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I work in arithmetic geometry. Recently, my research focuses on curves and their moduli spaces in positive characteristic from the point of view of fundamental groups, which are motived by the theory of anabelian geometry of curves over algebraically closed fields of characteristic $p > 0$.

Since the late 1990s, some developments of Florian Pop, Michel Raynaud, Mohamed Saïdi, and Akio Tamagawa showed evidence for very strong anabelian phenomena for curves over algebraically closed fields of characteristic p. In this situation, the Galois group of the base field is trivial, and the arithmetic fundamental group coincides with the geometric fundamental group, thus there is a total absence of a Galois action of the base field. This kinds of anabelian phenomenon go beyond Grothendieck's anabelian geometry, and this is the reason that we do not have an explicit description of the geometric fundamental group of any pointed stable curve in positive characteristic. Moreover, we may think that the anabelian geometry of curves over algebraically closed fields of characteristic p is a theory based on the following rough consideration: The geometric fundamental group of a pointed stable curve over an algebraically closed field of characteristic p must encode "moduli" of the curve.

Recently, I introduced a topological space which is called the moduli space of admissible fundamental groups of curves in positive characteristic, and posed the so-called "Homeomorphism Conjecture". This conjecture says that the moduli spaces of curves in positive characteristic can be reconstructed group-theoretically from the geometric fundamental groups of curves as topological spaces. Moreover, the Homeomorphism Conjecture gives us a new insight into the theory of the anabelian geometry of curves over algebraically closed fields of characteristic p based on the following philosophy:

> The anabelian properties of pointed stable curves over algebraically closed fields of characteristic p are equivalent to the topological properties of the moduli spaces of admissible fundamental groups.

In [1], [4], [7], I study the anabelian geometry of (possibly singular) pointed stable curves over algebraically closed fields of characteristic p. In particular, I proved the combinatorial Grothendieck conjecture for curves in positive characteristic, and formulated the weak Isom-version of the Grothendieck conjecture for arbitrary pointed stable curves over algebraically closed fields of characteristic p.

In [3], [5], [6], I study the Hasse-Witt invariants and the generalized Hasse-Witt invariants associated to a pointed stable curve in positive characteristic by using the theory of Raynaud-Tamagawa theta divisors. Those invariants play important roles to understand the structure of the geometric fundamental group of the curve.

In [2], [8], [9], [10], basing on the theory developed in the previous papers (in particular, [3], [4], [5], [7]), I introduced the moduli spaces of admissible fundamental groups, and formulated the Homeomorphism Conjecture. Moreover, I proved that the Homeomorphism Conjecture holds when the dimension of the moduli space of curves is 1. Furthermore, to solve the Homeomorphism Conjecture for higher dimensional moduli spaces, we formulated some new conjectures in [8] and [10]. In [10], we prove the group-theoretical specialization conjecture (one of the conjectures formulated in [10]) for curves of type $(0, n)$.

By the theory of the moduli spaces of admissible fundamental groups, fundamental groups can be considered as an analogue of local moduli of pointed stable curves. This insight gives me a new approach to seeing anabelian geometry. For example, in [11], with Y. Hoshi, we obtain a new proof of Mochizuki's famous theorem concerning the (Isom-version) Grothendieck's anabelian conjecture for curves over sub-p-adic fields without using Faltings' p-adic Hodge theory.

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