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Visualization of the Landau-Nakanishi surfaces for truss-bridge graphs

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Naofumi Honda* and Takahiro Kawai**

In this report we present several figures which describe the Landau-Nakanishi surfaces associated with the truss-bridge graph T_ℓ $(1 \leq \ell \leq 3)$ and its contractions such as the so-called ice-cream cone diagram, assuming the space-time dimension is 2. Our principal aim is to help the understanding of the reader of the paper

[HKS] N. Honda, T. Kawai and H. P. Stapp, On the geometric aspect of Sato's postulates on the S-matrix, to appear in RIMS *Kôkyûroku* Bessatsu (Preprint version: RIMS preprint No. 1810 available at http://www.kurims.kyoto-u.ac.jp/preprint/index.html),

but we believe they are interesting in their own right.

We note that all figures in this report are described with the help of a computer by the method described in

[HK] N. Honda and T. Kawai, A computer-assisted study of the Landau-Nakanishi geometry, RIMS *Kôkyûroku* No. 1861, 2013, pp. 100-110.

All the notations used in this report are the same as those used in [HKS].

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1. $L^{\times}(T_1)$ and $L^{\oplus}(T_1)$

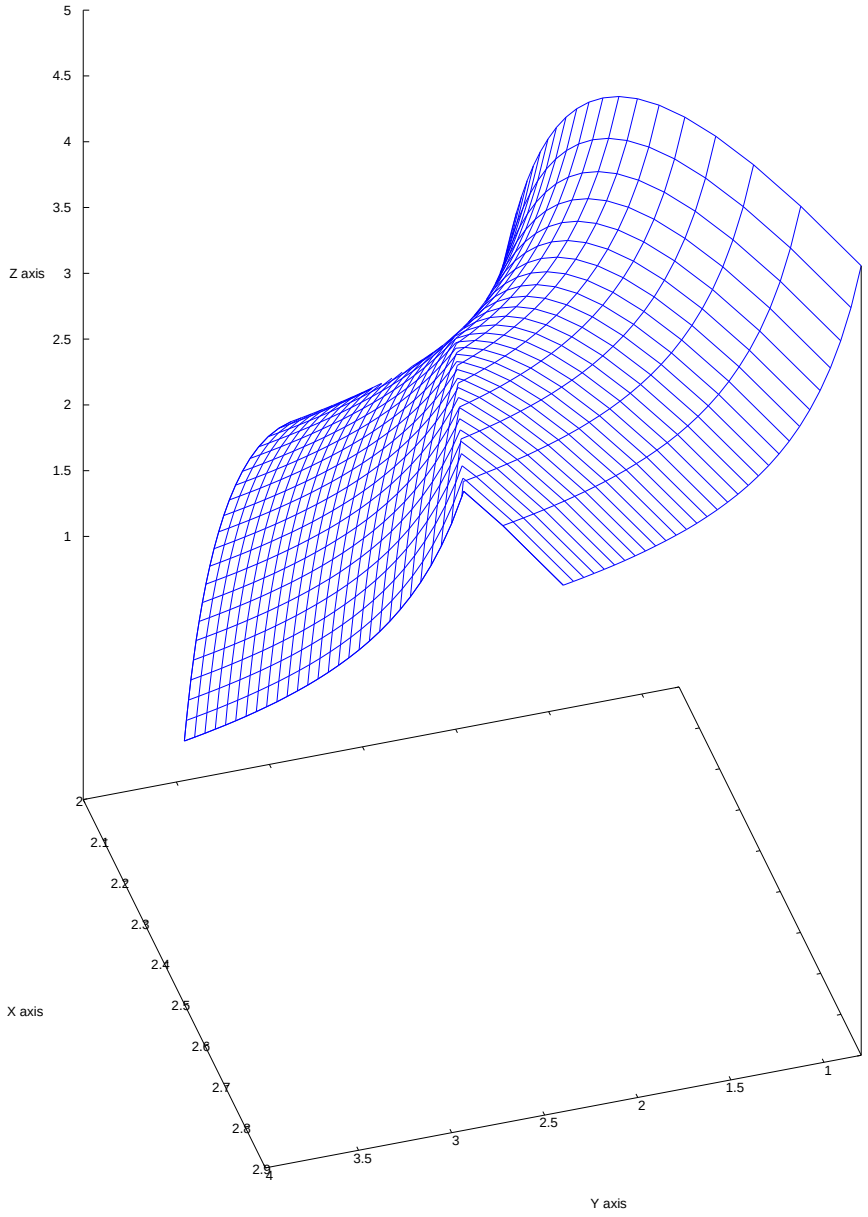


Figure 1.1. T_1, view$(116, 287)$

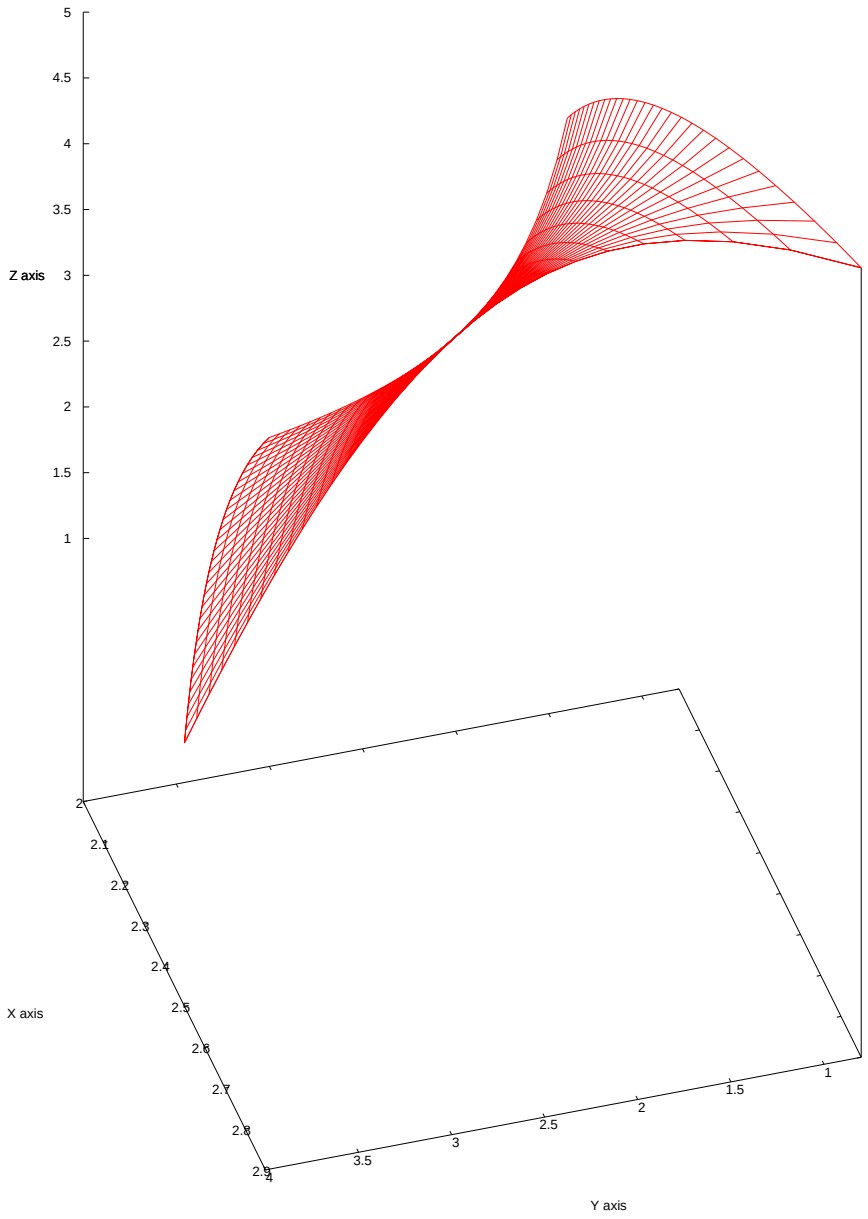


Figure 1.2. The positive part of T_1, view$(116, 287)$

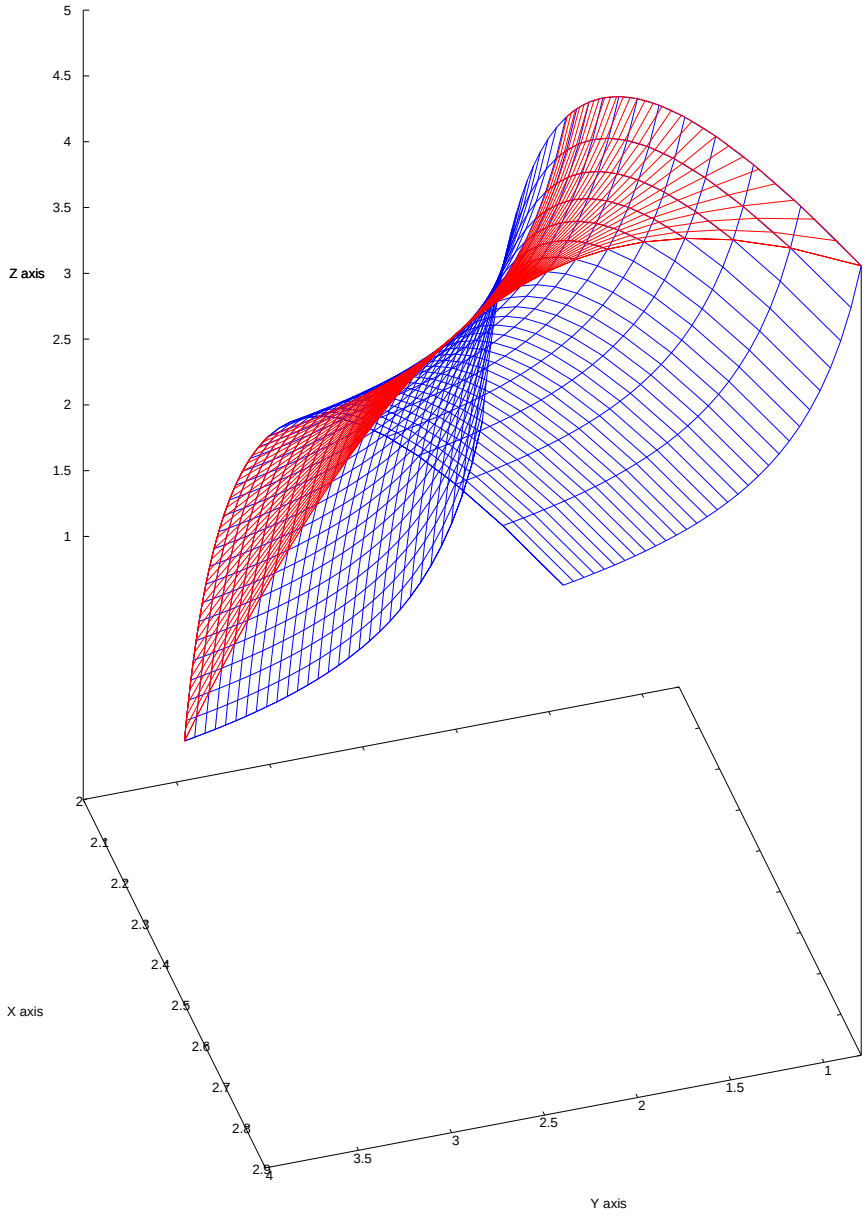


