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My research is based in probability theory, with a particular focus on random walks in random environments. Work in this area, which has been a major focus of probability over the last four decades, is motivated by the goal of understanding the interplay between the geometry of a space and the stochastic processes that live upon it.

In this report, I will discuss a recent project concerning the random walk on a certain critical percolation cluster. To introduce this, first consider bond percolation on the integer lattice $\mathbb{Z}^d$. In particular, independently for each edge of the lattice, retain the edge with probability p, and discard it otherwise. This procedure gives a random subgraph of $\mathbb{Z}^d$, and it is well known that there is a phase transition in the behaviour of this that takes place at a certain 'critical probability' $p_c(d)$. (A special case is $p_c(2) = 1/2$.) For $p > p_c(d)$, the percolation process admits a unique infinite cluster that is essentially space filling; this is the supercritical regime. And, for $p < p_c(d)$, the process admits only finite clusters; this is the subcritical regime. As for the critical value $p = p_c(d)$, it is widely expected that the large components of the graph have a non-trivial fractal structure, with volume growth exponent strictly lower than the Hausdorff dimension of the ambient space. My interest is in how random walks on these critical graphs behave. For context, in the supercritical regime, it is known that the random walks can be rescaled to give a Brownian motion on $\mathbb{R}^d$, just as would be the case on the lattice.

Now, in high dimensions ($d > 6$), there is a more precise conjecture that links critical percolation clusters to a certain random fractal tree called the continuum random tree. Whilst confirming this conjecture remains open, together with Eleanor Archer (Paris Nanterre), in [1], I showed that this prediction holds on another high-dimensional model, namely when the underlying graph $\mathbb{Z}^d$ is replaced by the random hyperbolic triangulation of the upper half plane. It is beyond the scope of this report to describe this model in detail, but nonetheless, this result supports the conjecture for the high-dimensional Euclidean case. Moreover, we were able to check the associated random walks converge to a natural diffusion on the limiting fractal. For this, we applied general theory that relates the convergence of processes to that of associated resistance metric measure spaces [2]. In this approach, one transfers questions about a random walk to those about an associated electrical network, which are often easier to study. Such a viewpoint has proved useful for other examples of random walks in random environments, and it is my goal to explore further applications of it. Moreover, in my research, I seek to develop further general tools that provide insights concerning other models arising in physics and elsewhere.

References

[1] E. Archer and D. A. Croydon, *Scaling limit of critical percolation clusters on hyperbolic random half-planar triangulations and the associated random walks*, preprint appears at arXiv:2208.12102 (2022).

[2] D. A. Croydon, *Scaling limits of stochastic processes associated with resistance forms*, Ann. Inst. Henri Poincaré Probab. Stat. **54** (2018), no. 4, 1939-1968.