetries of quantum field theories.

Let $\mathfrak{g}$ be a simple Lie algebra, f a nilpotent element and k a complex number. The universal affine $\mathcal{W}$-algebra $\mathcal{W}^k(\mathfrak{g}, f)$ is associated to the affine vertex algebra $V^k(\mathfrak{g})$ through a certain homological construction known as quantum Hamiltonian reduction (introduced by Feigin and Frenkel for principal nilpotent element and generalized by Kac, Roan and Wakimoto for arbitrary nilpotent element). Structure and representation theory of a given $\mathcal{W}$-algebra depends largely on the type of nilpotent element used in the construction, e.g. explicit formulae for generators and relations are not known generally, but only in the case of the minimal nilpotent element f.

One of the simplest examples of $\mathcal{W}$-algebras is the Bershadsky-Polyakov vertex algebra $\mathcal{W}_k = \mathcal{W}_k(sl_3, f_\theta)$, associated to $\mathfrak{g} = sl(3)$ and the minimal nilpotent element f_θ. T. Arakawa proved that $\mathcal{W}_k(sl_3, f_\theta)$ is rational for half-integer levels k, while in other cases it is a non-rational vertex algebra. More recently, for k admissible and non-integral, irreducible $\mathcal{W}_k$–modules were classified by D. Adamović and me in [1] for the boundary cases, and by Fehily, Kawasetsu and Ridout in full generality. They showed that every irreducible highest weight module for $\mathcal{W}_k$ is obtained as an image of the admissible modules for $L_k(sl(3))$ (which are classified by T. Arakawa). However, when we pass to integral k, these methods are no longer applicable, since in this case quantum hamiltonian reduction sends $L_k(sl(3))$–modules to zero.

In [2], Adamović and I were able to obtain a classification of irreducible positive energy modules $L(x, y)$ for $\mathcal{W}_k$ at integer levels, parametrizing the highest weights (x, y) as zeroes of certain polynomial functions (derived from the associated Zhu algebra). This is one of the first classification results of this type in literature for non-admissible, integral level affine $\mathcal{W}$-algebras.

Another topic which has received a great deal of attention lately is the notion of duality of $\mathcal{W}$-algebras. Let $\mathfrak{g}$ be a simple Lie algebra, and ${}^L\mathfrak{g}$ the Langlands dual Lie algebra of $\mathfrak{g}$. If f is a principal nilpotent element, the Feigin-Frenkel duality states that $\mathcal{W}^k(\mathfrak{g}, f) \simeq \mathcal{W}^l({}^L\mathfrak{g}, f)$, where $k \in \mathbb{C}$ is non-critical and l is the dual level. However, this type of symmetry does not hold in general for non-principal nilpotent elements, which leads us to the notion of Kazama-Suzuki type duality. Here the Langlands dual Lie algebra is replaced by a Lie superalgebra. The notion of Kazama-Suzuki dual was first introduced in the context of the duality of the $N = 2$ superconformal algebra and affine Lie algebra $\widehat{\mathfrak{sl}}(2)$, and later generalized for some other affine vertex algebras and $\mathcal{W}$–algebras.

Assume that U, V are vertex superalgebras. We say that V is the Kazama-Suzuki dual of U if there exist injective homomorphisms of vertex superalgebras $\varphi_1 : V \to U \otimes F$, $\varphi_2 : U \to V \otimes F_{-1}$, so that

$$V \cong \mathrm{Com}\left(\mathcal{H}^1, U \otimes F\right), \quad U \cong \mathrm{Com}\left(\mathcal{H}^2, V \otimes F_{-1}\right),$$

where $\mathcal{H}^1$ (resp. $\mathcal{H}^2$) is a rank one Heisenberg vertex subalgebra of $U \otimes F$ (resp. $V \otimes F_{-1}$), F is Clifford vertex superalgebra and F_{-1} is a lattice vertex superalgebra associated to the negative definite lattice $\mathbb{Z}\sqrt{-1}$. In [2], D. Adamović and I showed that the affine vertex superalgebra $L_{k'}(osp(1|2))$ is the Kazama-Suzuki dual of the Bershadsky-Polyakov algebra $\mathcal{W}_k(sl_3, f_\theta)$ for $k = 1$. Relaxed modules for $L_{k'}(osp(1|2))$ are mapped to the ordinary $\mathcal{W}_k$–modules, for which one expects it is easier to obtain the tensor category structure and fusion rules. As a consequence of this duality, we were able to obtain a free-field realization of $\mathcal{W}_k$ and their highest weight modules.

参考文献

[1] D. Adamović, A. Kontrec, *Classification of irreducible modules for Bershadsky-Polyakov algebra at certain levels*, J. Algebra Appl., Vol. 20 (2021), no. 6, 2150102

[2] D. Adamović, A. Kontrec, *Bershadsky-Polyakov vertex algebras at positive integer levels and duality*, Transformation Groups (2022), https://doi.org/10.1007/s00031-022-09721-z