Figure 1.3. T_1, total view at $(116, 287)$

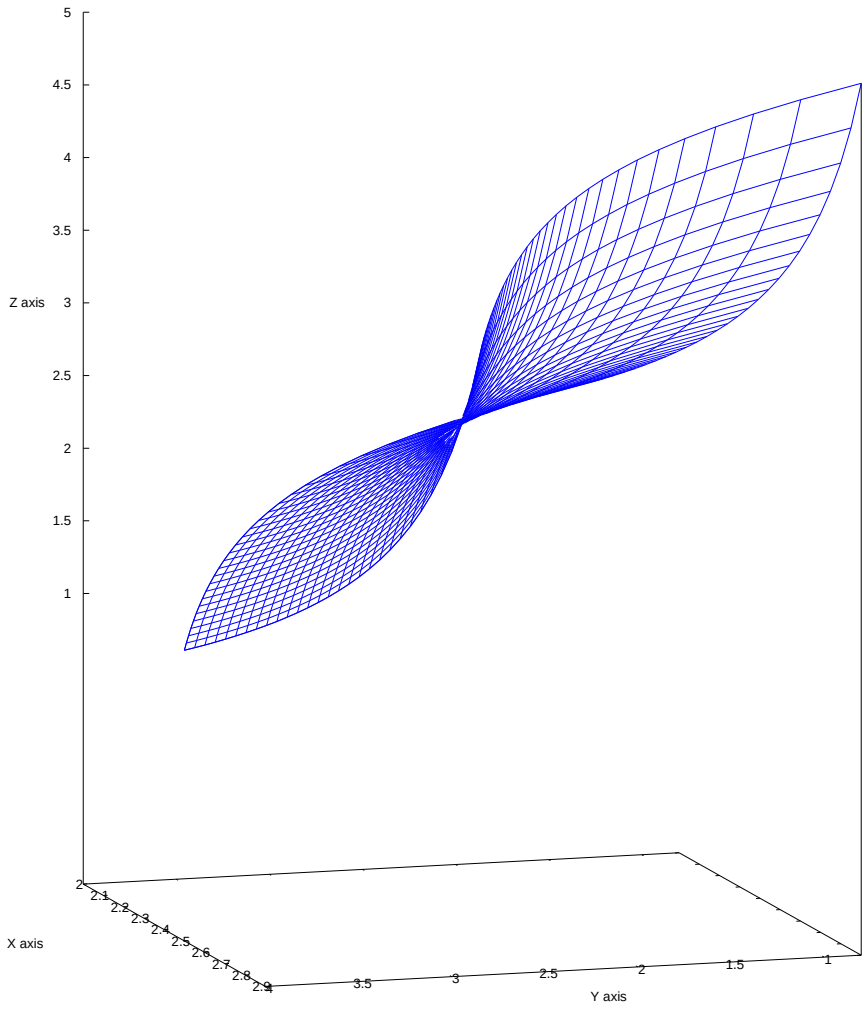


Figure 1.4. T_1, view$(97, 287)$

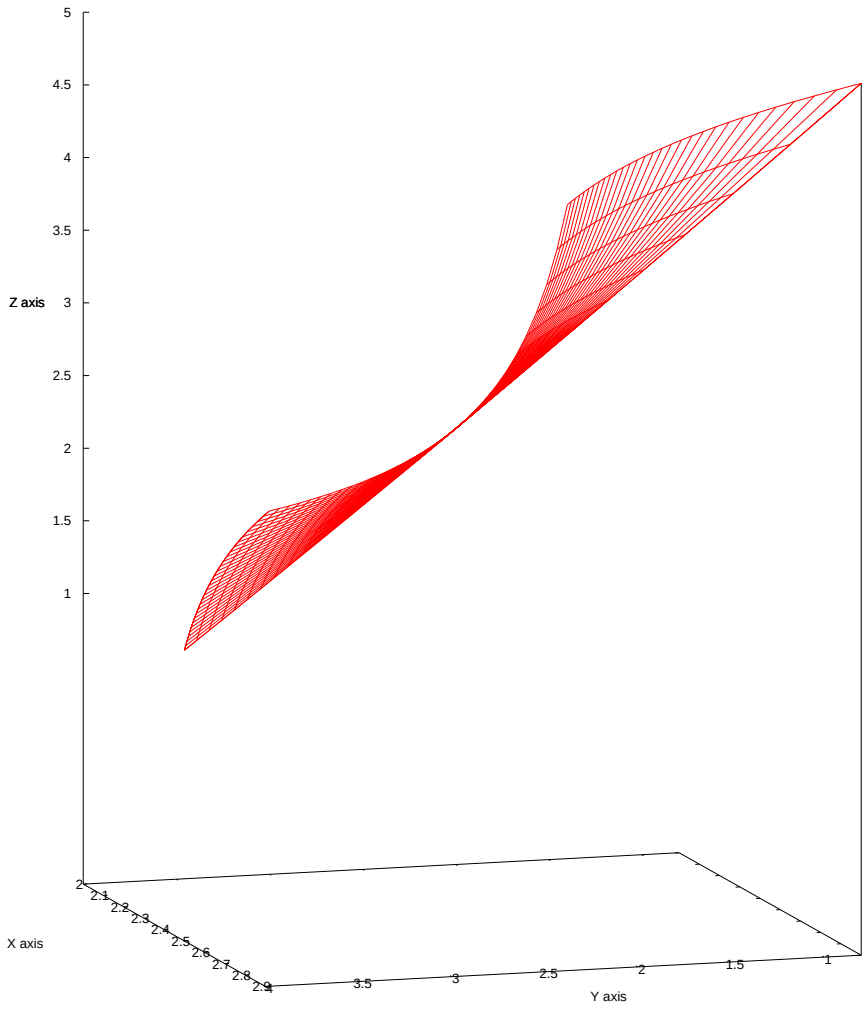


Figure 1.5. The positive part of T_1, view$(97, 287)$

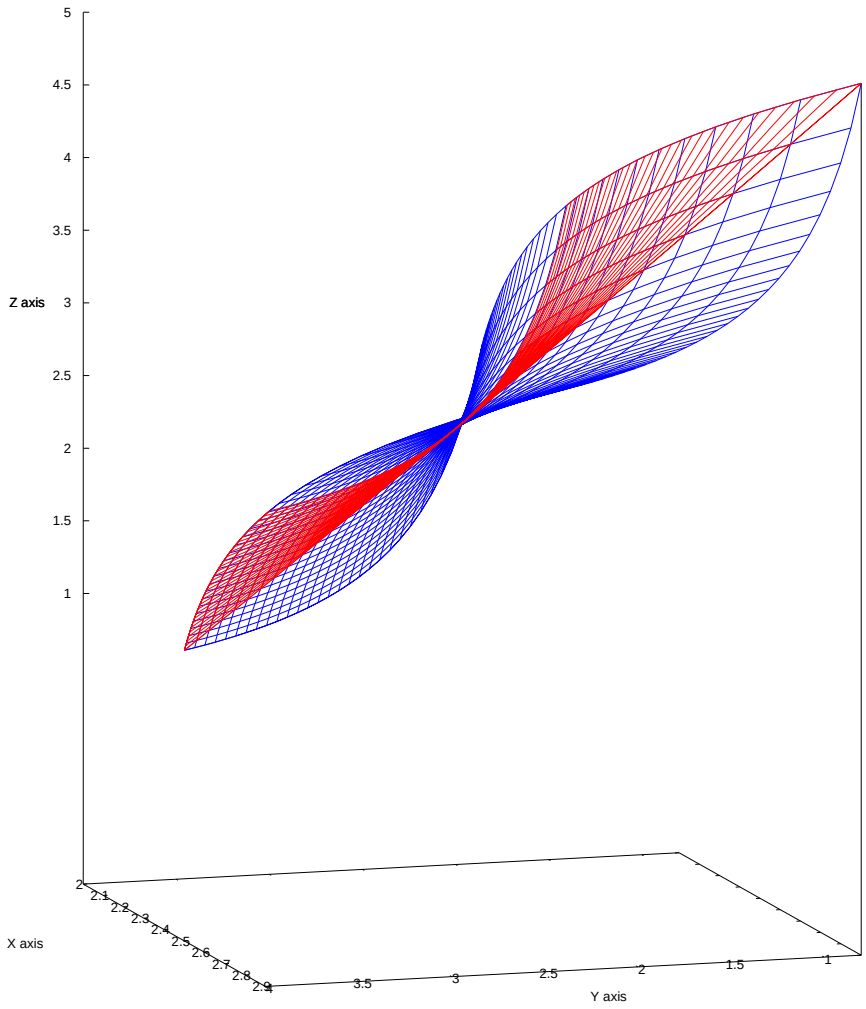


Figure 1.6. T_1, total view at $(97, 287)$

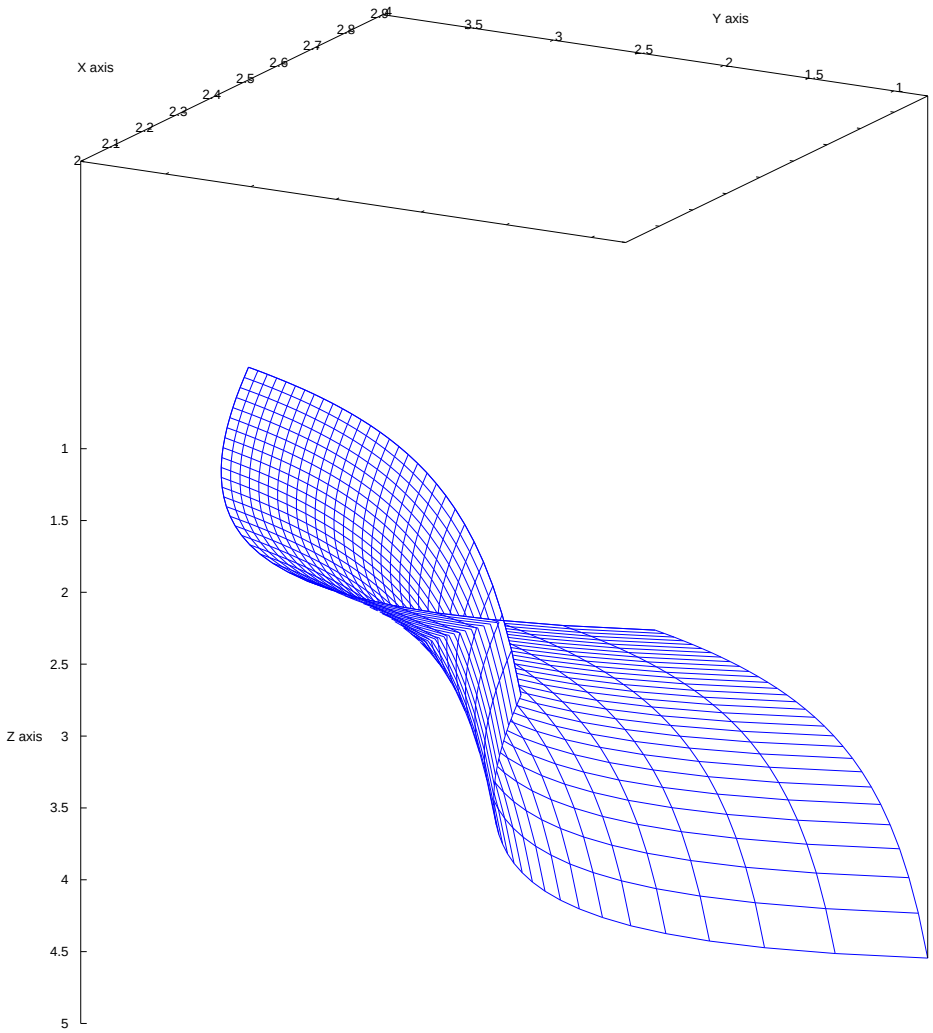


Figure 1.7. T_1, view$(281, 299)$

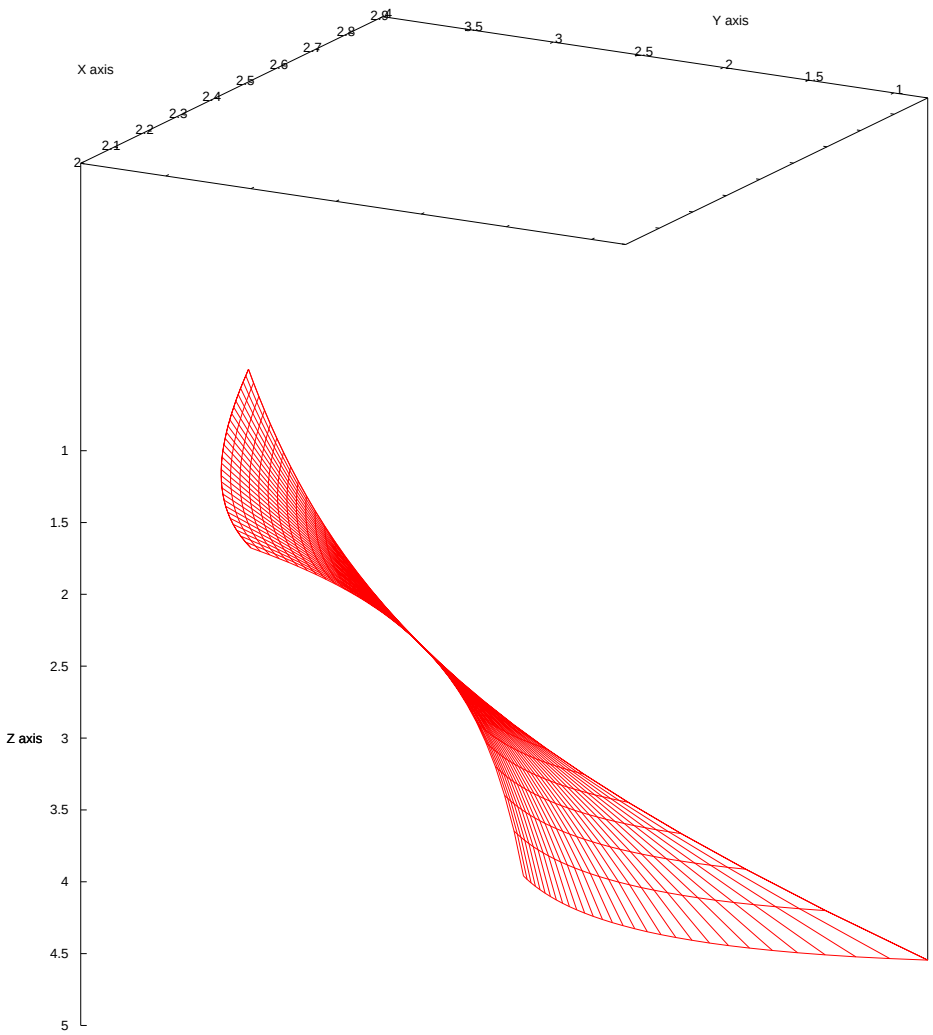


Figure 1.8. The positive part of T_1, view$(281, 299)$

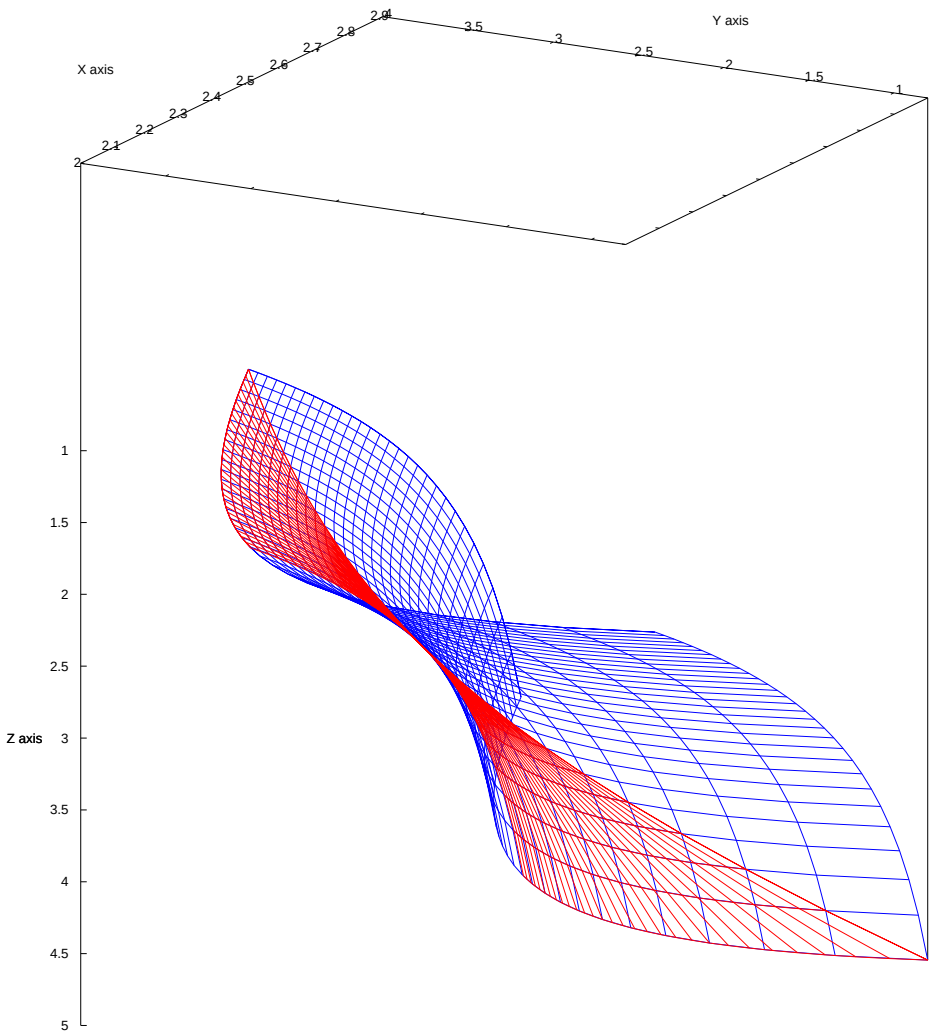


Figure 1.9. T_1, total view at $(281, 299)$

2.1 $L^{\times}(T_2)$ and $L^{\oplus}(T_2)$

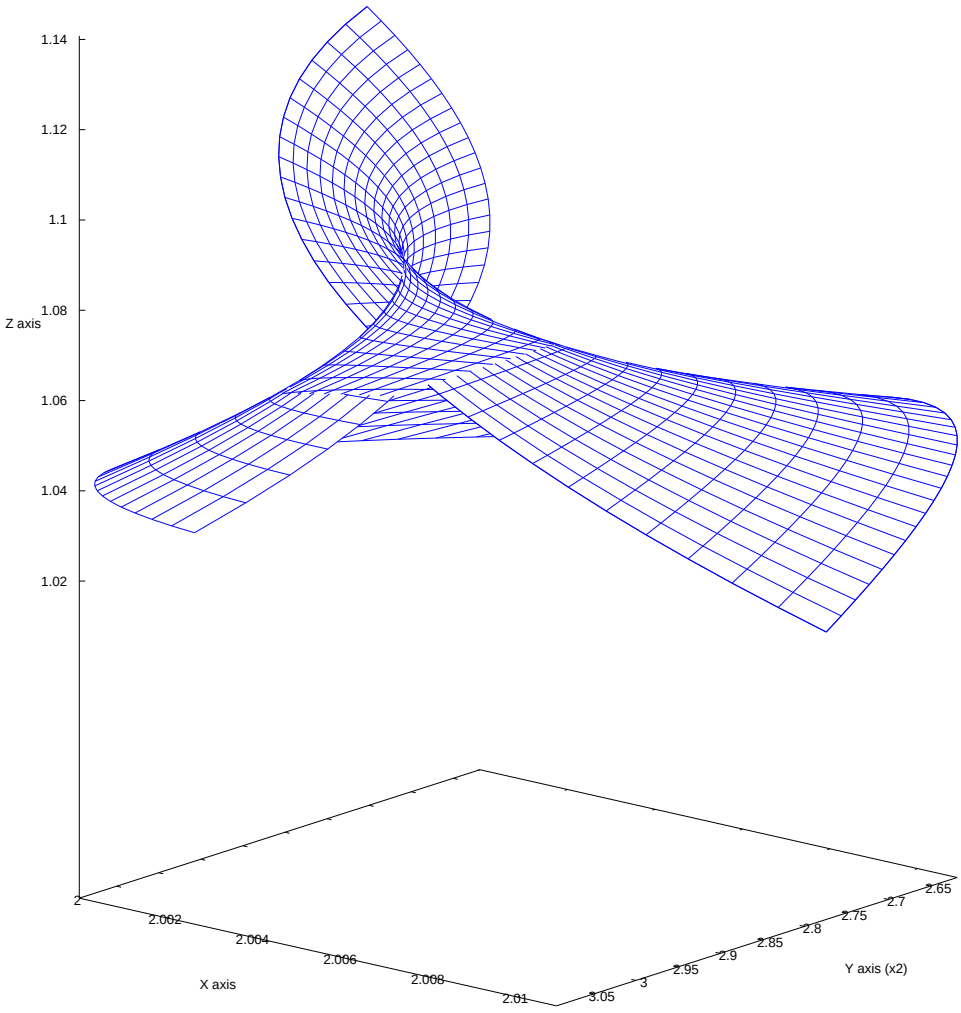


Figure 2.1. T_2, $(a, b) = (0.7, 0.95)$, view$(101, 320)$

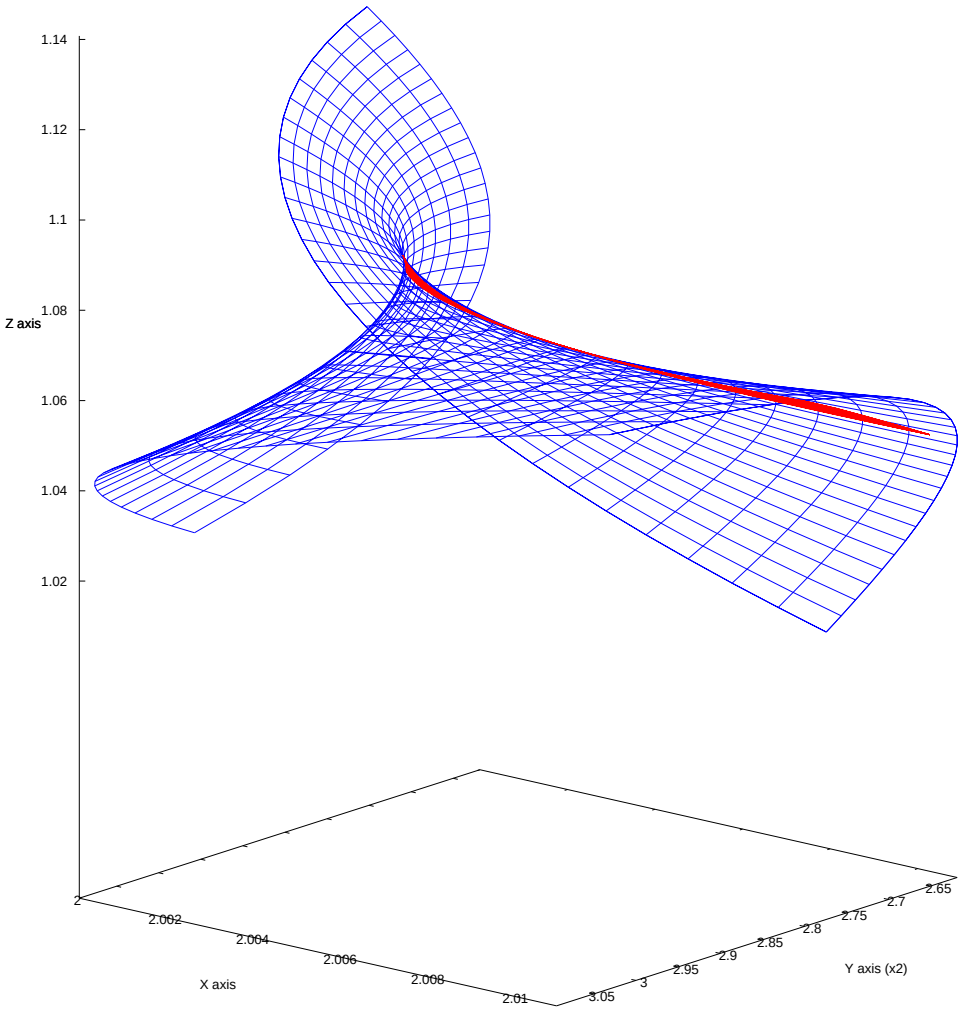


Figure 2.2. T_2 with a positive part (red), $(a, b) = (0.7, 0.95)$, view$(101, 320)$

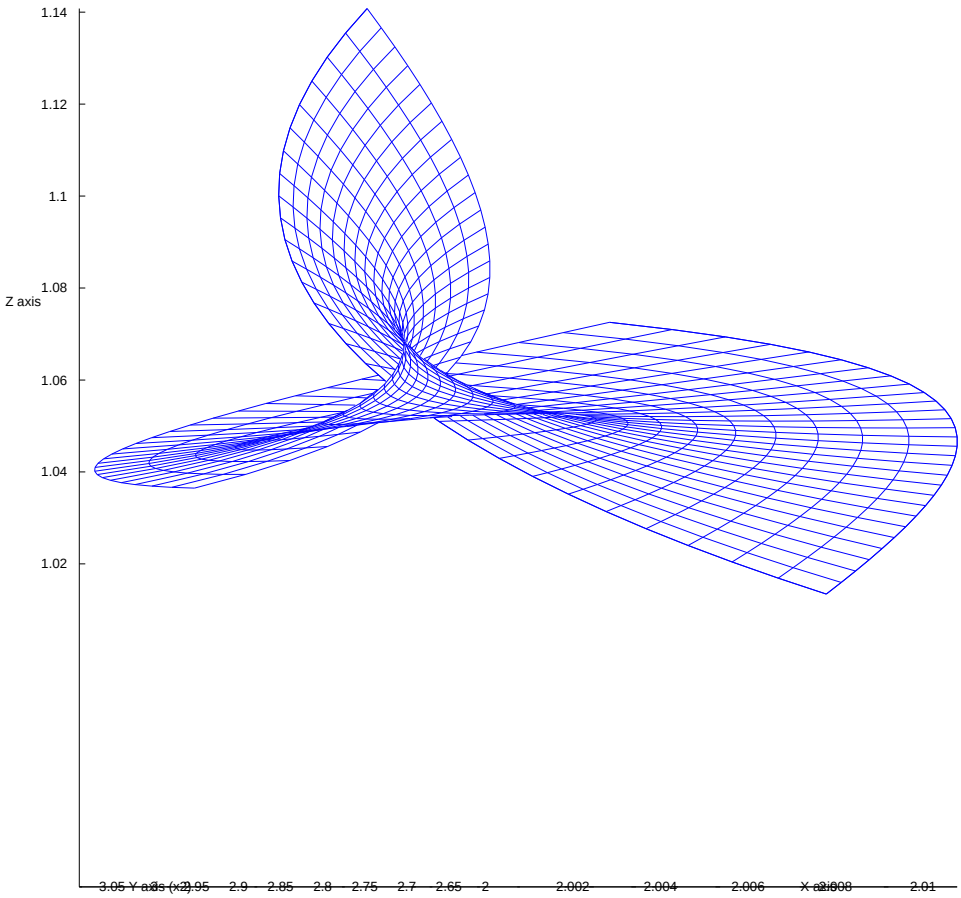


Figure 2.3. T_2, $(a, b) = (0.7, 0.95)$, view$(90, 320)$

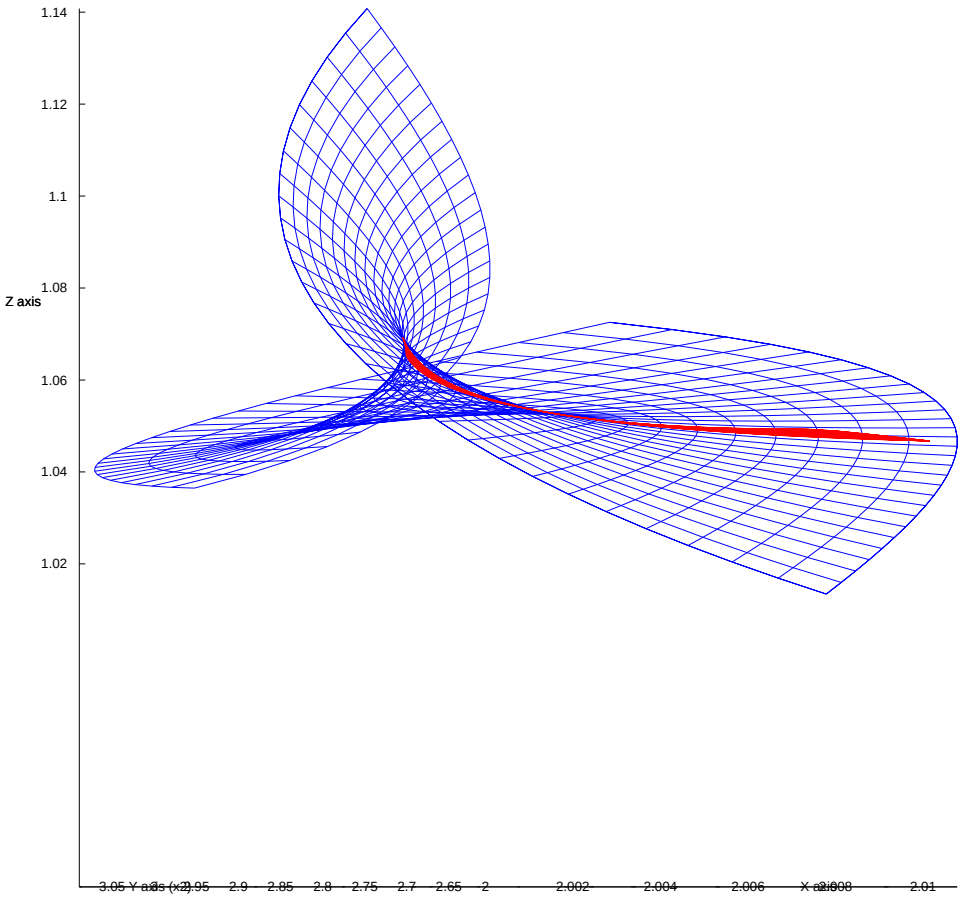


Figure 2.4. T_2 with a positive part (red), $(a, b) = (0.7, 0.95)$, view$(90, 320)$

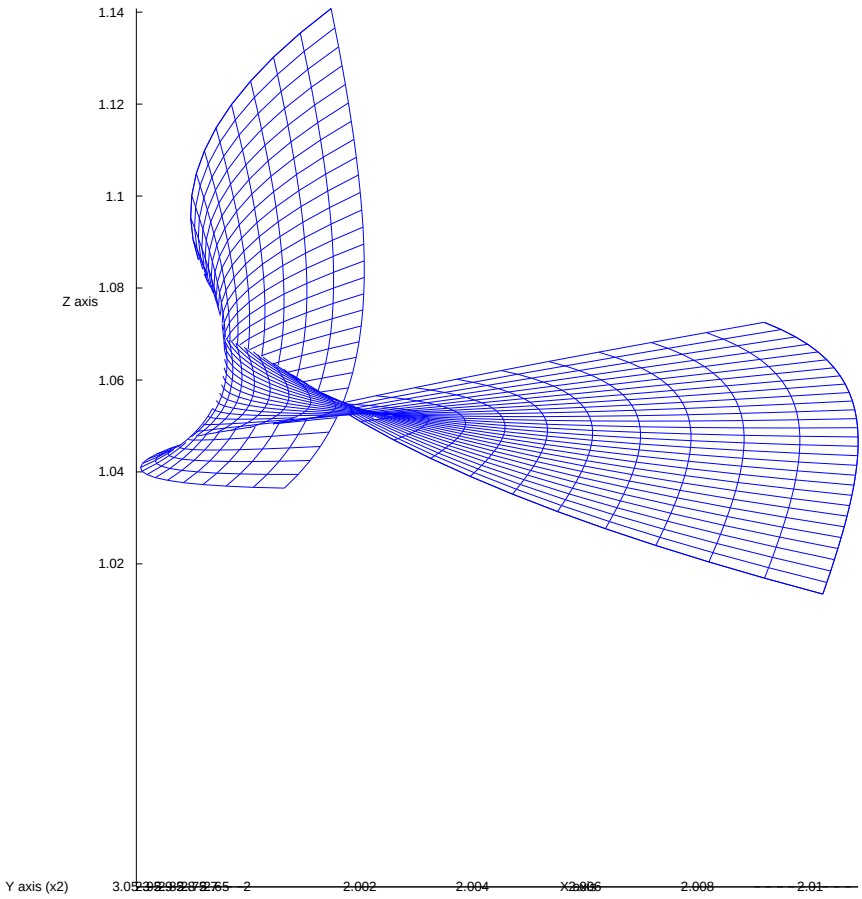
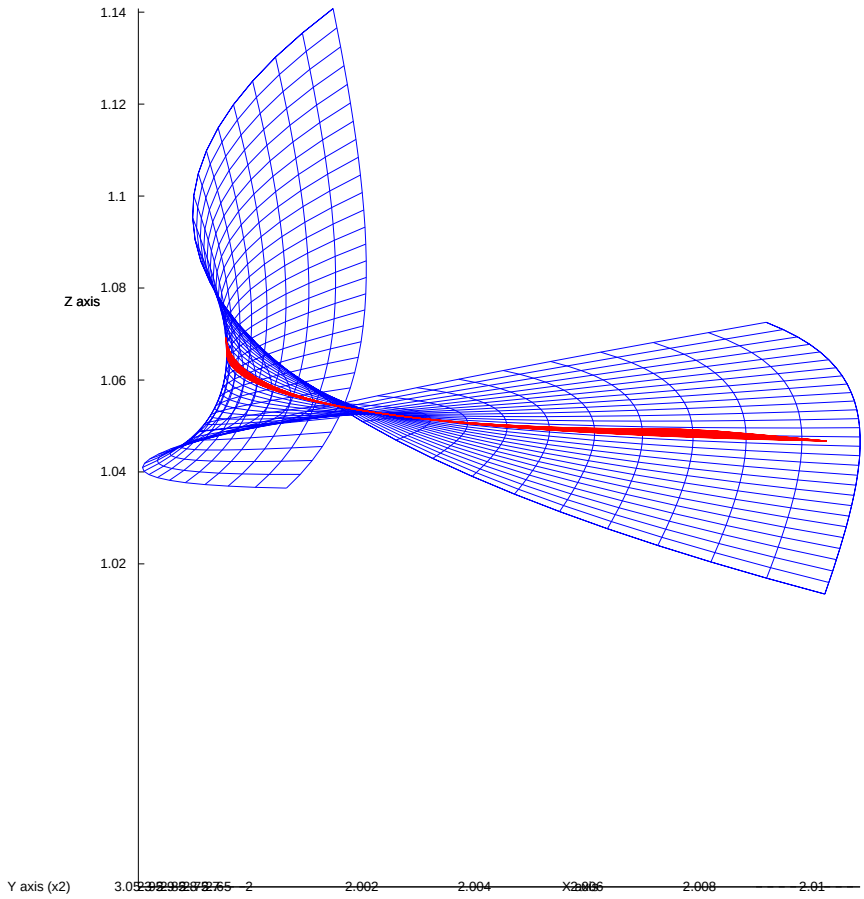


Figure 2.5. T_2, $(a, b) = (0.7, 0.95)$, view$(90, 350)$



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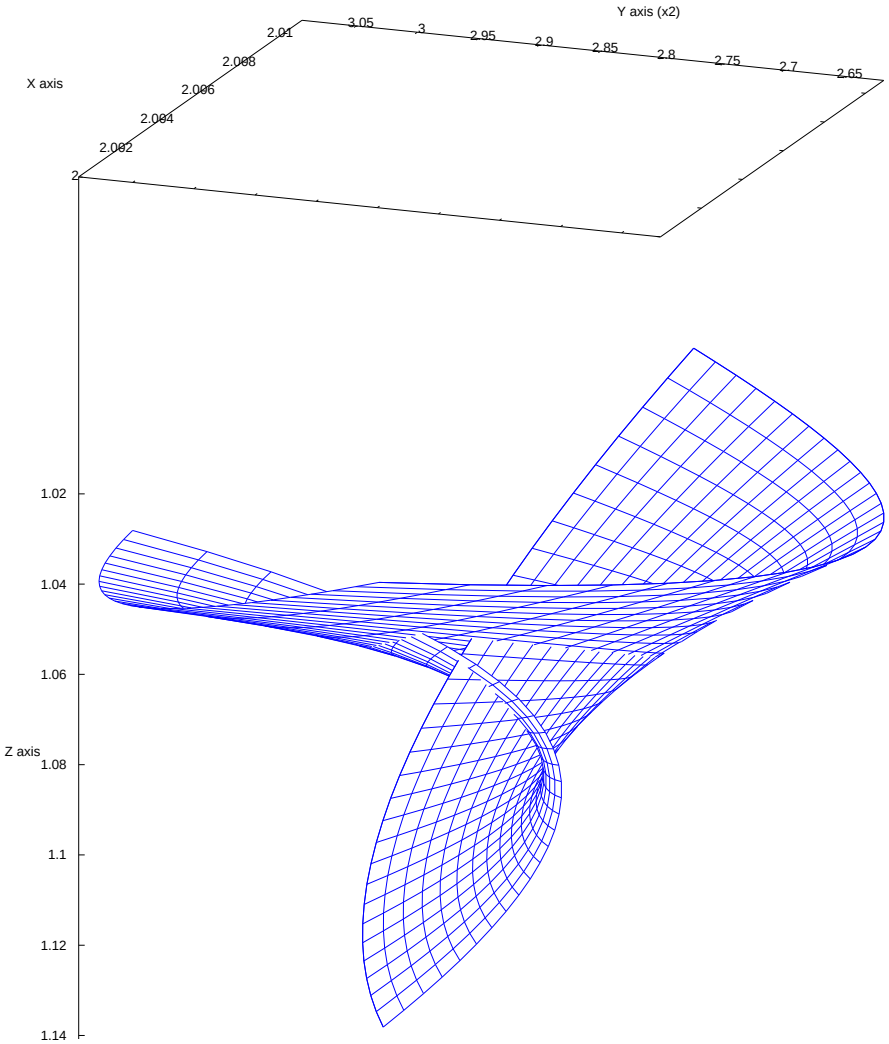

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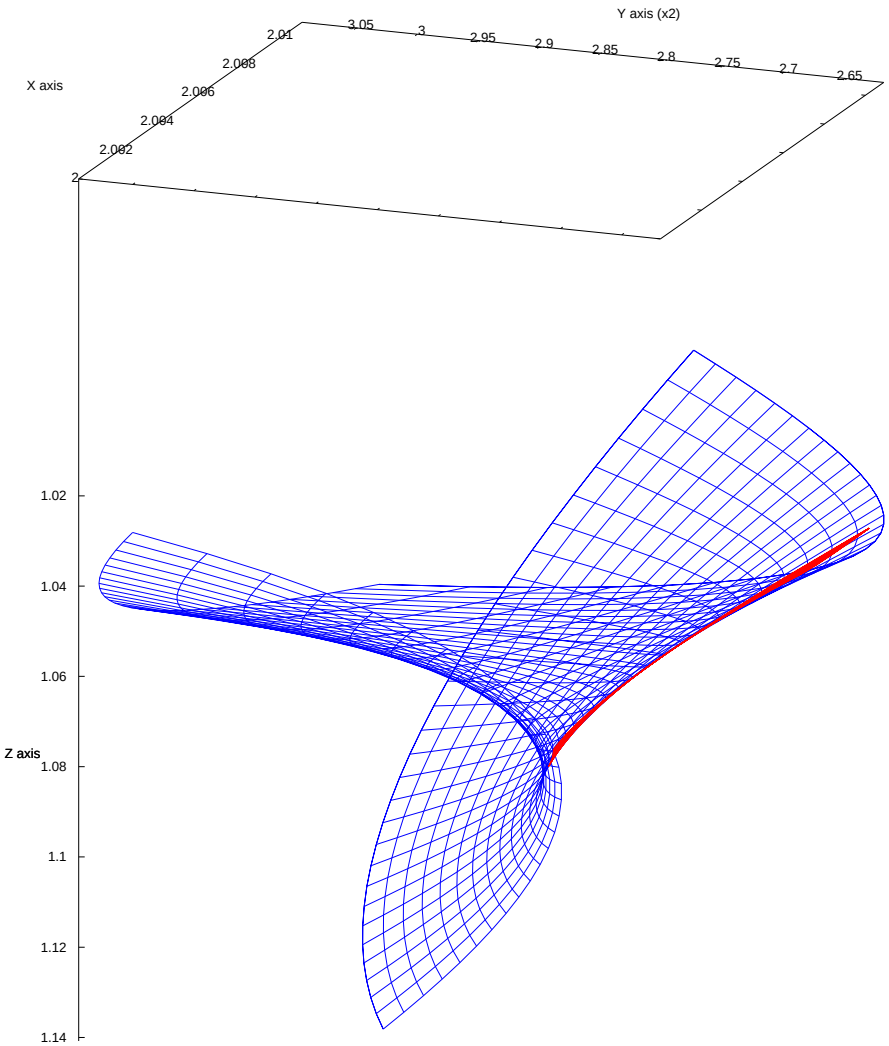

Figure 2.8. T_2 with a positive part (red), $(a, b) = (0.7, 0.95)$, view$(281, 291)$

