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My research centers around homotopy theory and the way it interacts with arithmetic and algebraic geometry. I study this interaction from several perspectives, and I highlight a few in this report.

A powerful tool in this subject is the theory of deformation infinity-categories such as synthetic spectra, which informally interpolate between stable homotopy theory and number theory [1]. In a current joint project with William Balderrama, we are working on an unstable analogue of this construction, which one could call *synthetic spaces* [2]. These are expected to have a variety of applications, in particular:

1. enable "cofibre of τ" ([3]) calculational techniques in the study of the unstable Adams spectral sequence,

2. give a purely topological description of the unstable cellular motivic category, building on the upcoming work of Asok-Bachmann-Hopkins on the Wilson space conjecture.

As we show in our upcoming work, such unstable deformations can be constructed in a variety of contexts. Keeping track of the resulting functoriality is a particularly subtle problem, and involves a non-trivial amount of $(\infty, 2)$-category theory.

A different instance of the relationship between homotopy theory and algebraic geometry is given by the weight filtration, which is a canonical filtration on the cohomology of algebraic variety arising from resolution of singularities and algebraic compactification. In previous joint work with Peter Haine we were able to show that over the complex numbers, such a weight filtration exists in the spectral context; that is, as a canonical filtration on the stable homotopy type itself [4].

In a joint paper with Toni Annala from this year, we were able to extend this construction into mixed characteristic, constructing in particular a canonical weight filtration on logarithmic prismatic and syntomic cohomology [5].

In chromatic homotopy theory proper, one problem I've been particularly interested is the Hahn-Wilson conjecture on spectra of fp-type, which describes them as generated by a particularly simple example known as the truncated Brown-Peterson spectrum. This is related due to the proof of the higher analogues of Quillen-Lichtenbaum due to Hahn and Wilson, which gave interesting

new examples of spectra of fp-type. In joint work with David Lee, we are able to prove a chromatically localized variant of the Hahn-Wilson conjecture, and deduce the original conjecture at height one [6]. Along the way, we are also able to show the existence of $K(n)$-local finite complexes with particularly regular rings of homotopy groups.

[1] P. Pstragowski, *Synthetic spectra and the cellular motivic category*, Invent. Math. **232** (2023), no. 2, pp. 553–681.

[2] W. Balderrama, P. Pstragowski, *Unstable synthetic deformations*, To appear

[3] R. Burklund, D. C. Isaksen, and Z. Xu, *Classical stable homotopy groups of spheres via F_2-synthetic methods*, Peking Math. J. (2025), pp. 1–23.

[4] P. J. Haine and P. Pstragowski, *Spectral weight filtrations*, arXiv preprint arXiv:2309.15072 (2023).

[5] T. Annala and P. Pstragowski, *A note on weight filtrations at the characteristic*, arXiv preprint arXiv:2502.19626 (2025).

[6] D. J. Lee and P. Pstragowski, *The monochromatic Hahn–Wilson conjecture*, arXiv preprint arXiv:2410.08029 (2024).