## 特定助教 Wojciech Porowski (Anabelian geometry)

My research is based in anabelian geometry over global and local fields. As a brief summary of my previous work: in [1] it is shown how one can reconstruct group theoretically the notion of the local height of a rational point on an elliptic curve over a local field from the group theoretic data consisting of the étale fundamental group of a once-punctured elliptic curve and a given section, whereas [2] presents a group theoretic criterion for determining the reduction type of an elliptic curve over a $p$-adic local field from the data of the geometrically maximal pro-$p$ quotient of the fundamental group of a once-punctured elliptic curve.

My current projects concern the notion of a Galois section, namely a splitting of the fundamental exact sequence associated to a geometrically connected variety $X$ over a number field $K$. Every $K$-rational point of $X$ gives rise to such a splitting and there is a well-known conjecture (Grothendieck Section Conjecture) which describes precisely the remaining sections in the case of a hyperbolic curve $X$ as a set of so-called cuspidal sections. In this generality, this conjecture is widely open.

Since last year I have been considering the following problem: suppose that two local sections $s$ and $t$ are locally conjugate at all places $v$ from some 'large' subset $\Omega$ of all places, does it imply that they are globally conjugate? This question may be thought of as a sort of local-global principle for Galois sections. I managed to prove that the answer is indeed positive when the subset $\Omega$ has density one; the proof of this result appears in the preprint [3]. In fact, the situation considered in [3] is somewhat more general: we replace the section $t$ by a finite collection of sections $t_1, \ldots t_n$ for some $n \geq 1$ and assume that for every $v \in \Omega$ there exists $1 \leq i \leq n$ such that $s$ and $t_i$ are locally conjugate at $v$. This type of assumption is referred to as a finite covering of a section $s$. Then the main result of [3] is that for some $1 \leq i \leq n$ sections $s$ and $t_i$ are globally conjugate. The main input of the proof is a modification of a certain argument of Stoll and the vanishing of centralizers of sections for hyperbolic curves.

Another problem that I considered is whether a global Galois section $s$ that is locally cuspidal for all valuations $v$ from a 'large' subset of valuations $\Omega$ must be globally cuspidal. Here I have been able to prove that if $\Omega$ is assumed to be of 'strongly positive density' (i.e., it has positive density after restricting to any finite field extension of $K$) then indeed the section $s$ must be a cuspidal section. A preprint containing this result is currently under preparation. Also, as a work in progress, I am trying to strengthen the main result of [3] by considering sets $\Omega$ of strongly positive density.


[1] W. Porowski, Anabelian reconstruction of the Néron-Tate local height function, European Journal of Mathematics 8, 1172–1195, (2022)

[2] W. Porowski, Pro-$p$ criterion of good reduction of punctured elliptic curves, Publications of the Research Institute for Mathematical Sciences, 59, No. 4, 843–882, (2023).

[3] W. Porowski, Locally conjugate Galois sections, preprint, arXiv 2312.08005.