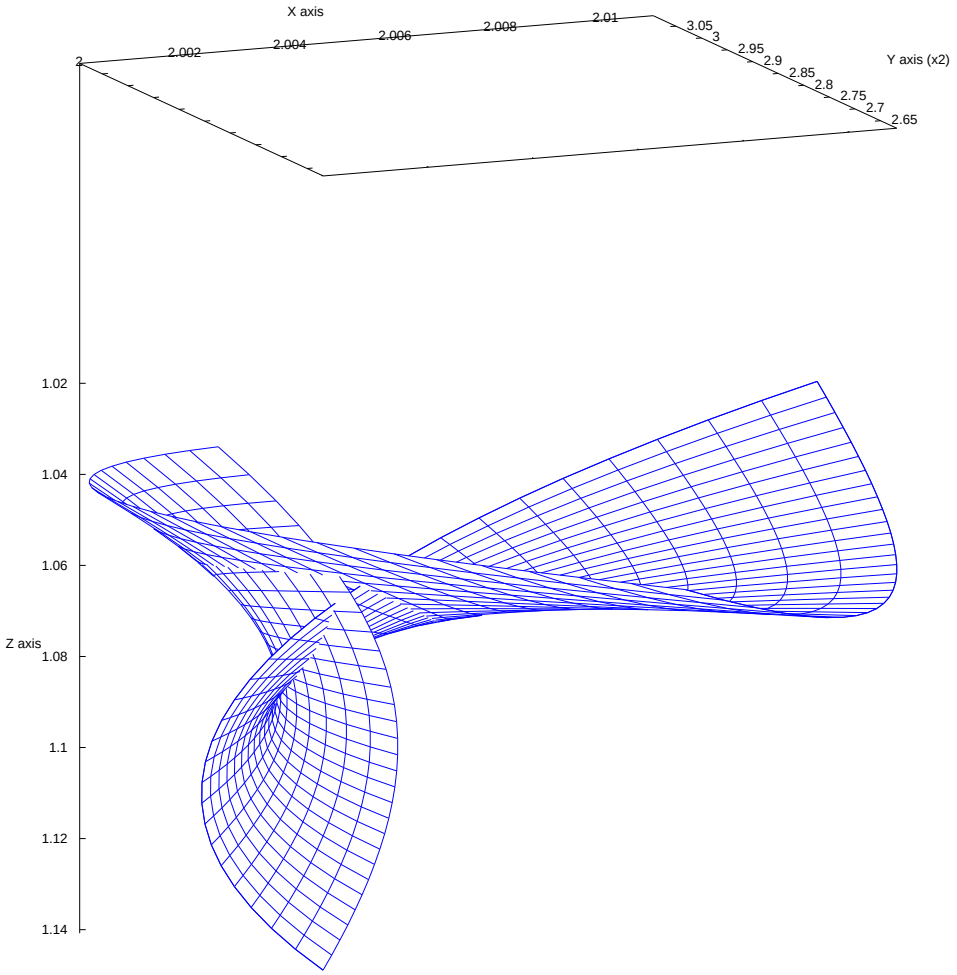

Figure 2.9. T_2, $(a, b) = (0.7, 0.95)$, view$(278, 337)$

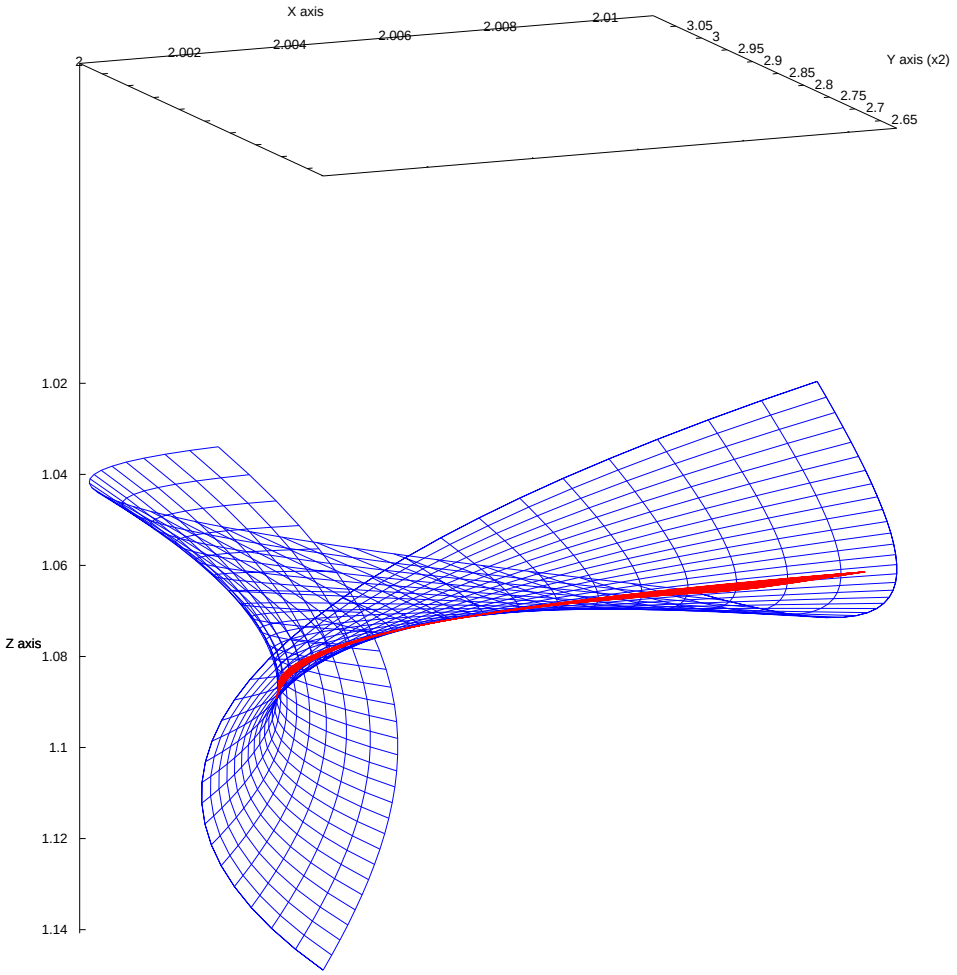

Figure 2.10. T_2 with a positive part (red), $(a, b) = (0.7, 0.95)$, view$(278, 337)$

2.2 $L^{\times}(T_2)$, $L^{\oplus}$(the ice-cream cone diagram) and $3PT$

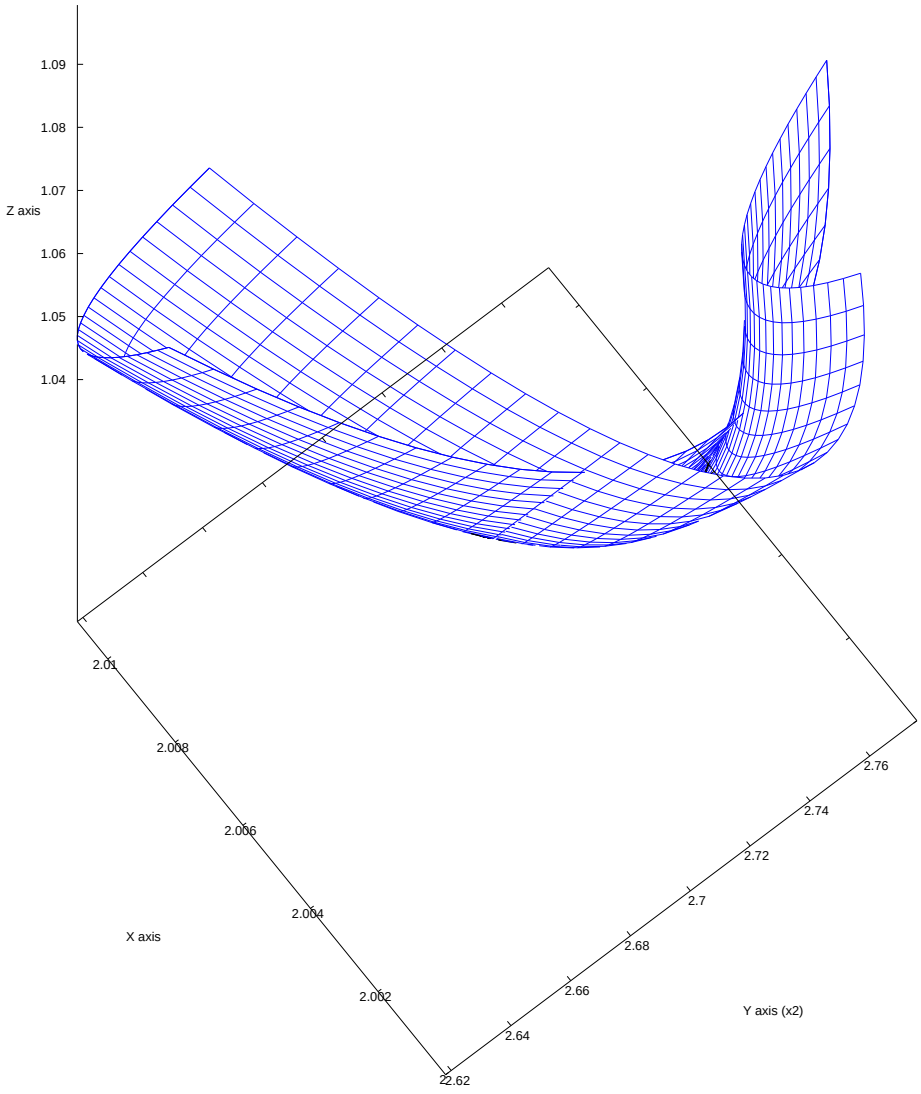

Figure 2.11. T_2, $(a, b) = (0.7, 0.95)$, view$(133, 128)$

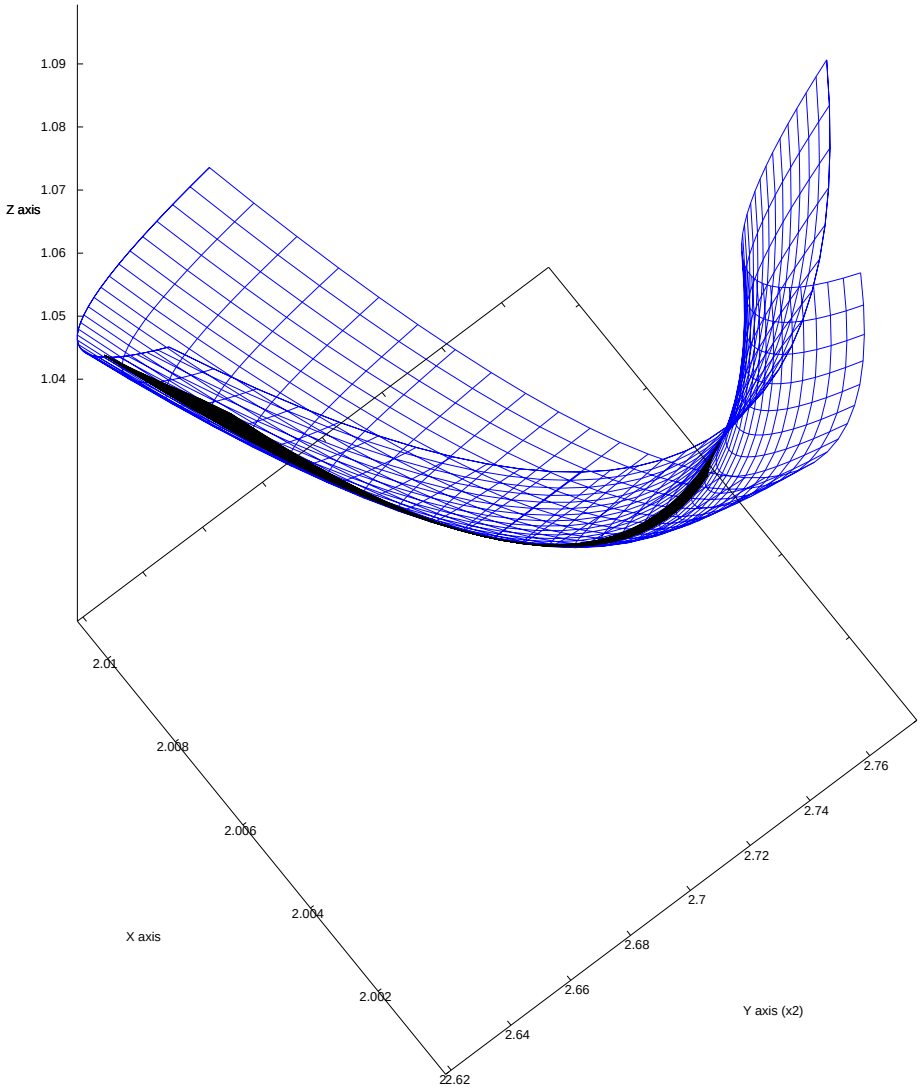

Figure 2.12. T_2 and its positive part (black) with a transparent mode, $(a, b) = (0.7, 0.95)$, view$(133, 128)$

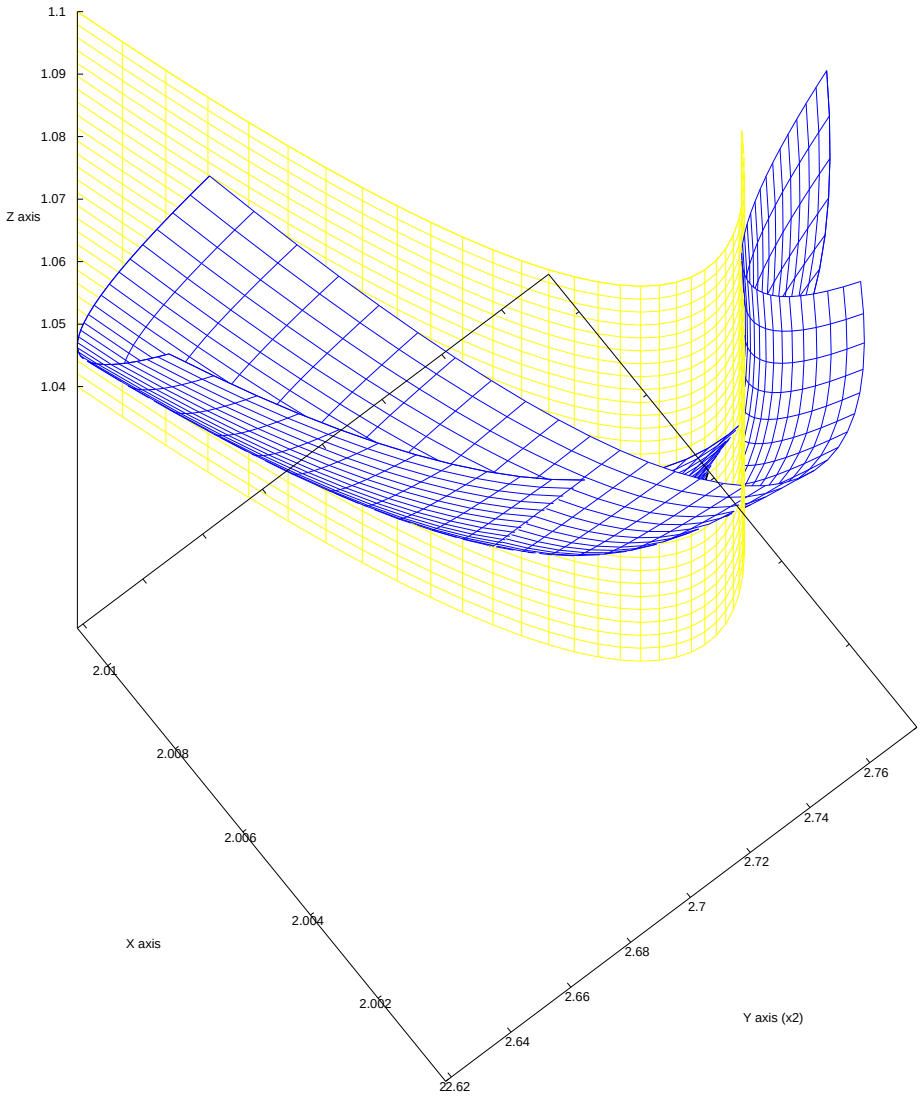

Figure 2.13. T_2 (blue) and the ice-cream cone (yellow),
$(a, b) = (0.7, 0.95)$, view$(133, 128)$

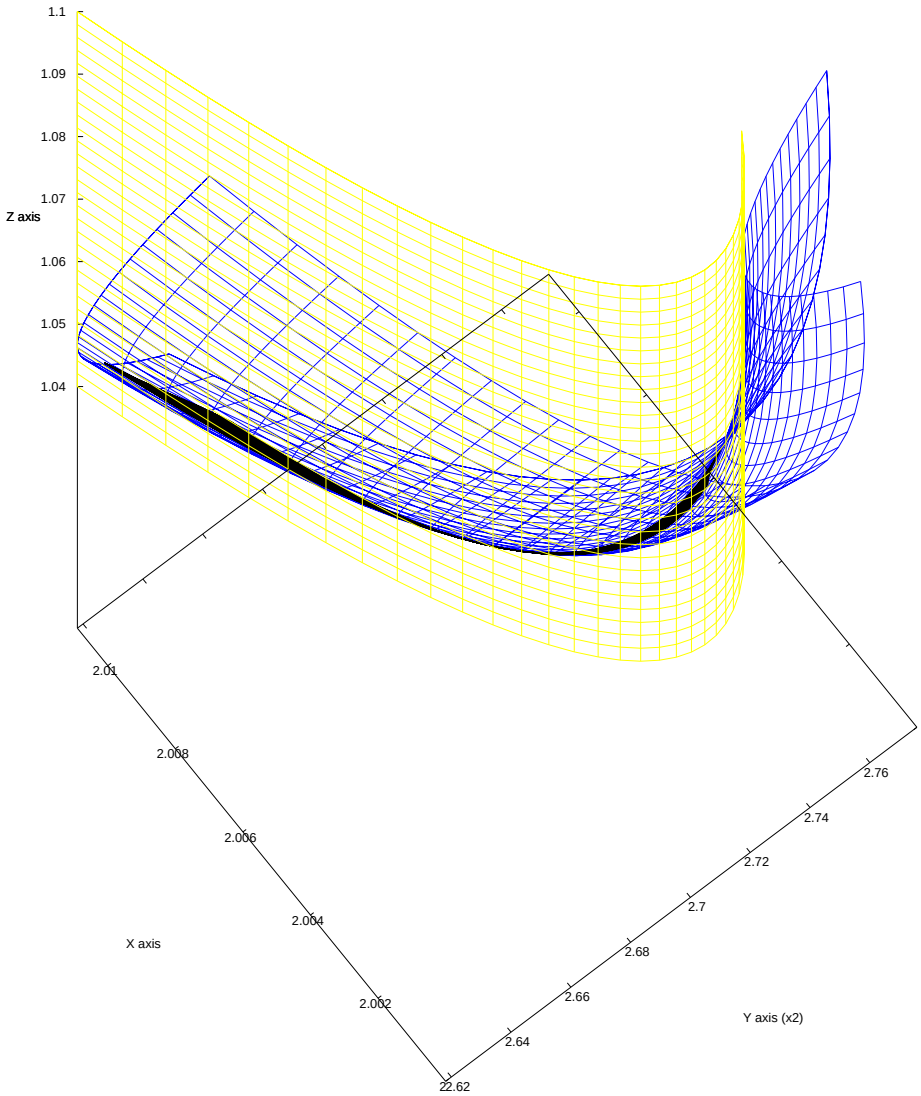

Figure 2.14. T_2 (blue) and the ice-cream cone (yellow), a transparent mode, $(a, b) = (0.7, 0.95)$, view$(133, 128)$

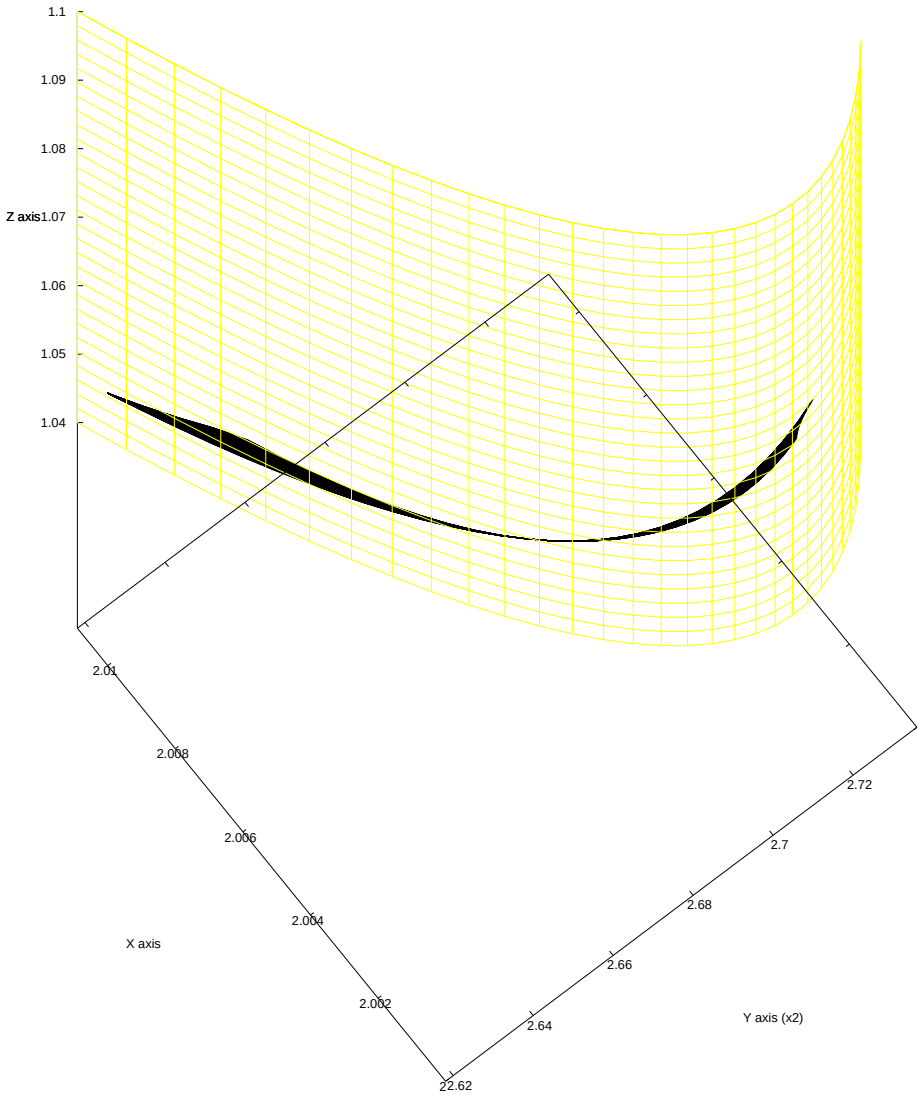

Figure 2.15. The positive part of T_2 (black) and the ice-cream cone (yellow), $(a, b) = (0.7, 0.95)$, view$(133, 128)$

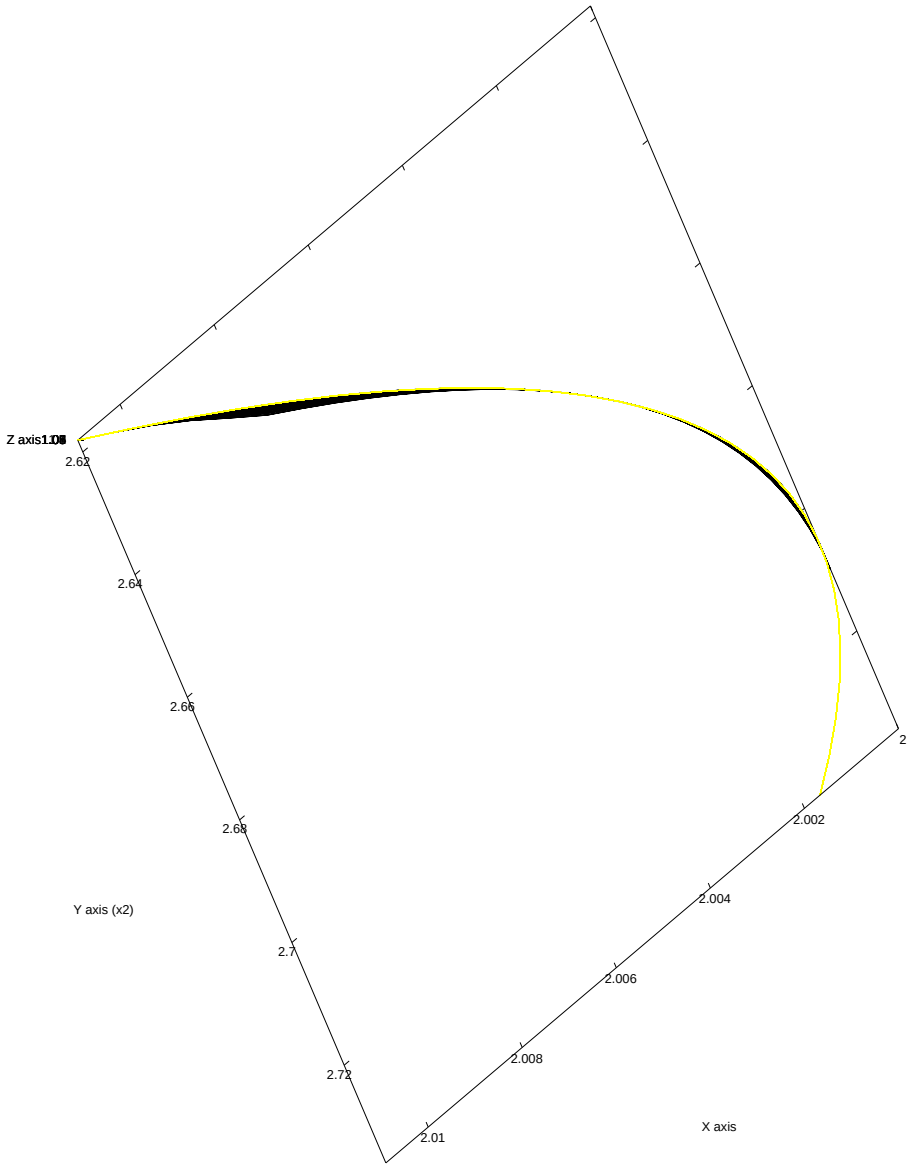

Figure 2.16. The positive part of T_2 (black) and the ice-cream cone (yellow), $(a, b) = (0.7, 0.95)$, view$(0, 149)$

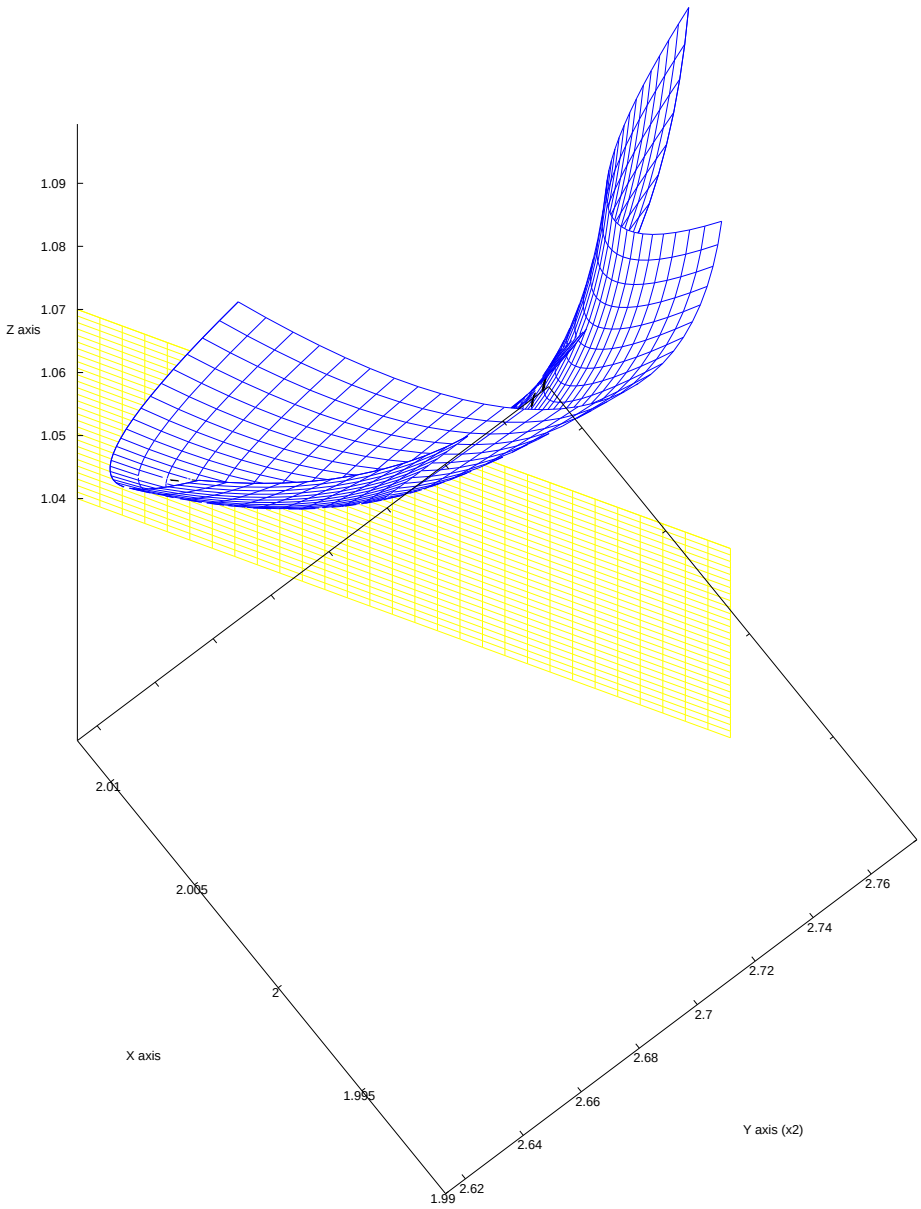

Figure 2.17. T_2 (blue) and $3PT$ (yellow), $(a, b) = (0.7, 0.95)$, view$(133, 128)$

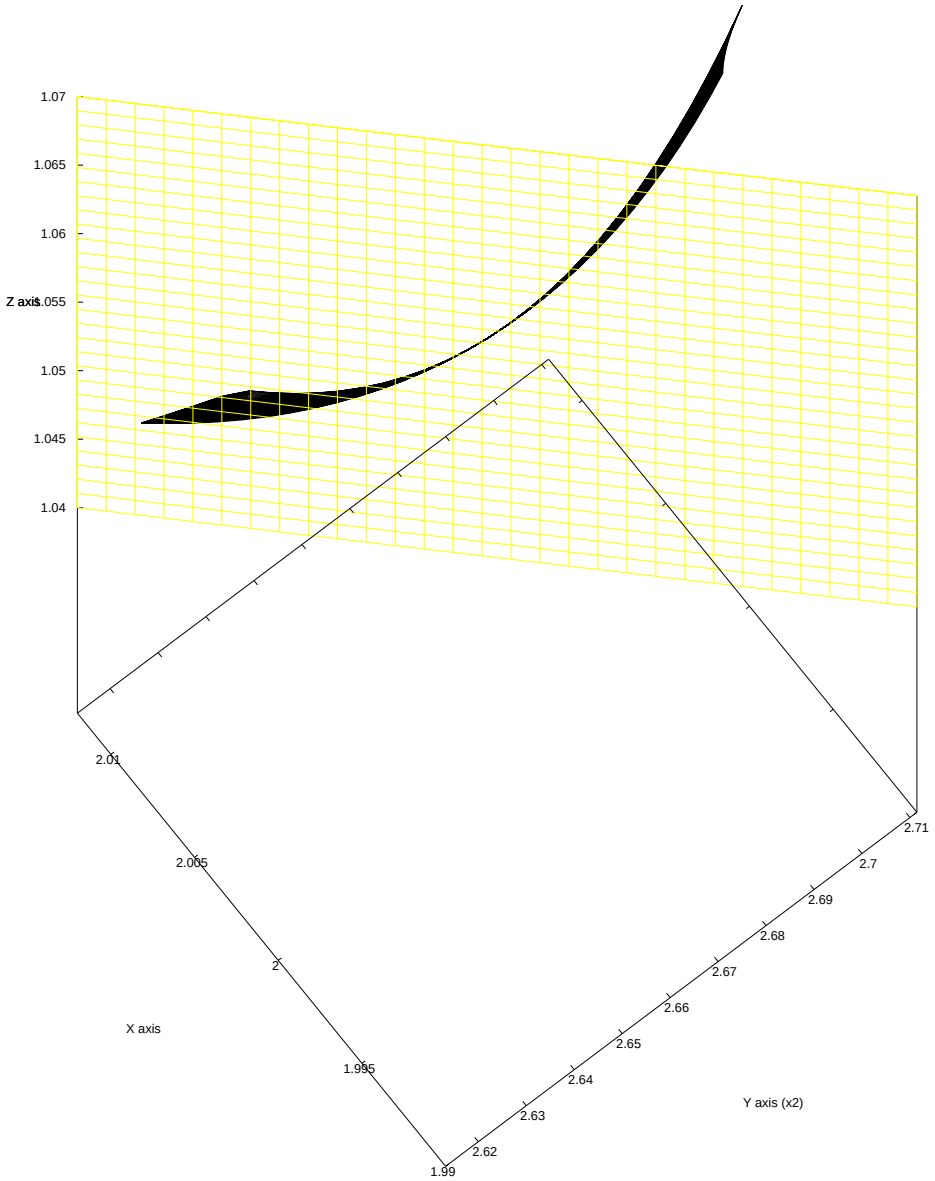

Figure 2.18. The positive part of T_2 (black) and $3PT$ (yellow), $(a, b) = (0.7, 0.95)$, view$(133, 128)$

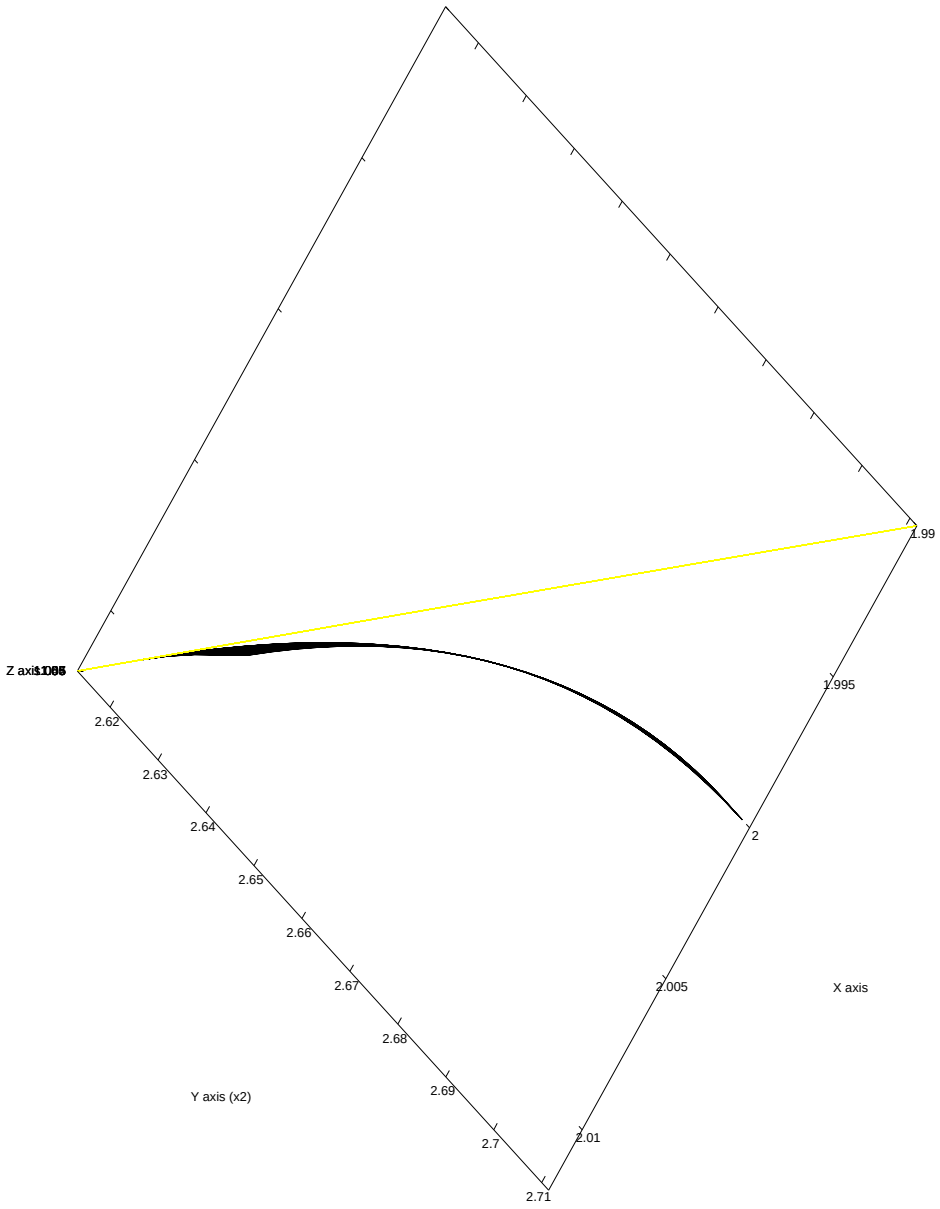

Figure 2.19. The positive part of T_2 (black) and $3PT$ (yellow), $(a, b) = (0.7, 0.95)$, view$(0, 128)$

3.1 $L^{\times}(T_3)$ and $L^{\oplus}(T3)$

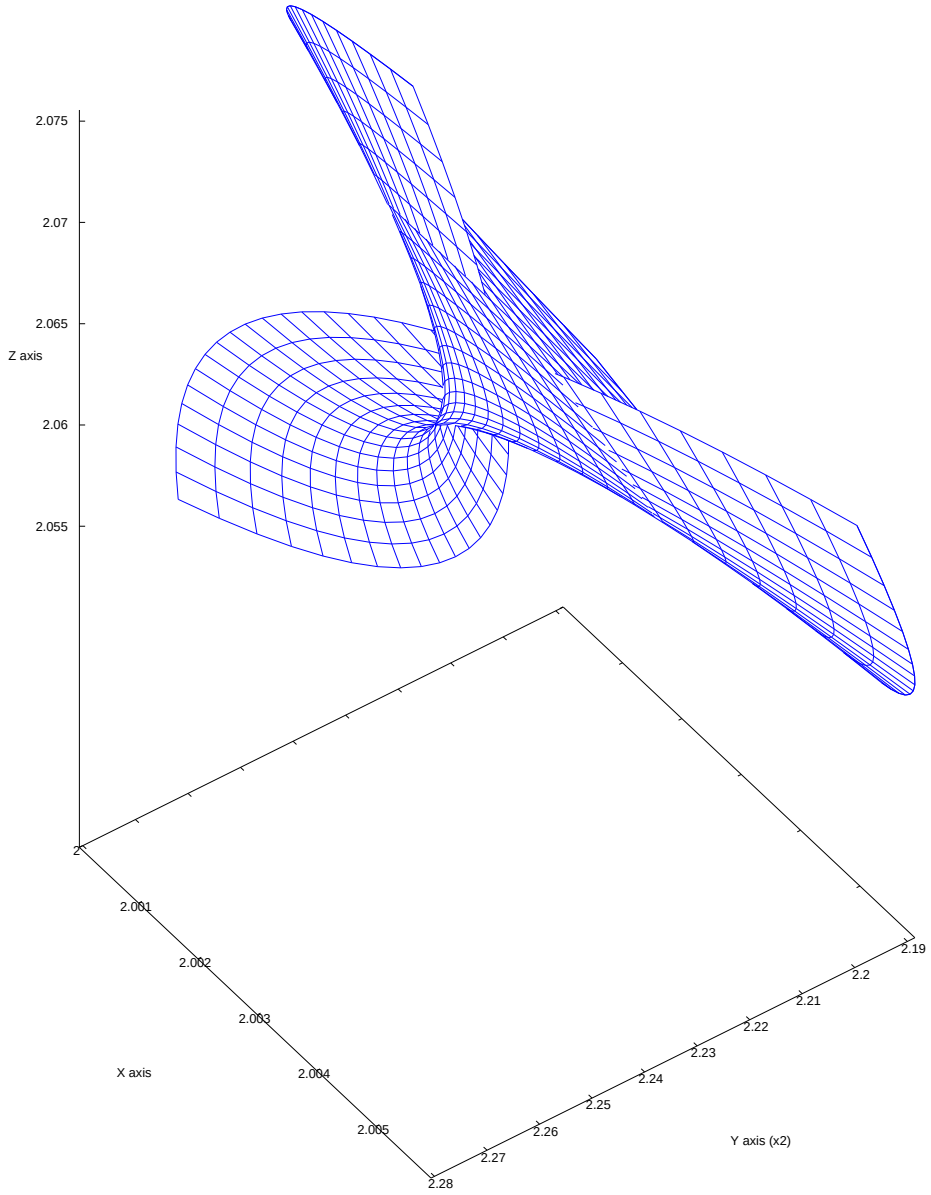

Figure 3.1. T_3, $(a, b) = (0, 9, 1.95)$, view$(119, 306)$

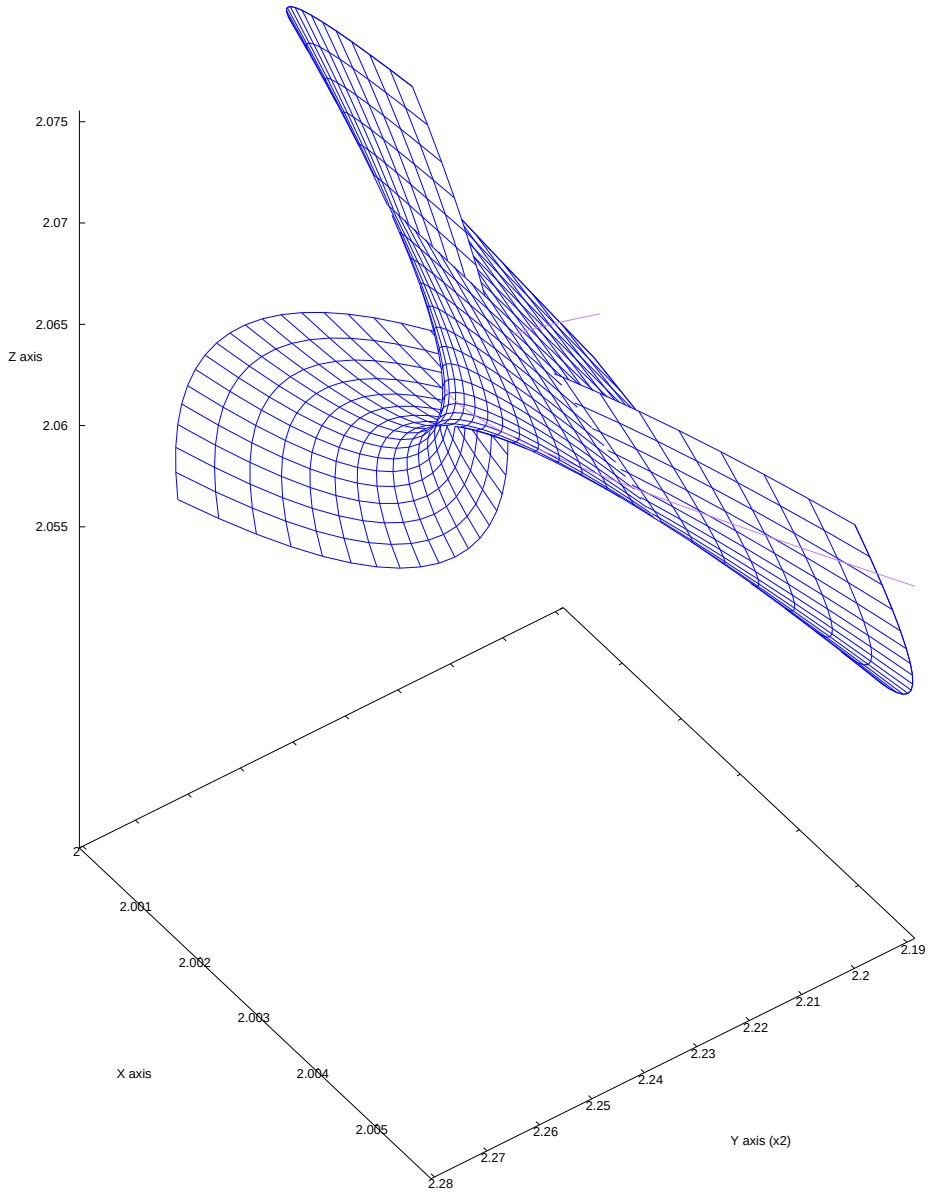

Figure 3.2. T_3 (blue) and the codim 2 component (purple), $(a, b) = (0, 9, 1.95)$, view$(119, 306)$

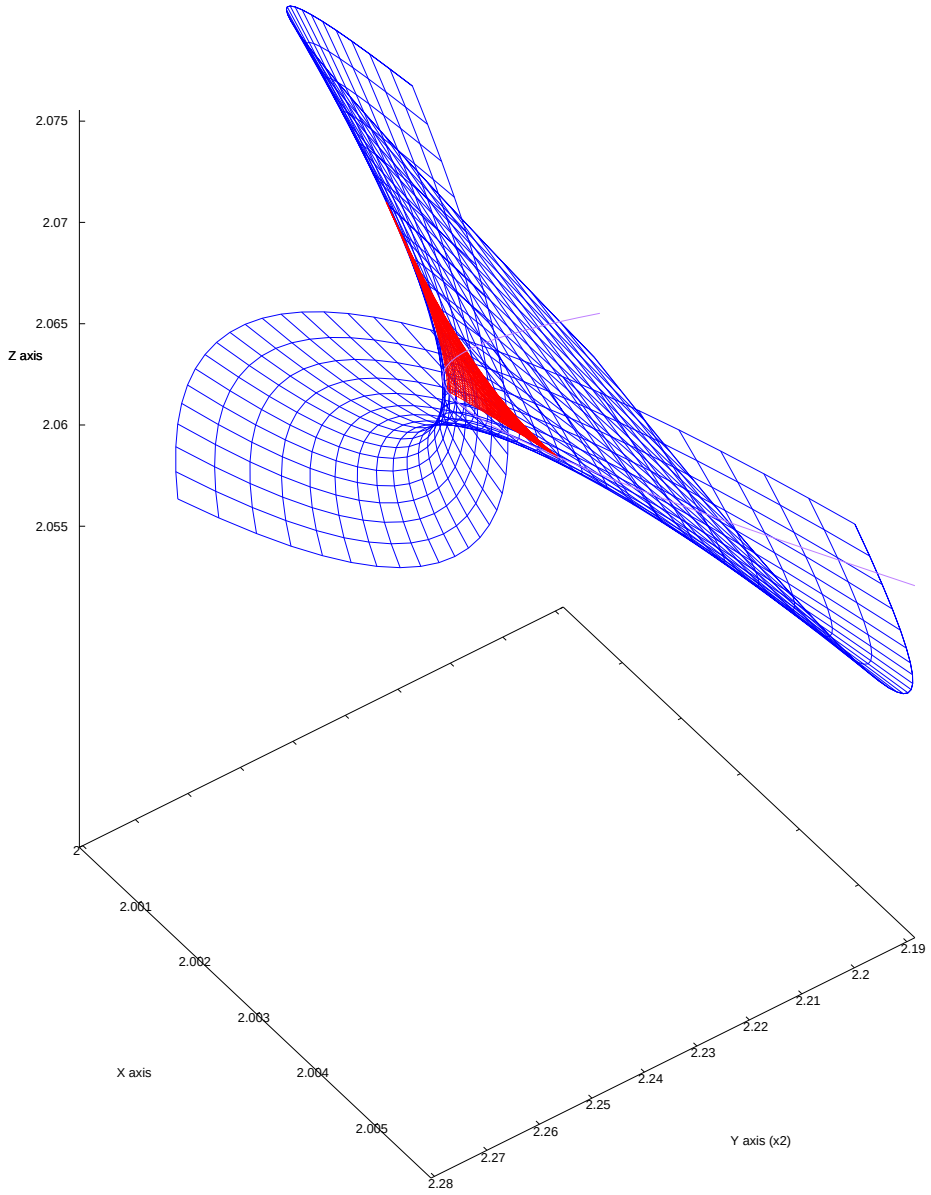

Figure 3.3. T_3 (blue) and its positive part (red), $(a, b) = (0, 9, 1.95)$, view$(119, 306)$

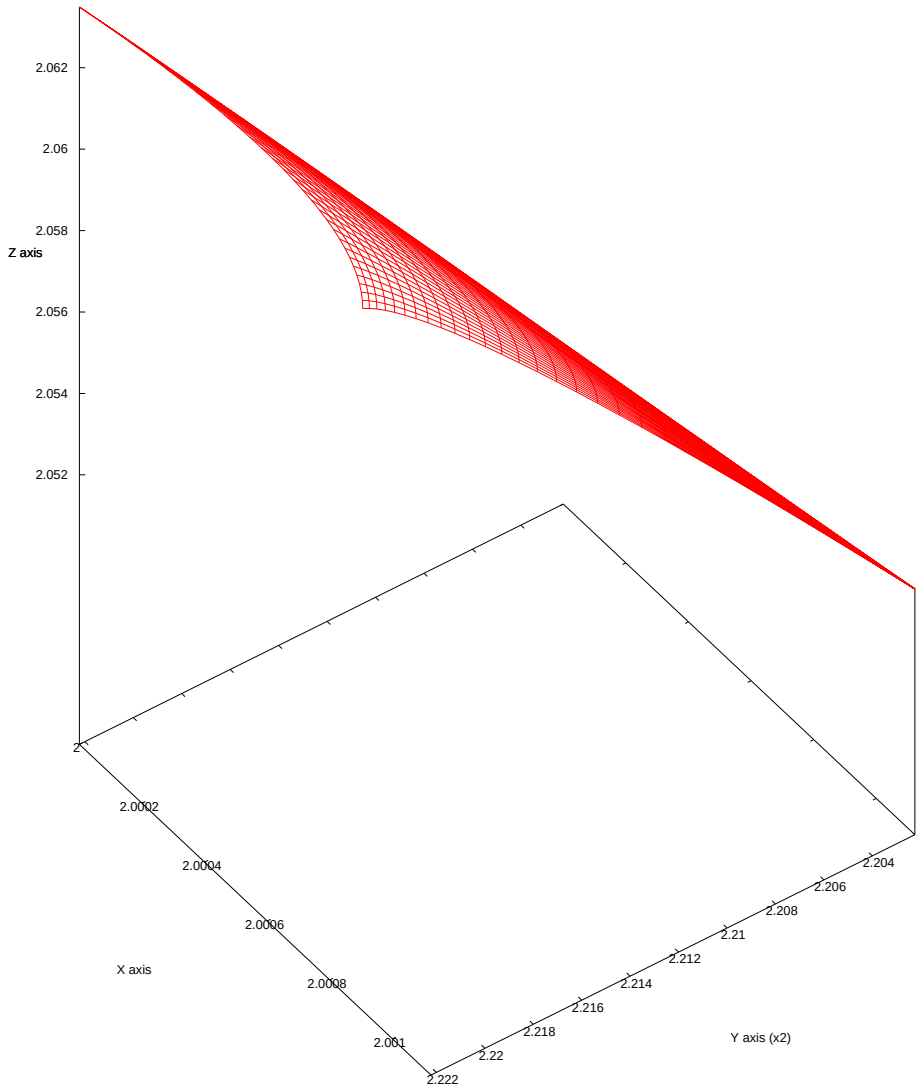

Figure 3.4. The positive part of T_3 (red), $(a, b) = (0, 9, 1.95)$, view$(119, 306)$

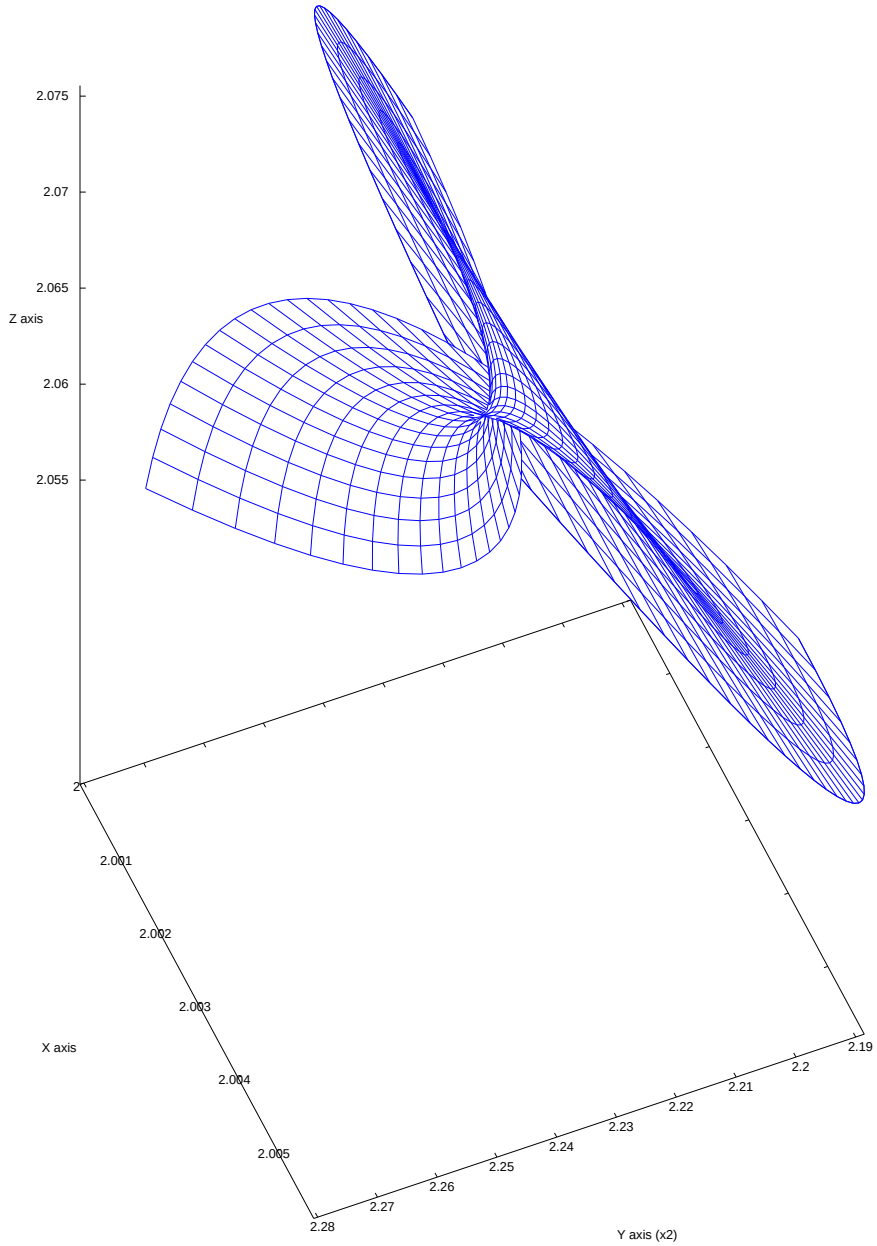

Figure 3.5. T_3, $(a, b) = (0, 9, 1.95)$, view$(124, 293)$

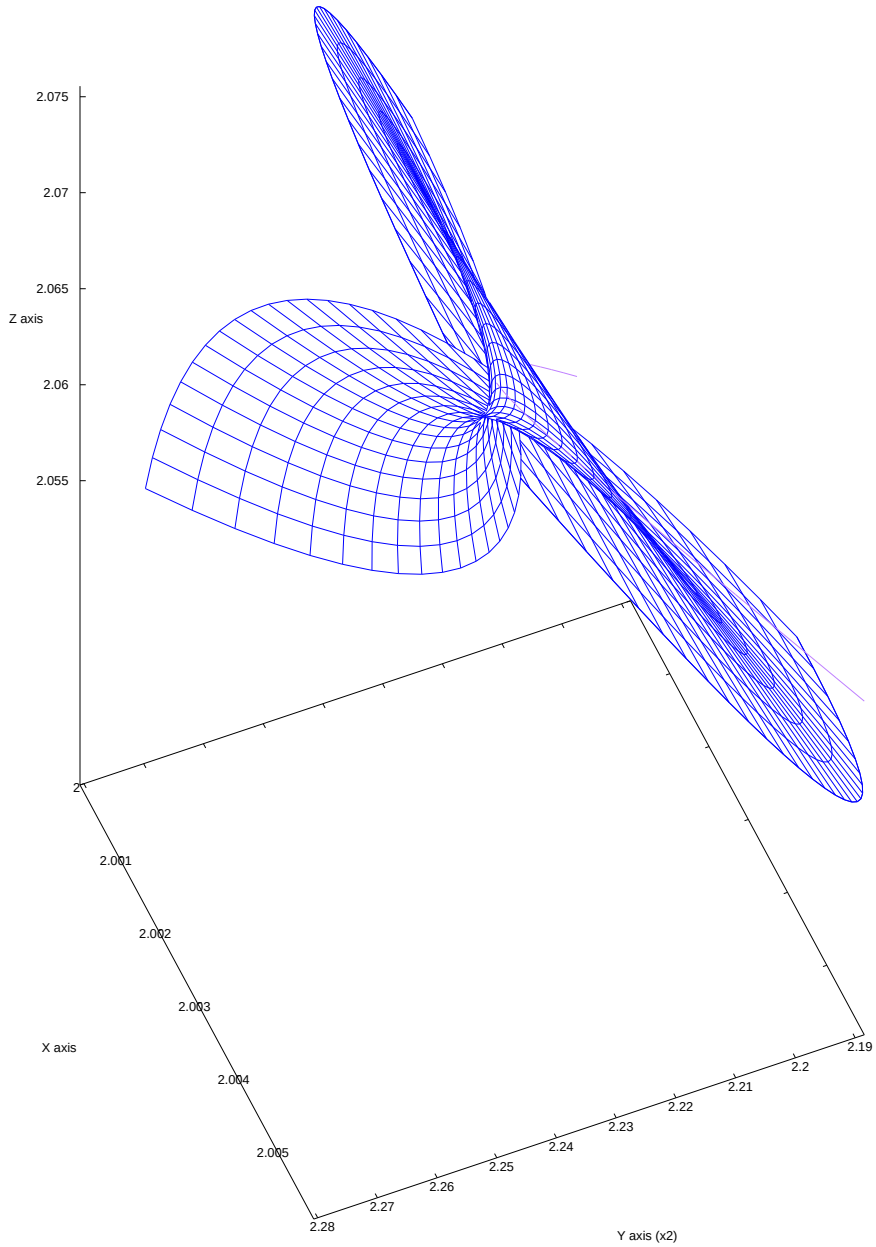

Figure 3.6. T_3 (blue) and the codim 2 component (purple), $(a, b) = (0, 9, 1.95)$, view$(124, 293)$

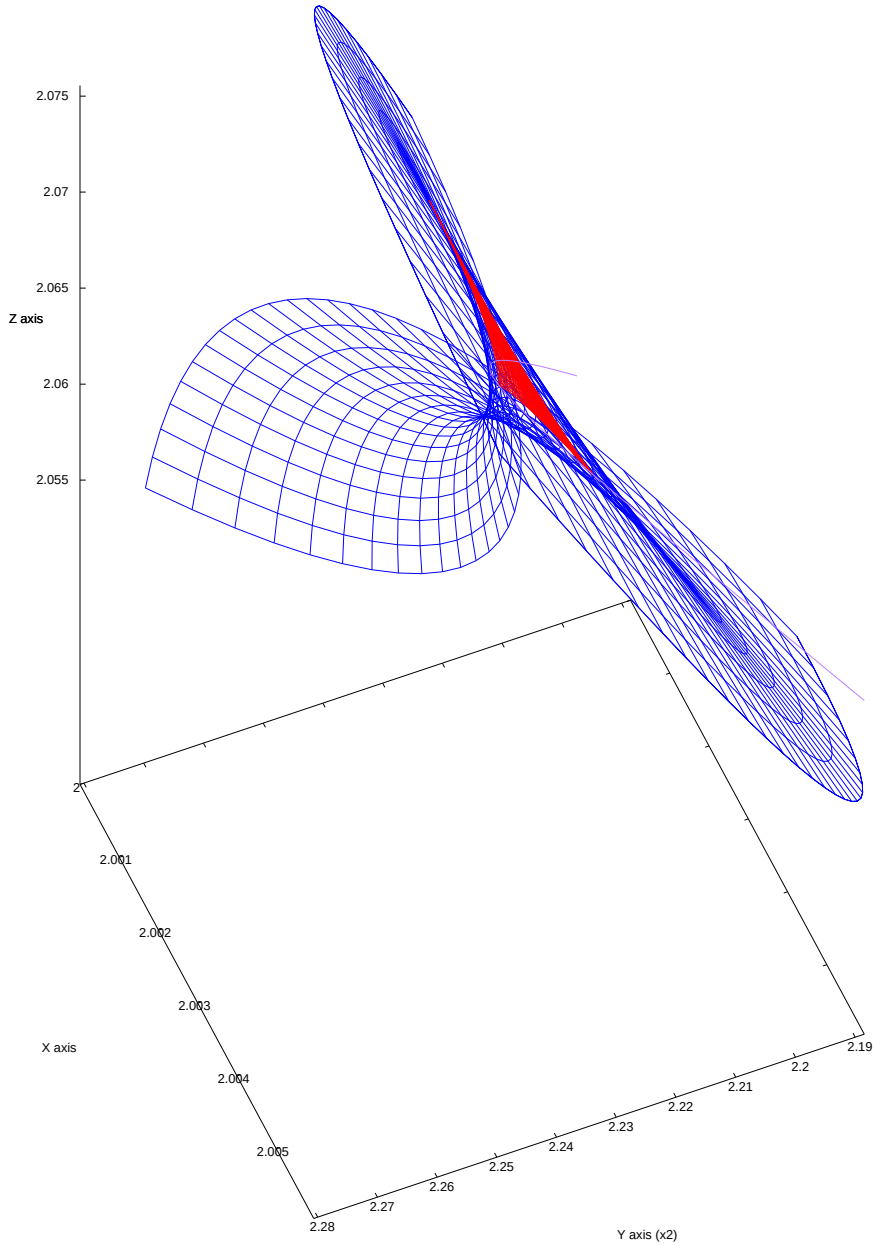

Figure 3.7. T_3 (blue) and its positive part (red), $(a, b) = (0, 9, 1.95)$, view$(124, 293)$

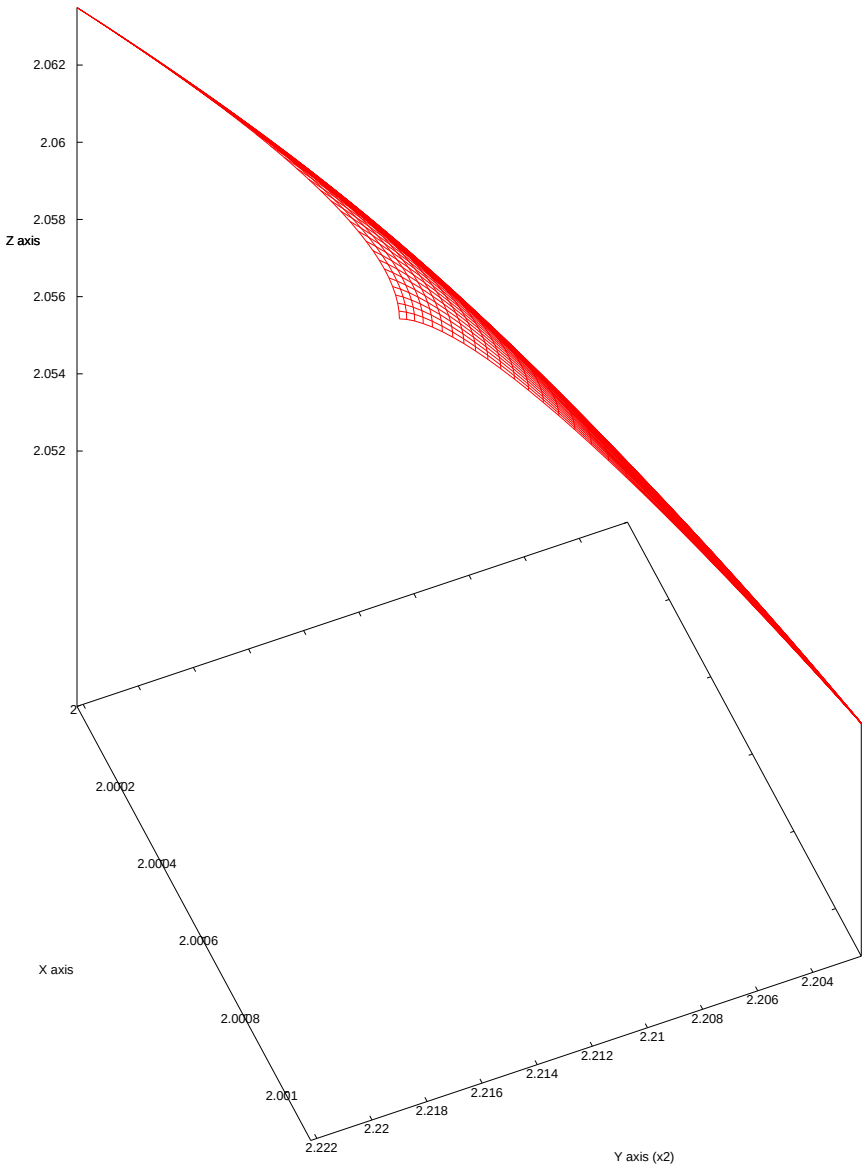

Figure 3.8. The positive part of T_3 (red), $(a, b) = (0, 9, 1.95)$, view$(124, 293)$

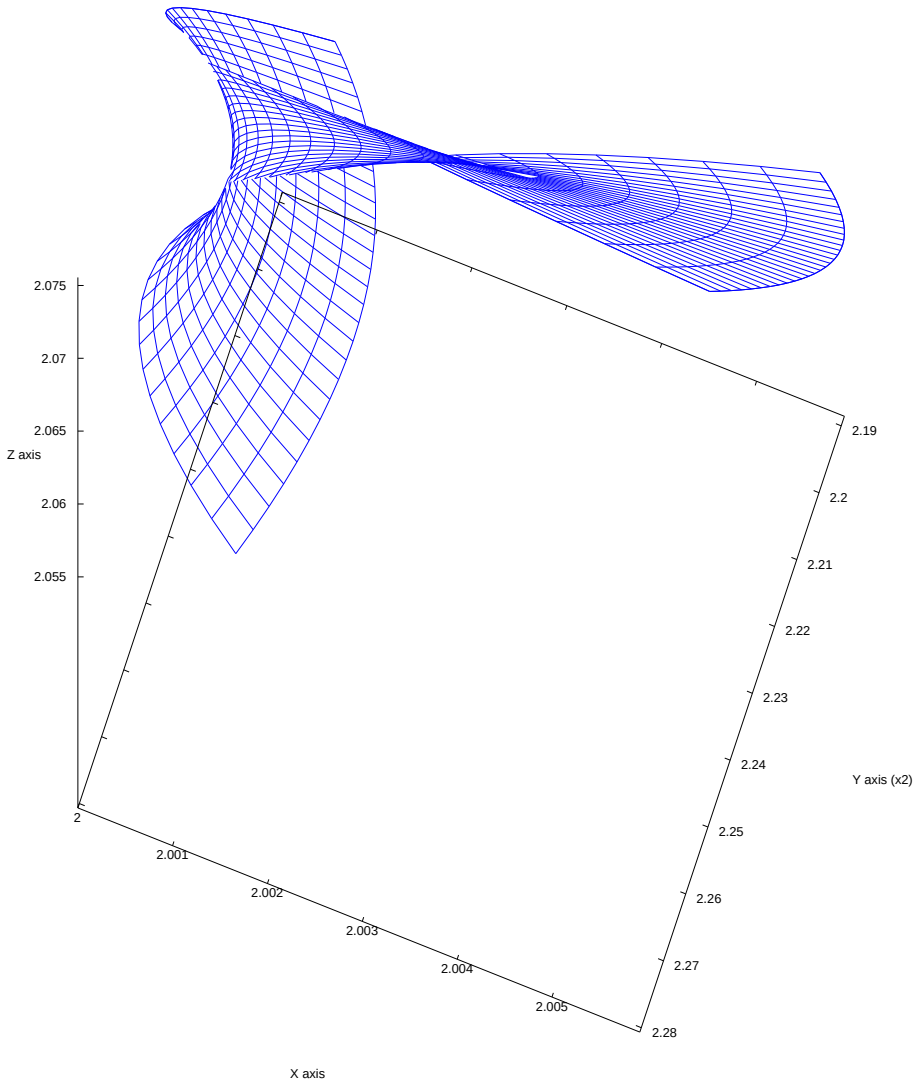

Figure 3.9. T_3, $(a, b) = (0, 9, 1.95)$, view$(141, 340)$

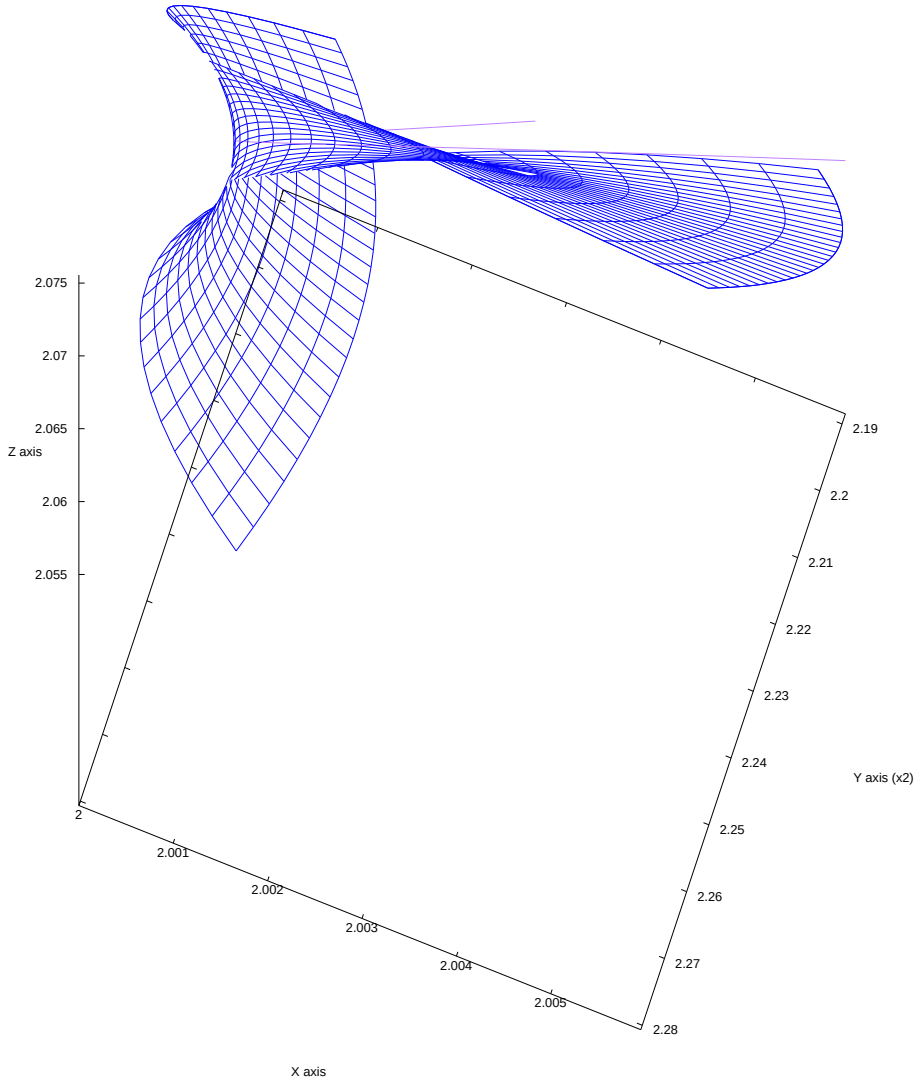

Figure 3.10. T_3 (blue) and the codim 2 component (purple), $(a, b) = (0, 9, 1.95)$, view$(141, 340)$

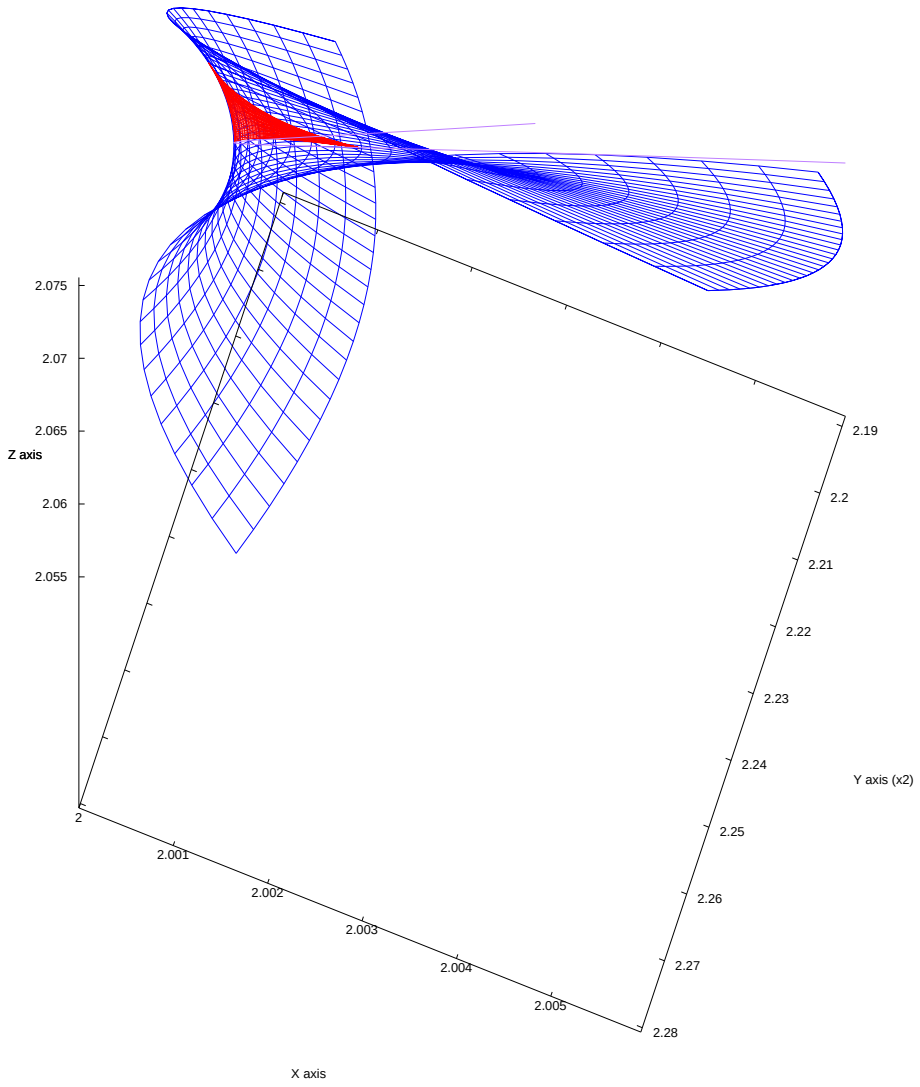

Figure 3.11. T_3 (blue) and its positive part (red), $(a, b) = (0, 9, 1.95)$, view$(141, 340)$

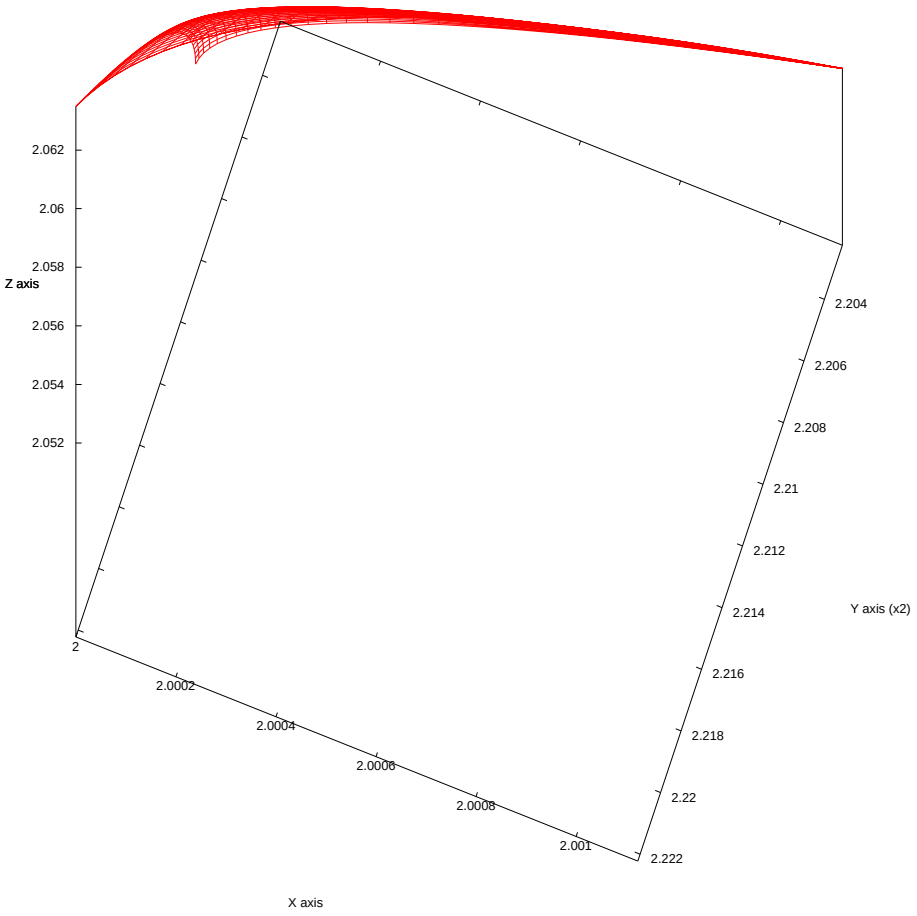

Figure 3.12. The positive part of T_3 (red), $(a, b) = (0, 9, 1.95)$, view$(141, 340)$

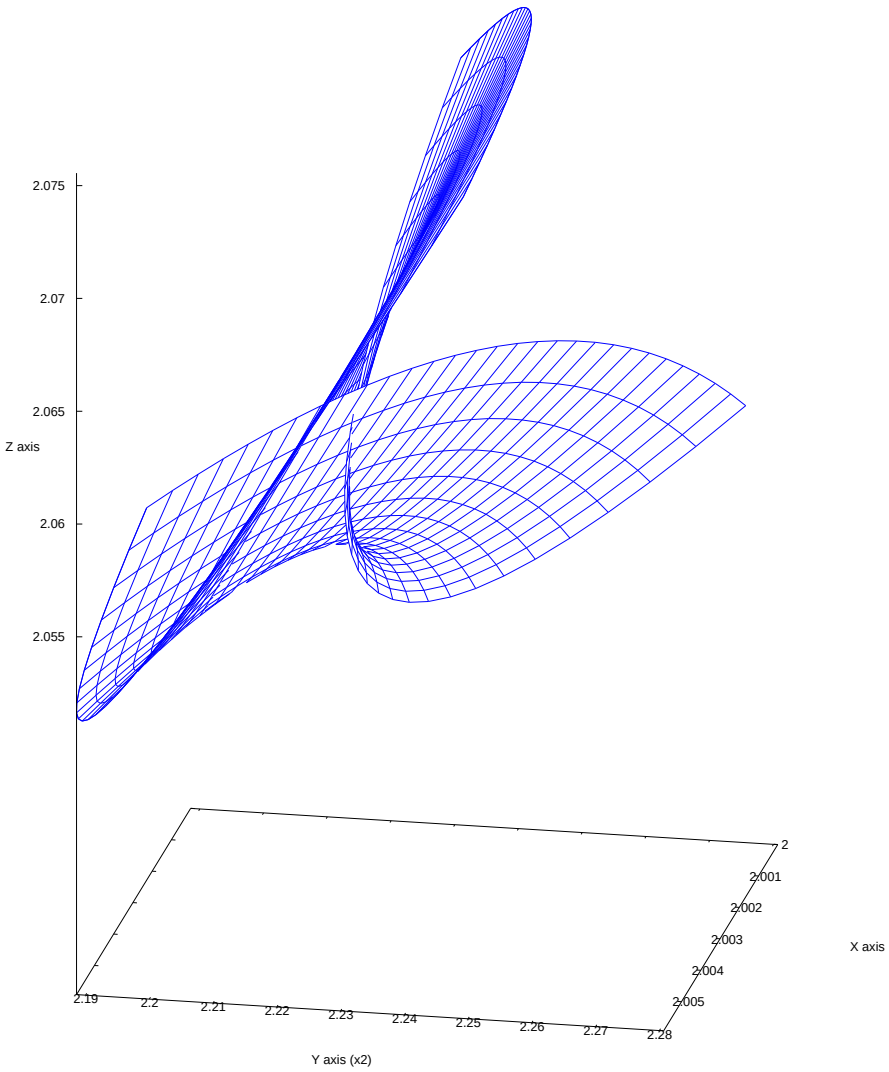

Figure 3.13. T_3, $(a, b) = (0, 9, 1.95)$, view$(77, 101)$

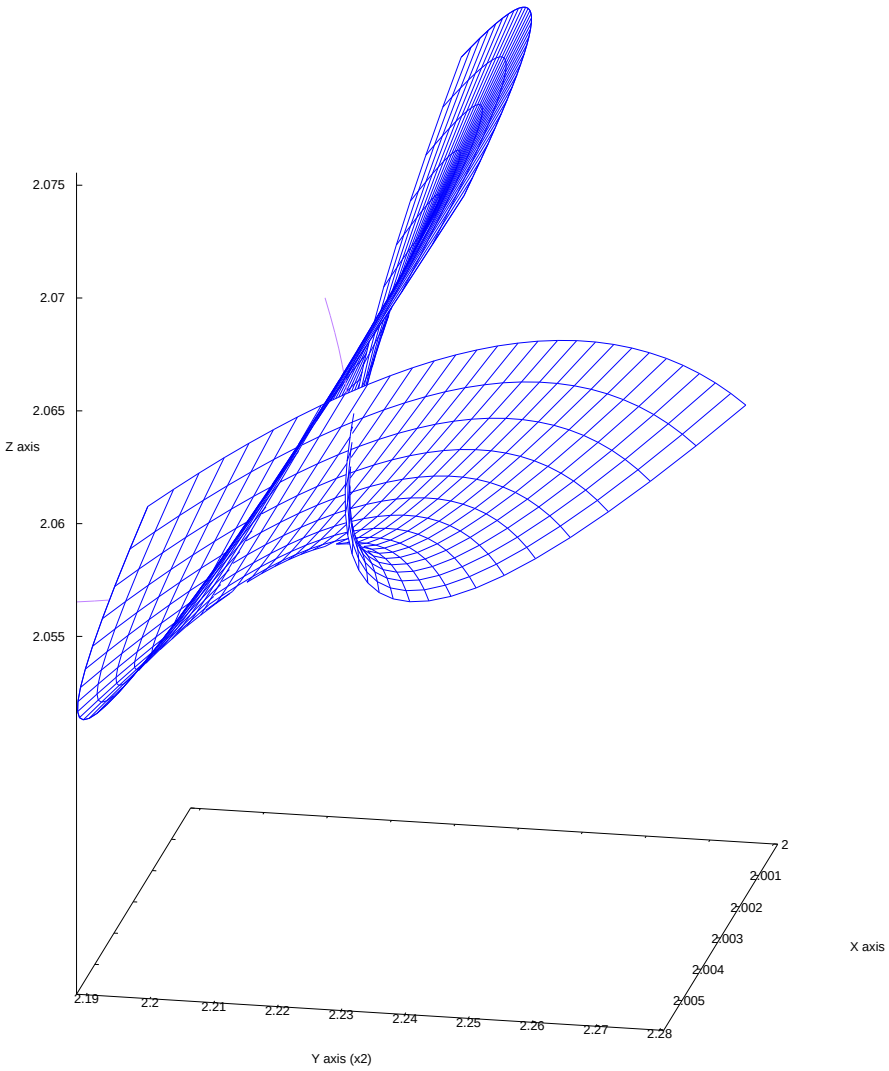

Figure 3.14. T_3 (blue) and the codim 2 component (purple), $(a, b) = (0, 9, 1.95)$, view$(77, 101)$

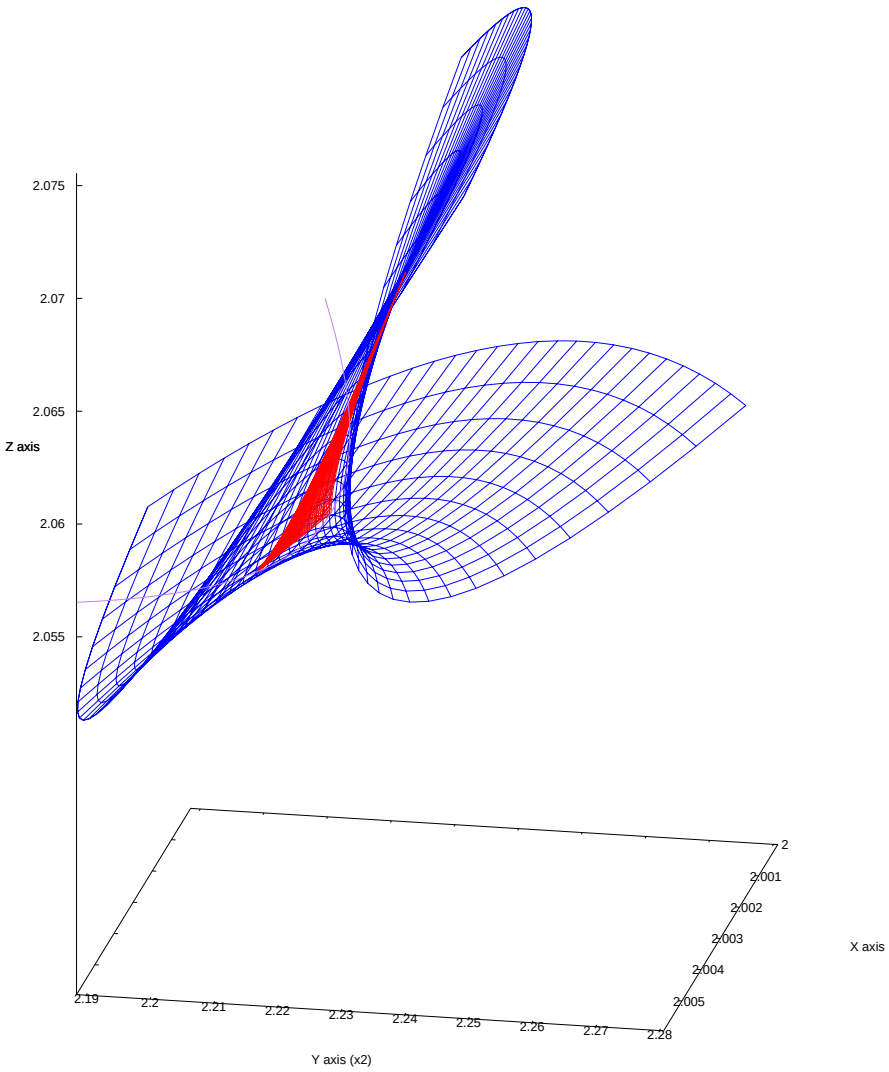

Figure 3.15. T_3 (blue) and its positive part (red), $(a, b) = (0, 9, 1.95)$, view$(77, 101)$

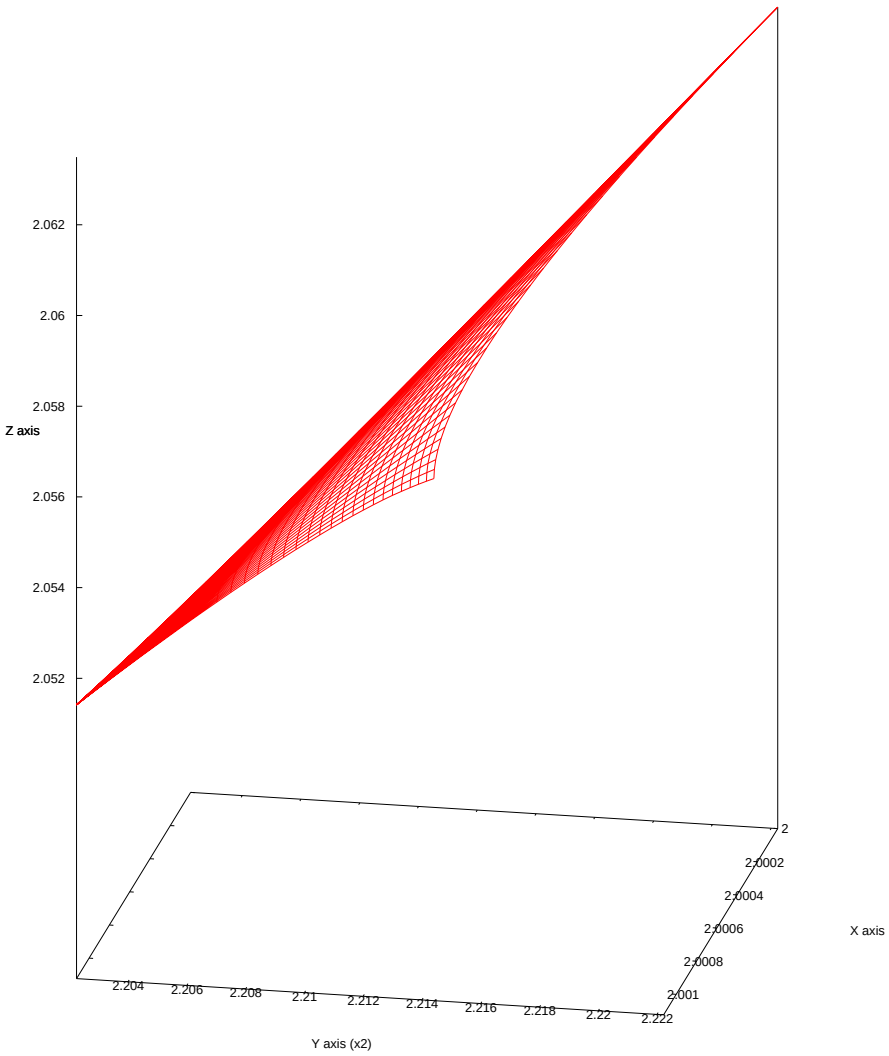

Figure 3.16. The positive part of T_3 (red), $(a, b) = (0, 9, 1.95)$, view$(77, 101)$

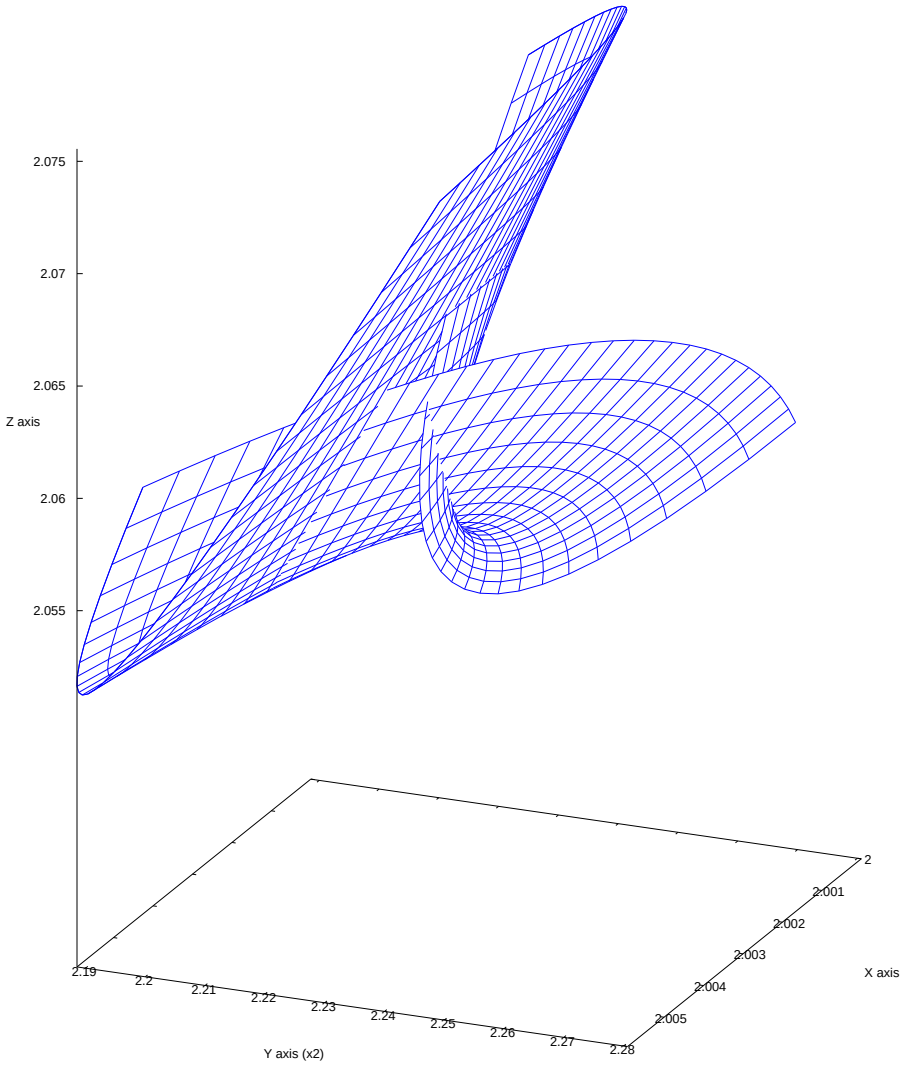

Figure 3.17. T_3, $(a, b) = (0, 9, 1.95)$, view$(76, 113)$

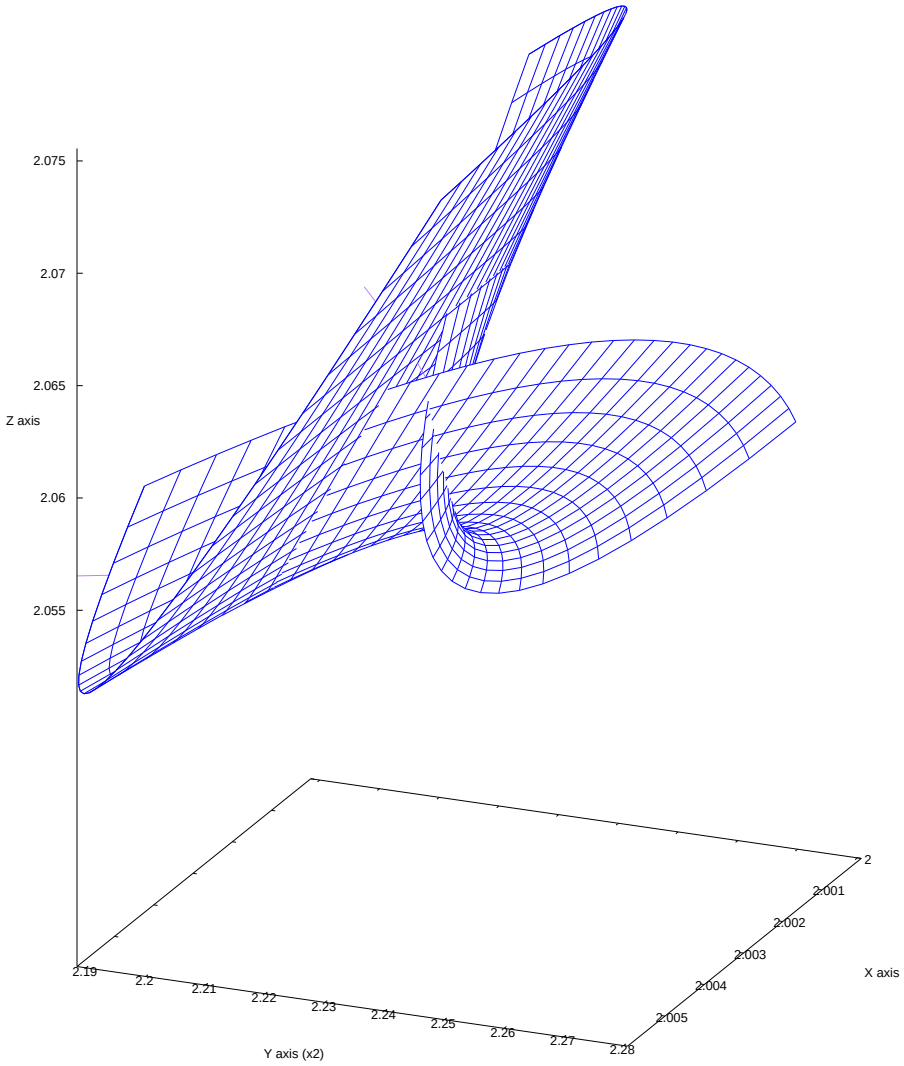

Figure 3.18. T_3 (blue) and the codim 2 component (purple), $(a, b) = (0, 9, 1.95)$, view$(76, 113)$

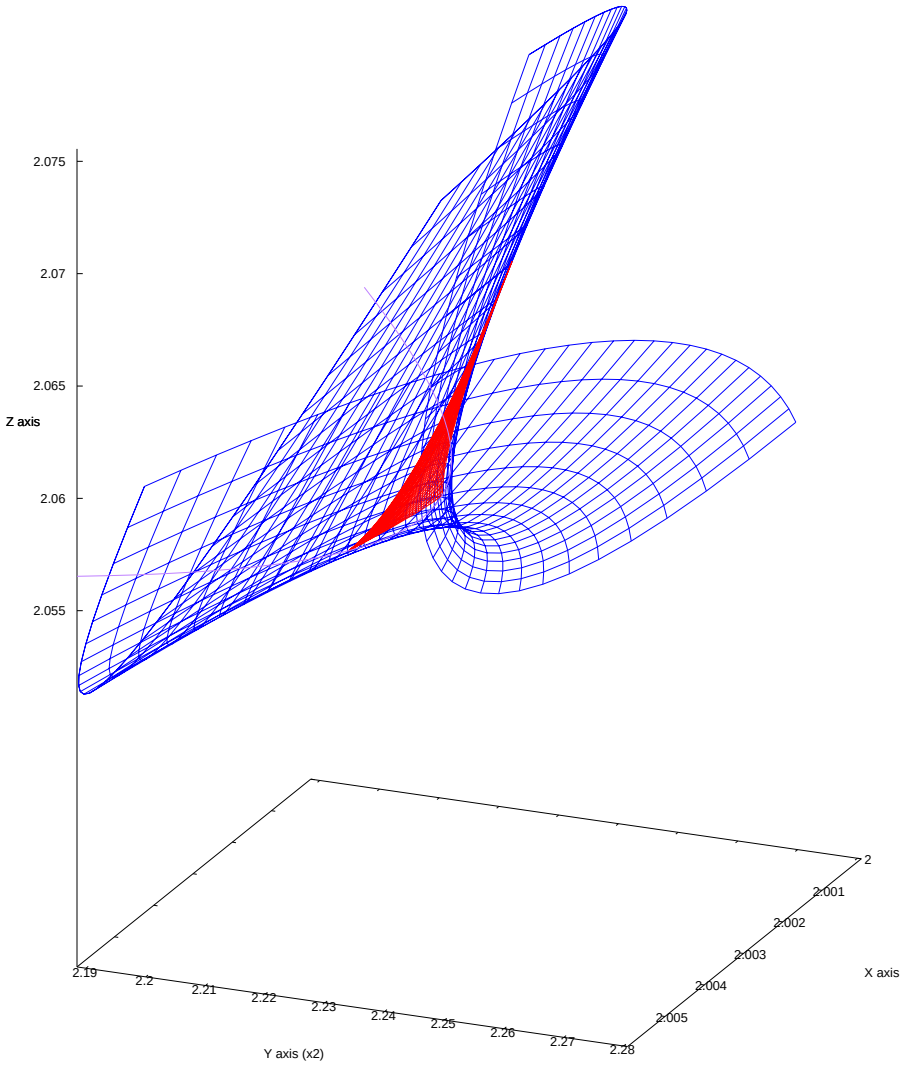

Figure 3.19. T_3 (blue) and its positive part (red), $(a, b) = (0, 9, 1.95)$, view$(76, 113)$

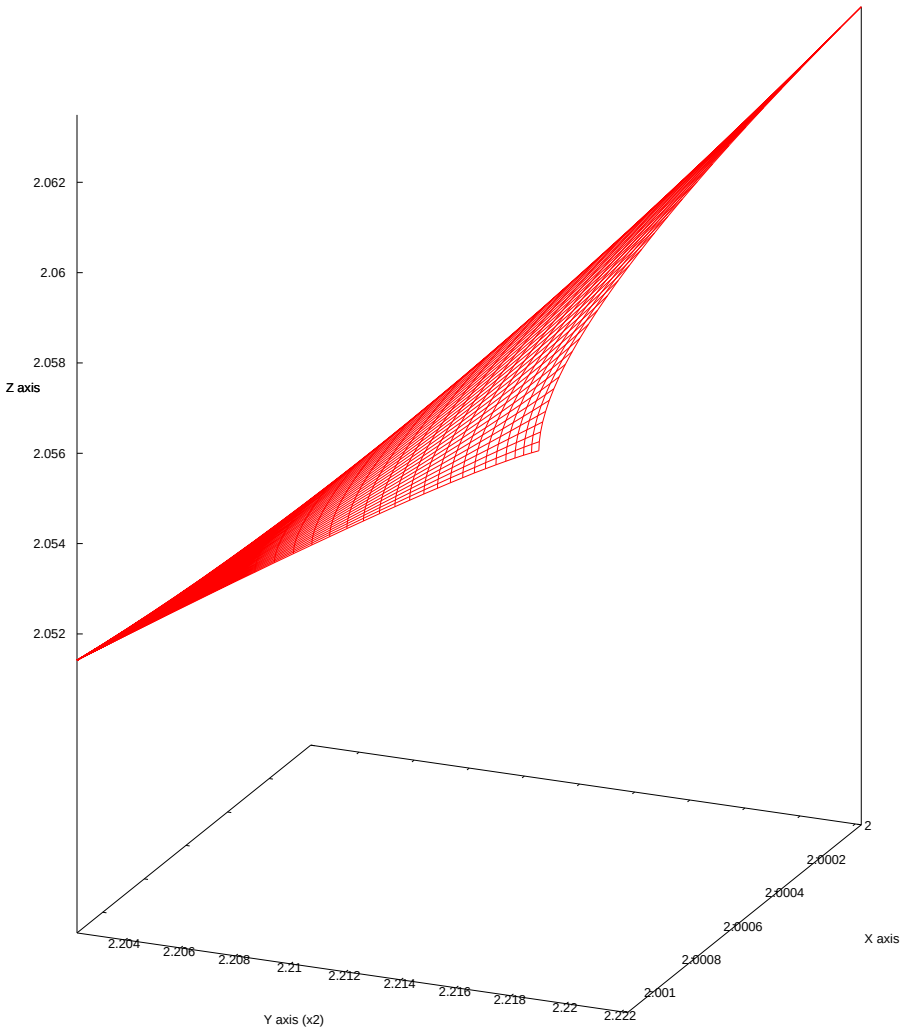

Figure 3.20. The positive part of T_3 (red), $(a, b) = (0, 9, 1.95)$, view$(76, 113)$

3.2 Landau-Nakanishi surfaces associated with T_3 and its contractions outside the exceptional set N

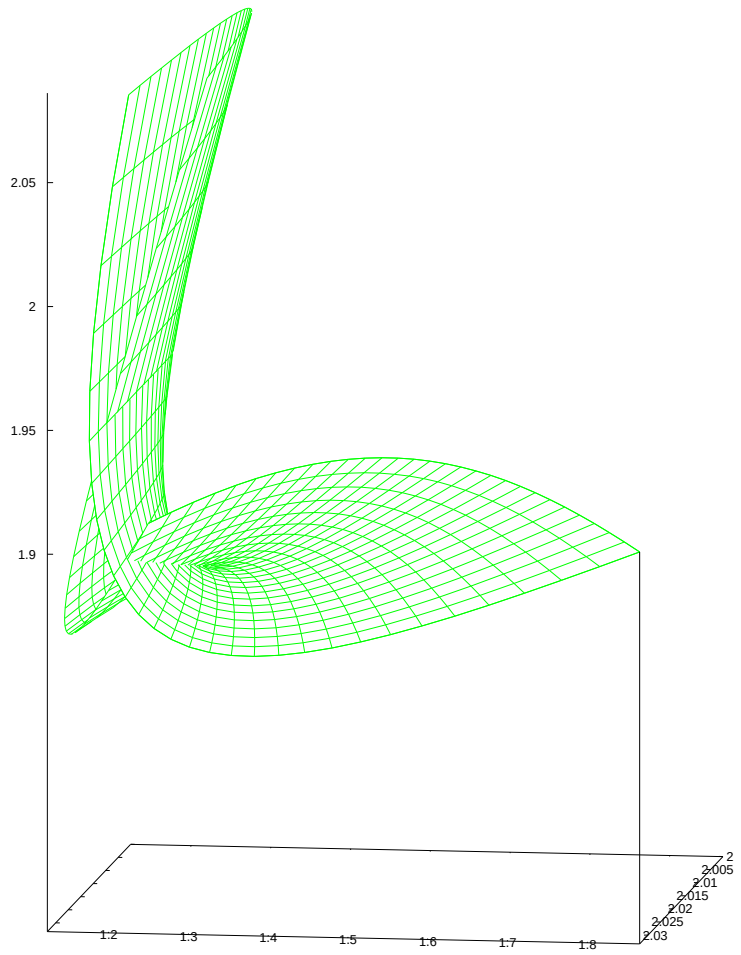

Figure 3.21. T_3 (green), $(a, b) = (0.85, 2.15)$, view$(84, 98)$

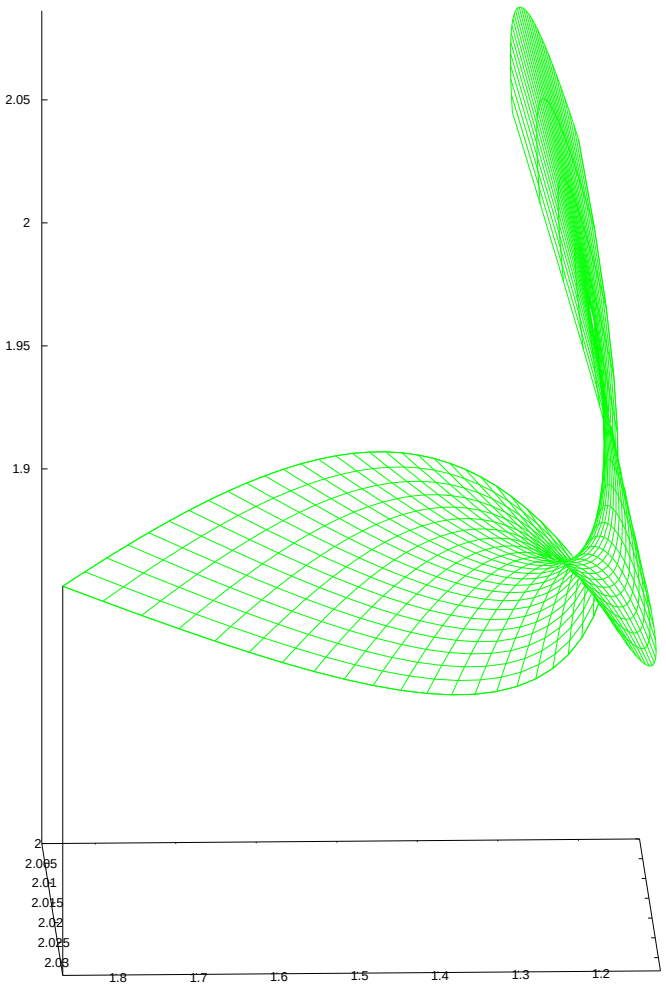

Figure 3.22. T_3 (green), $(a, b) = (0.85, 2.15)$, view$(99, 272)$

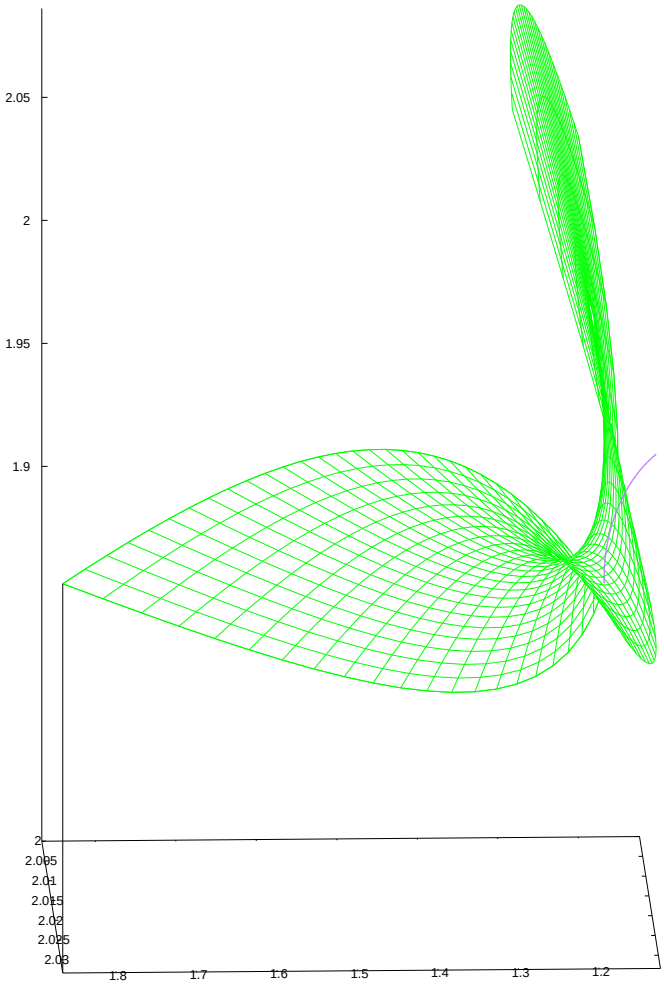

Figure 3.23. T_3 (green) and the codim 2 component (purple), $(a, b) = (0.85, 2.15)$, view$(99, 272)$.

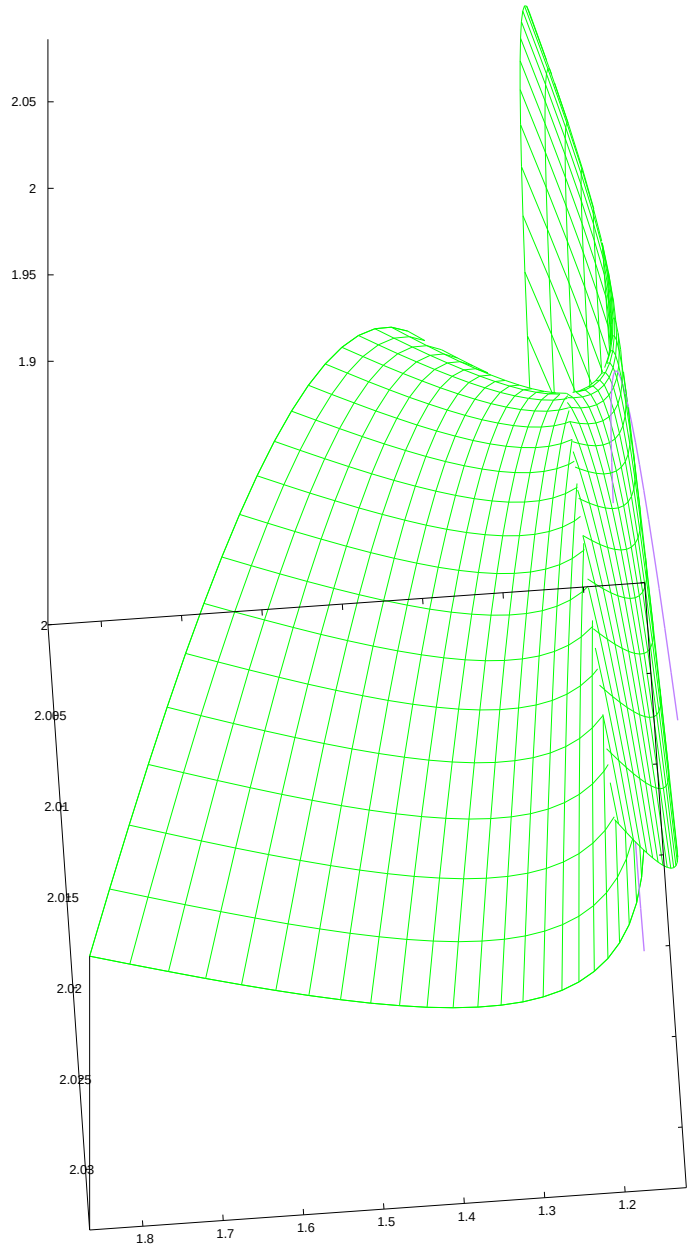

Figure 3.24. T_3 (green) and the codim 2 component (purple), $(a, b) = (0.85, 2.15)$, view$(136, 274)$.

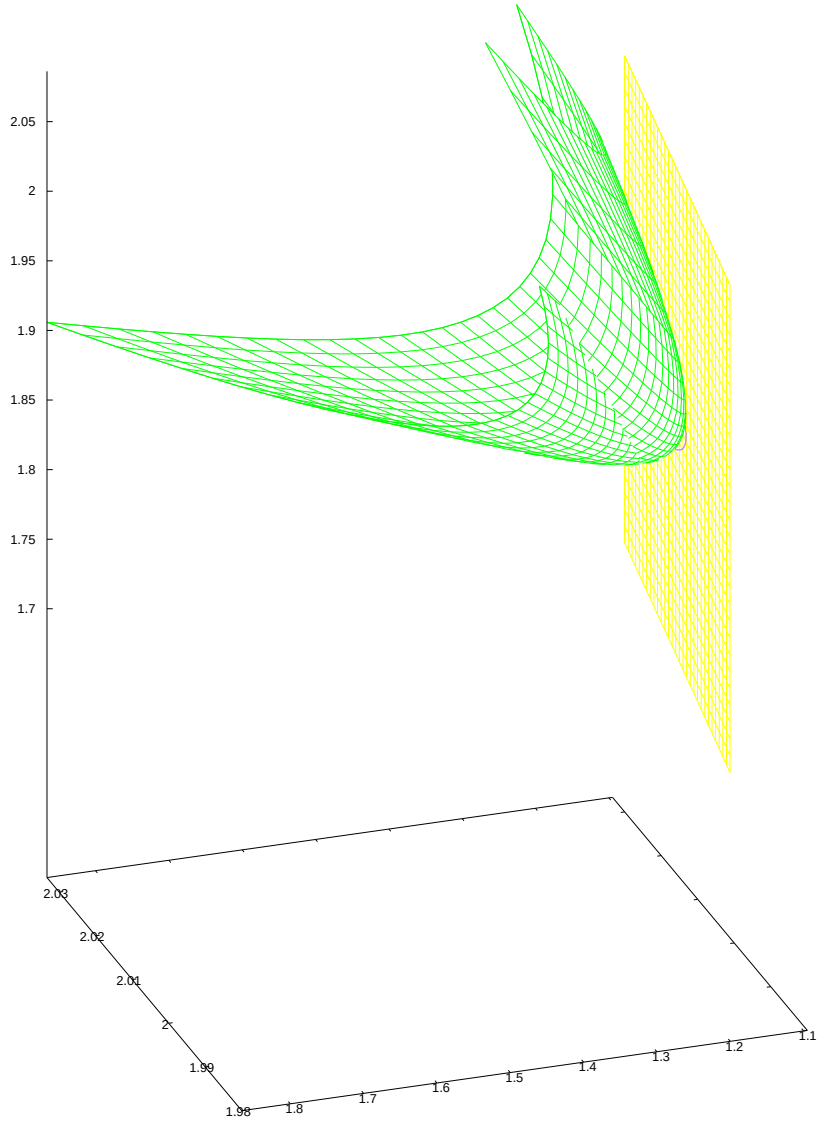

Figure 3.25. T_3 (green) and $3PT$ (yellow), $(a, b) = (0.85, 2.15)$, view$(73, 251)$.

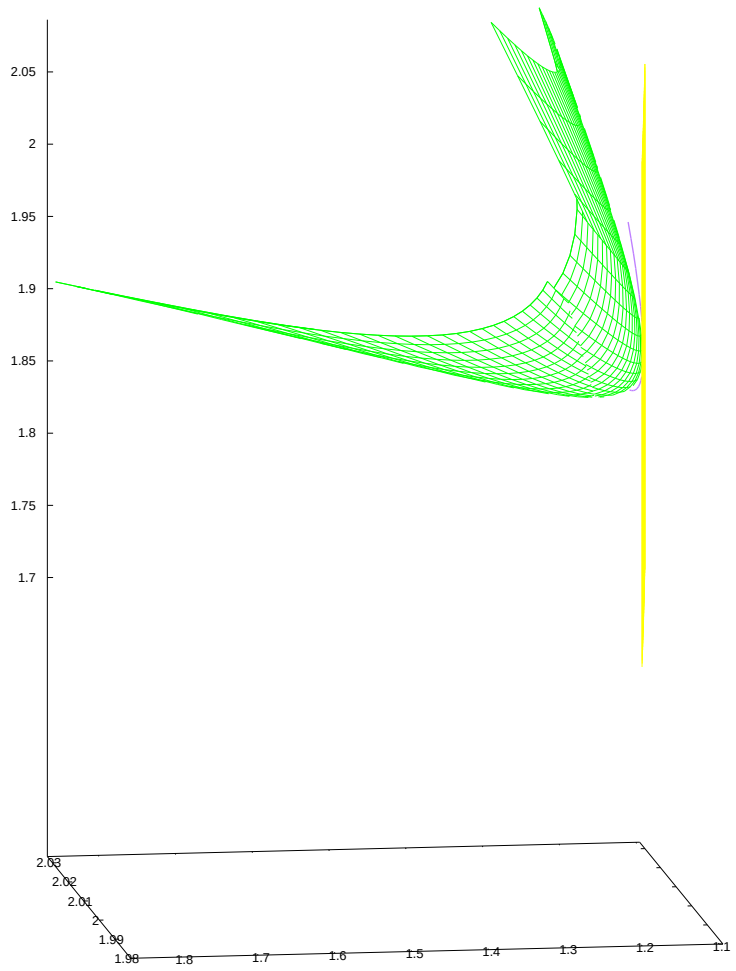

Figure 3.26. T_3 (green) and $3PT$ (yellow), $(a, b) = (0.85, 2.15)$, view$(83, 262)$.

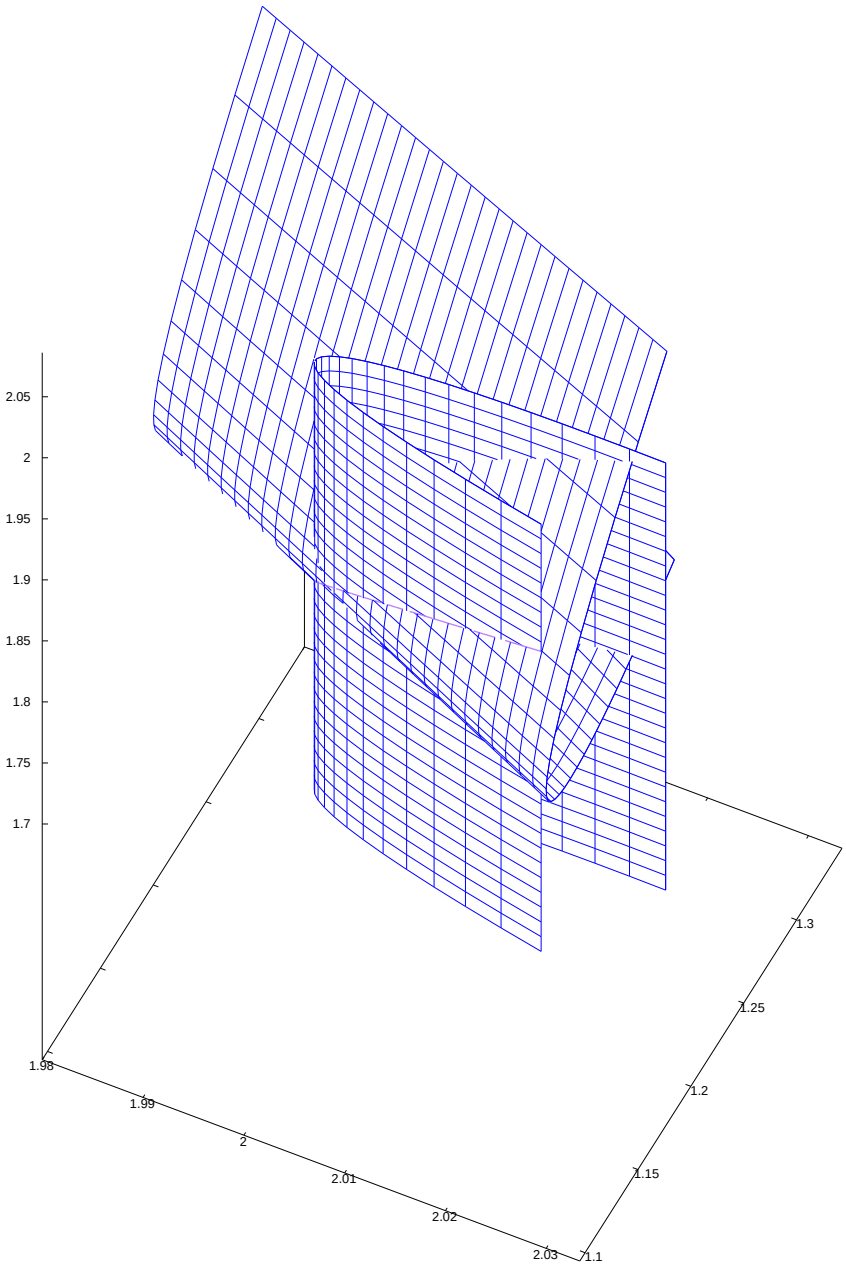

Figure 3.27. The ice-cream cones (blue) and the codim 2 component (purple), $(a, b) = (0.85, 2.15)$, view$(57, 26)$.

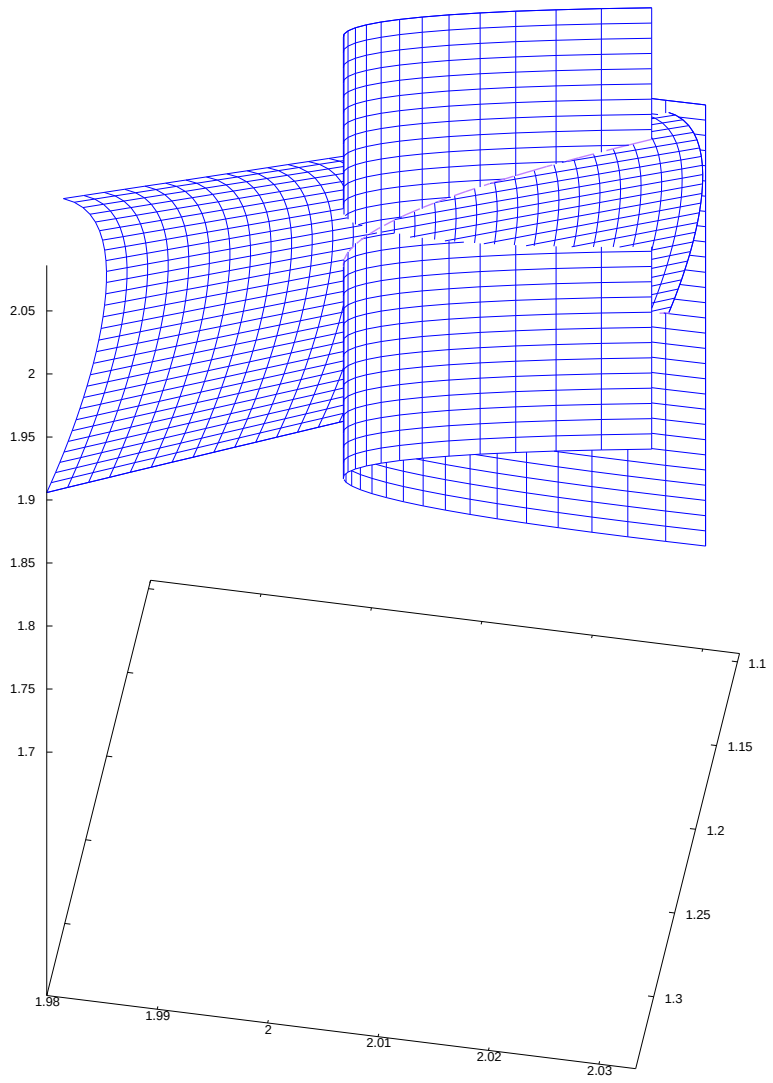

Figure 3.28. The ice-cream cones (blue) and the codim 2 component (purple), $(a, b) = (0.85, 2.15)$, view$(120, 350)$.

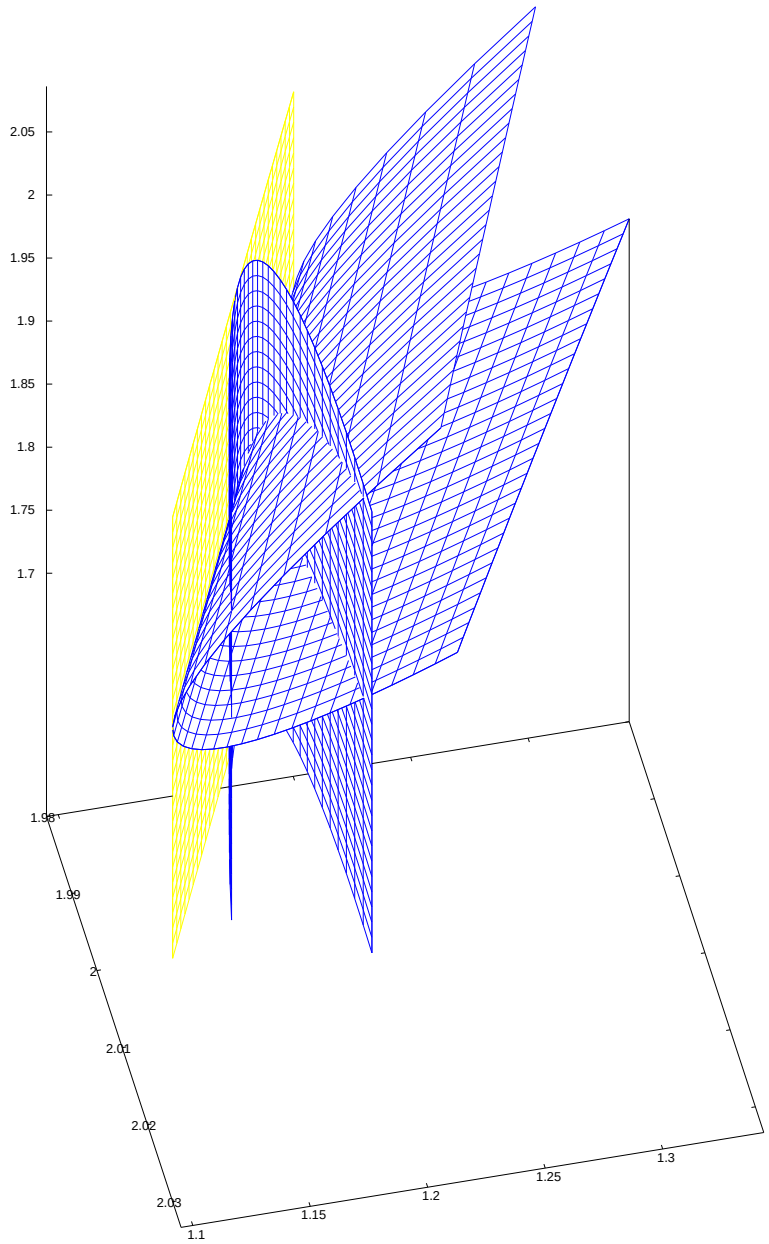

Figure 3.29. The ice-cream cones (blue) and $3PT$ (yellows), $(a, b) = (0.85, 2.15)$, view$(60, 77)$.

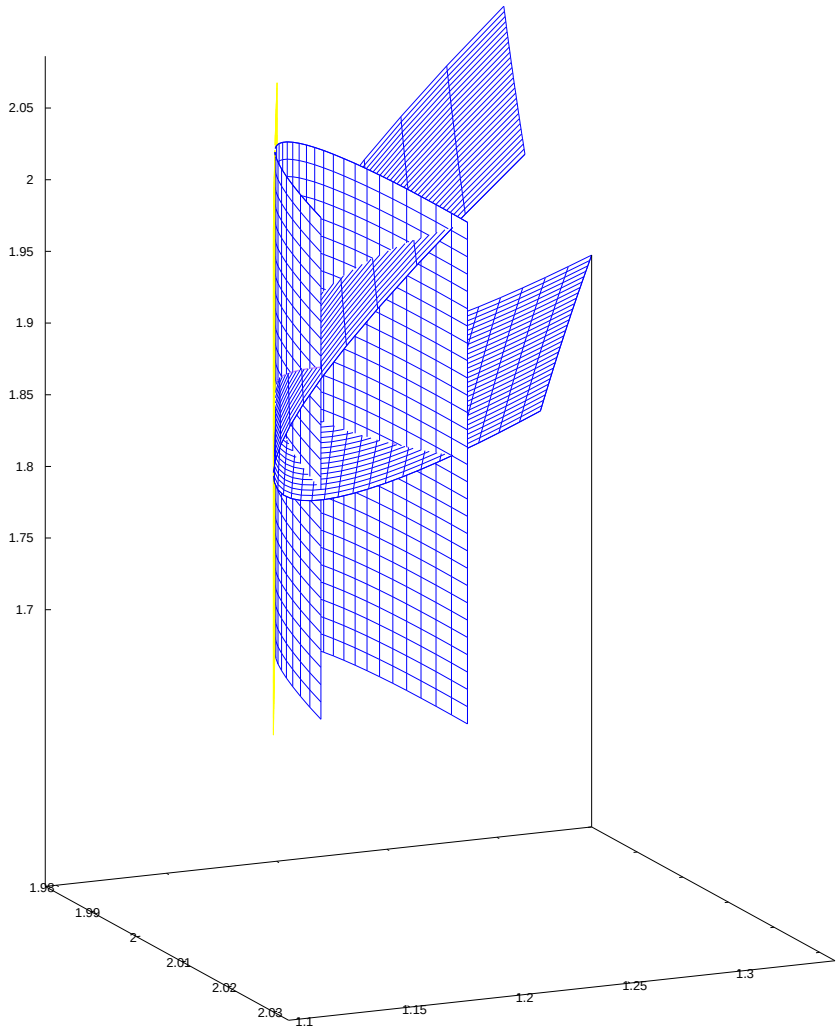

Figure 3.30. The ice-cream cones (blue) and $3PT$ (yellows), $(a, b) = (0.85, 2.15)$, view$(80, 66)$.

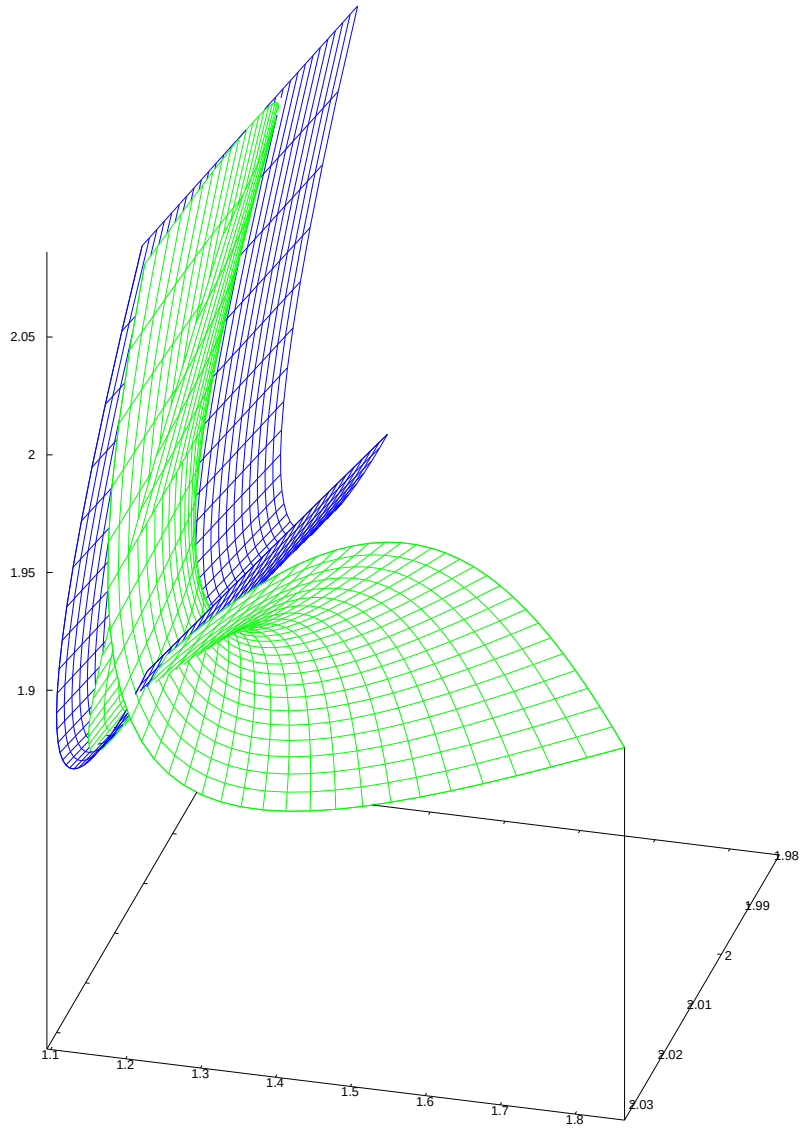

Figure 3.31. The right ice-cream cone (blue) and T_3 (green), $(a, b) = (0.85, 2.15)$, view$(71, 105)$.

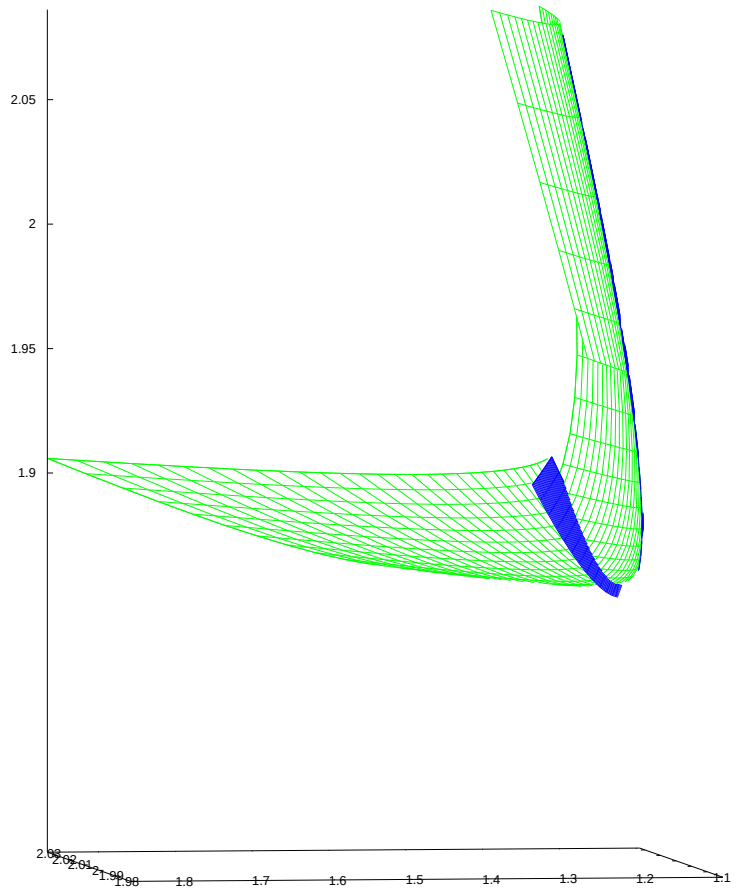

Figure 3.32. The right ice-cream cone (blue) and T_3 (green), $(a, b) = (0.85, 2.15)$, view$(88, 262)$.

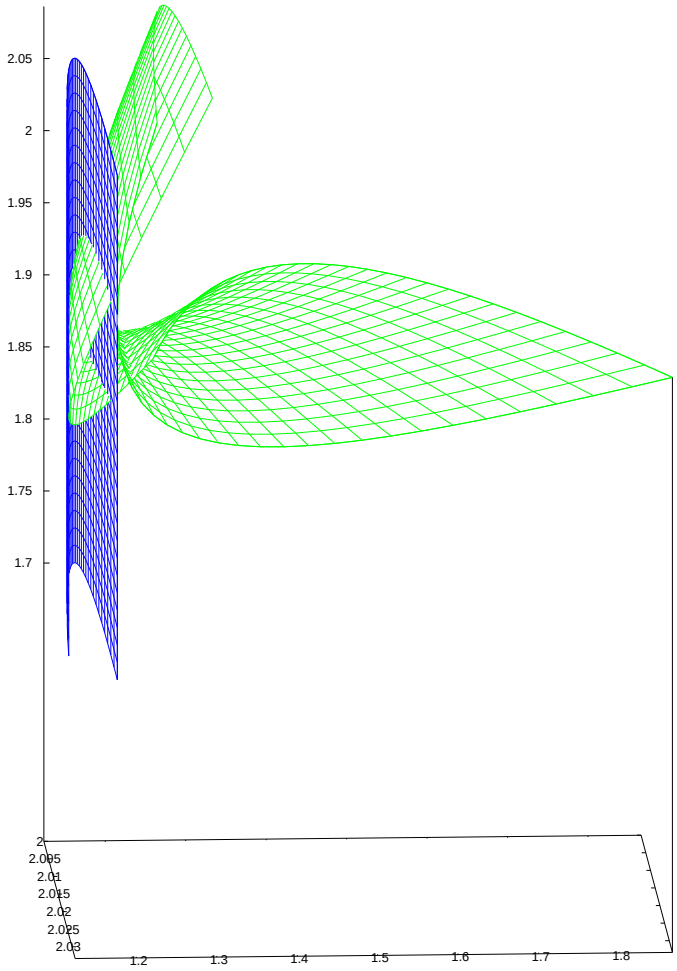

Figure 3.33. The left ice-cream cone (blue) and T_3 (green), $(a, b) = (0.85, 2.15)$, view$(82, 87)$.

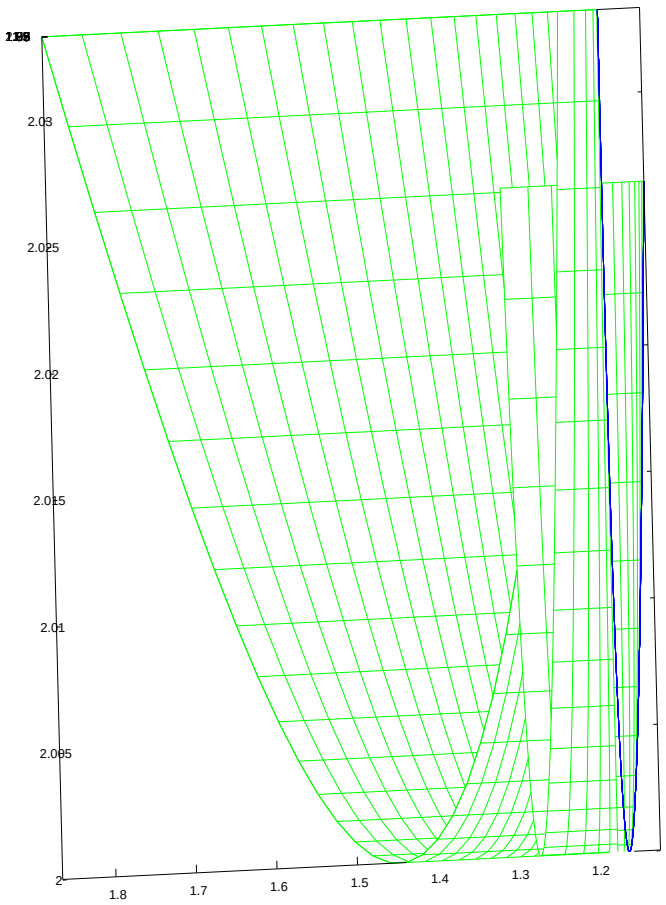

Figure 3.34. The left ice-cream cone (blue) and T_3 (green), $(a, b) = (0.85, 2.15)$, view$(0, 268)$.

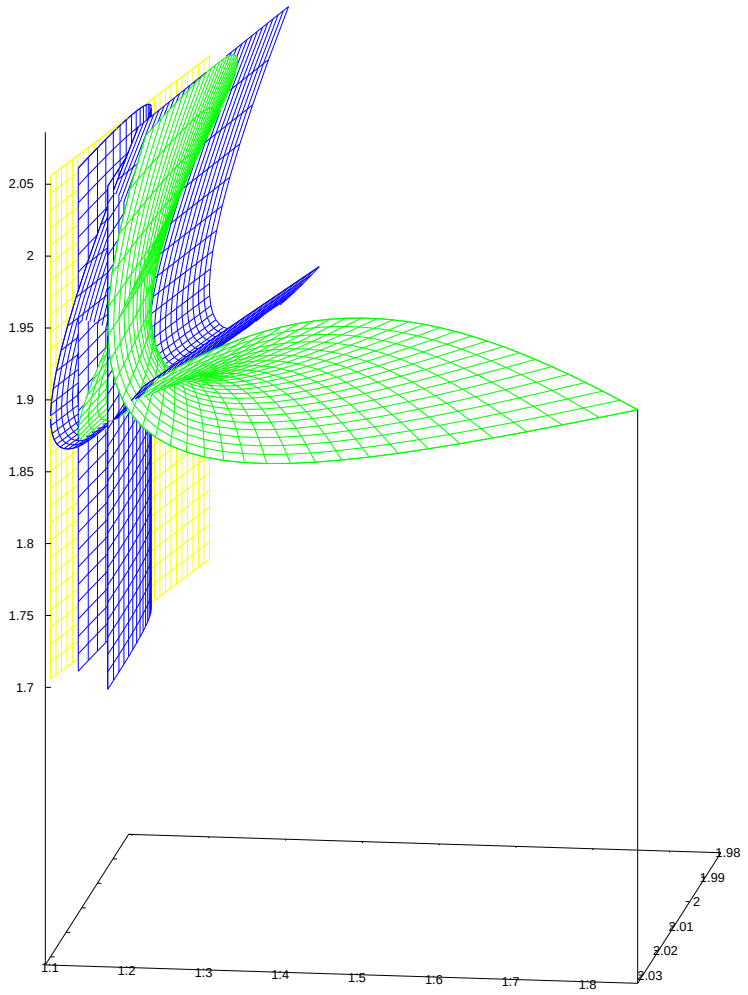

Figure 3.35. The ice-cream cones (blue), $3PT$ (yellow) and T_3 (green), $(a, b) = (0.85, 2.15)$, view$(81, 98)$.

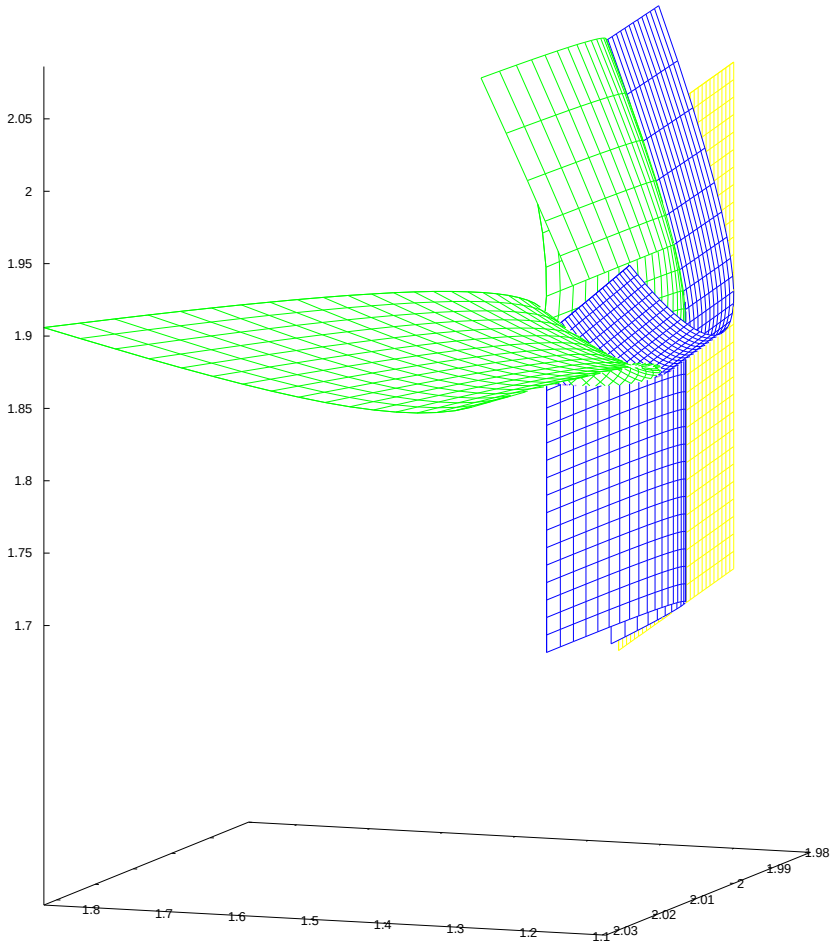

Figure 3.36. The ice-cream cones (blue), $3PT$ (yellow) and T_3 (green), $(a, b) = (0.85, 2.15)$, view$(96, 250)$.

3.3 Landau-Nakanishi surfaces associated with T_3 and its contractions near the exceptional set N

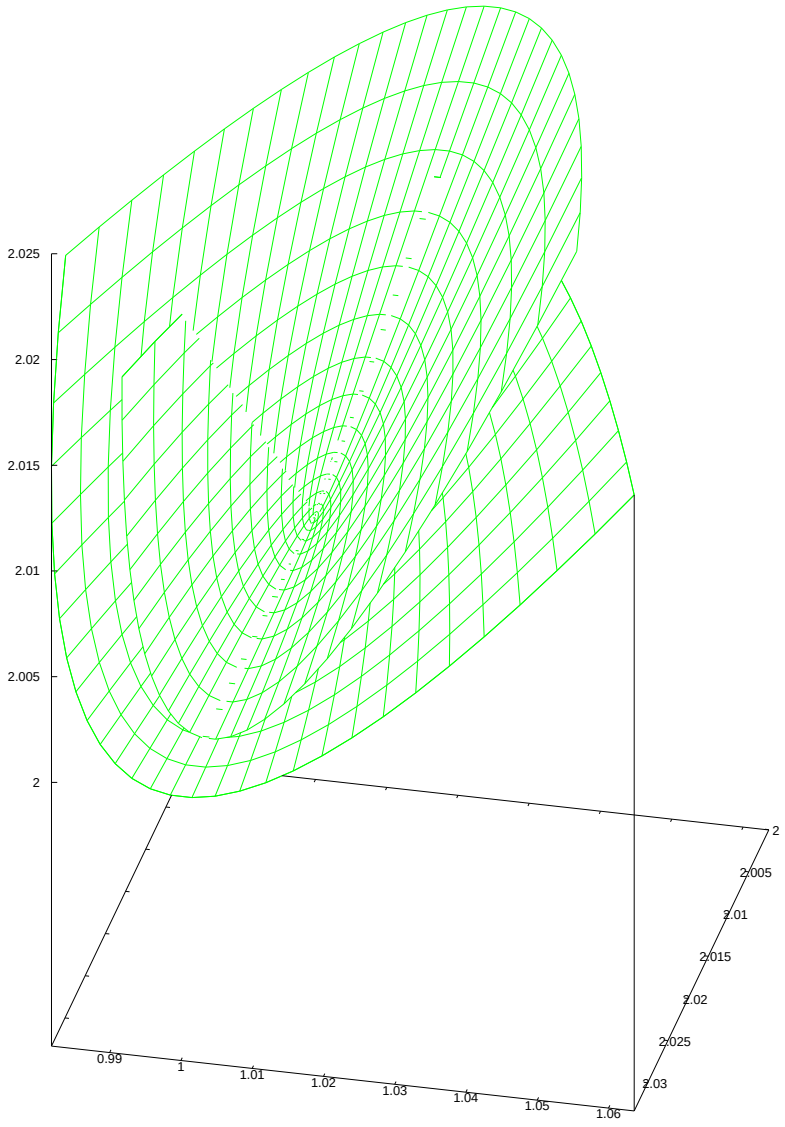

Figure 3.37. T_3 (green), $(a, b) = (1.0, 2.0)$, view$(73, 103)$

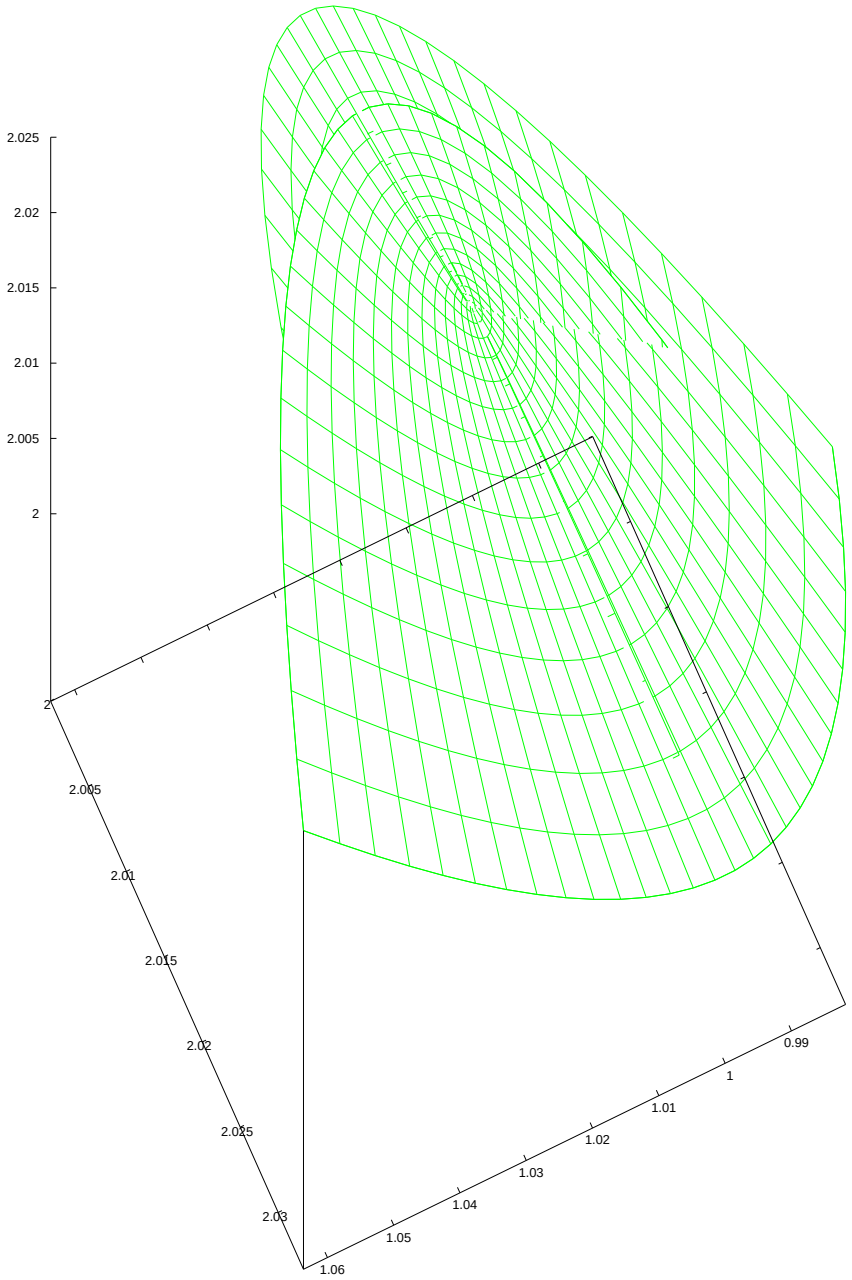

Figure 3.38. T_3 (green), $(a, b) = (1.0, 2.0)$, view$(138, 295)$

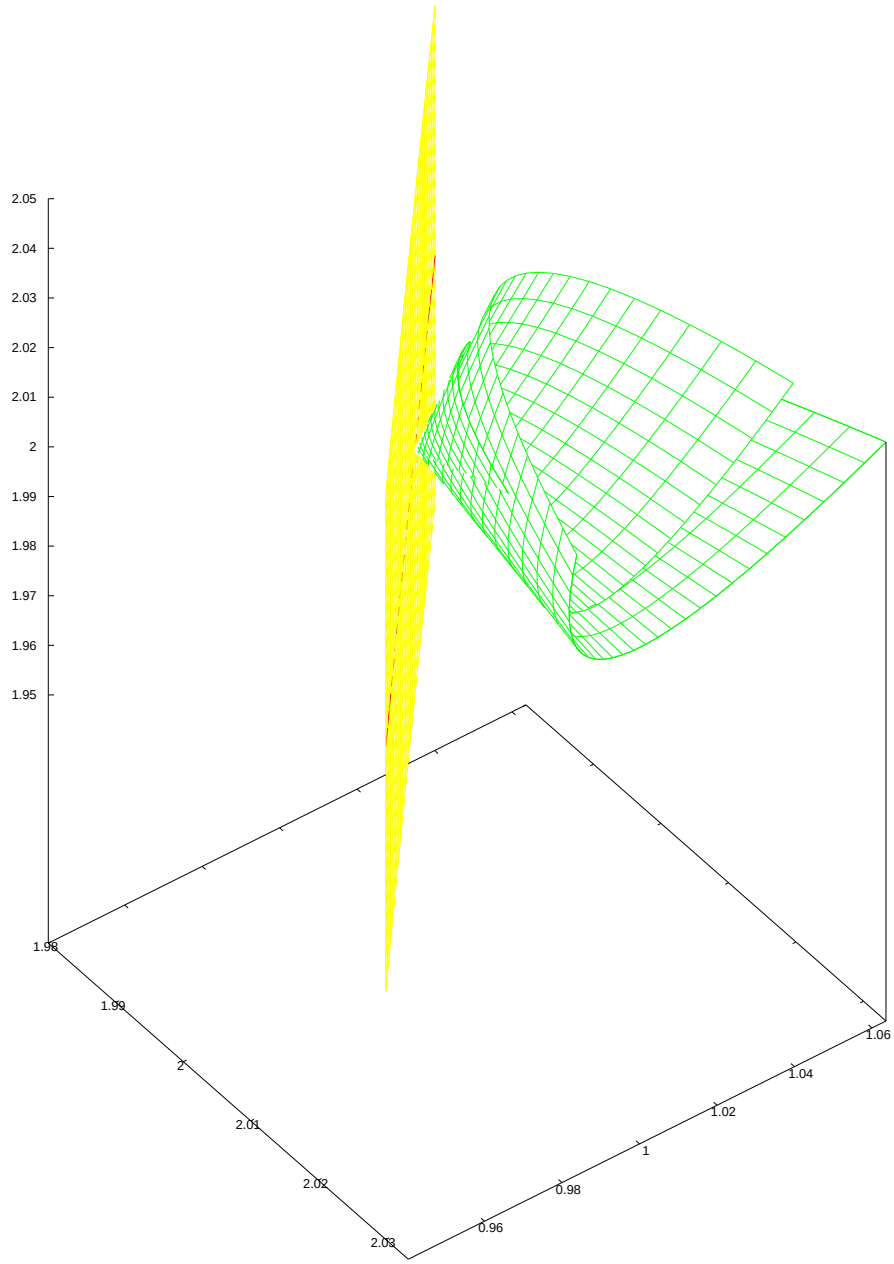

Figure 3.39. T_3 (green) and $3PT$ (yellow), $(a, b) = (1.0, 2.0)$, view$(62, 53)$

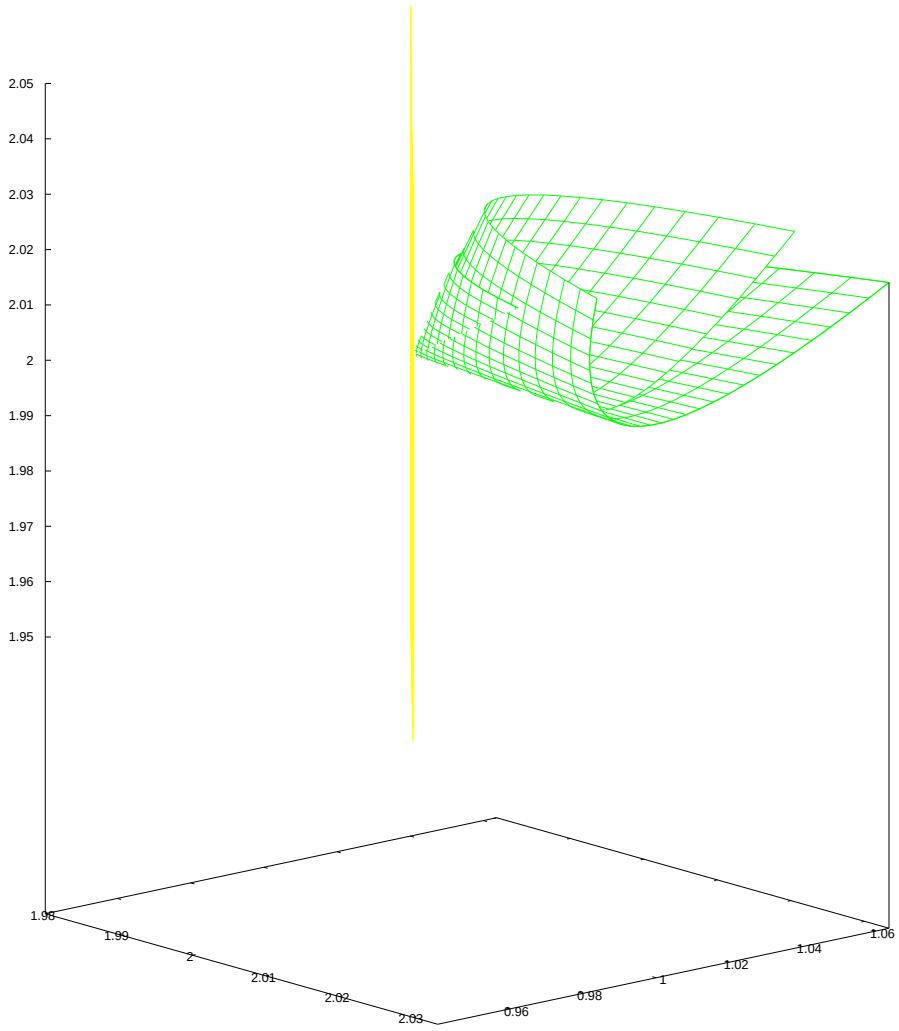

Figure 3.40. T_3 (green) and $3PT$ (yellow), $(a, b) = (1.0, 2.0)$, view$(80, 49)$

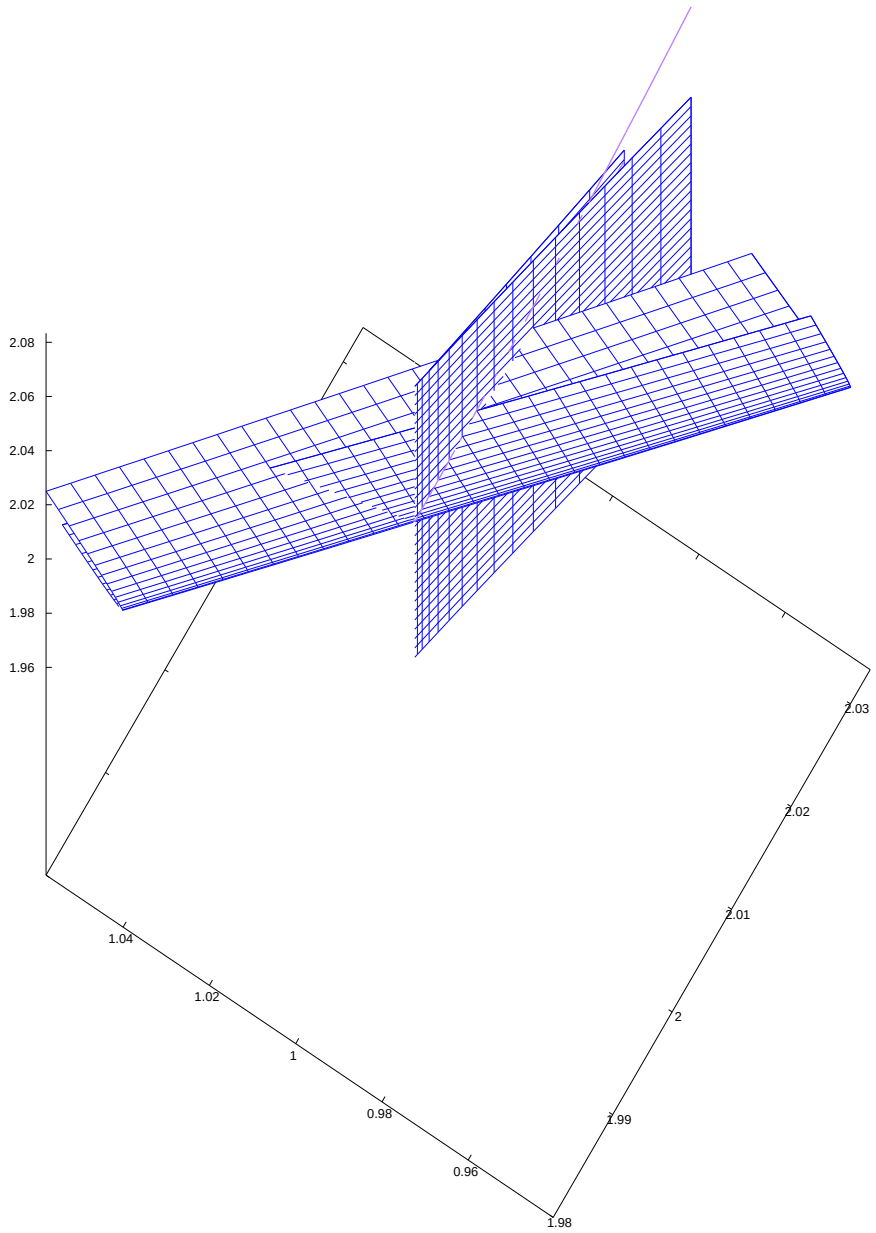

Figure 3.41. The ice-cream cones (blue) and the codim 2 component (purple), $(a, b) = (1.0, 2.0)$, view$(40, 302)$

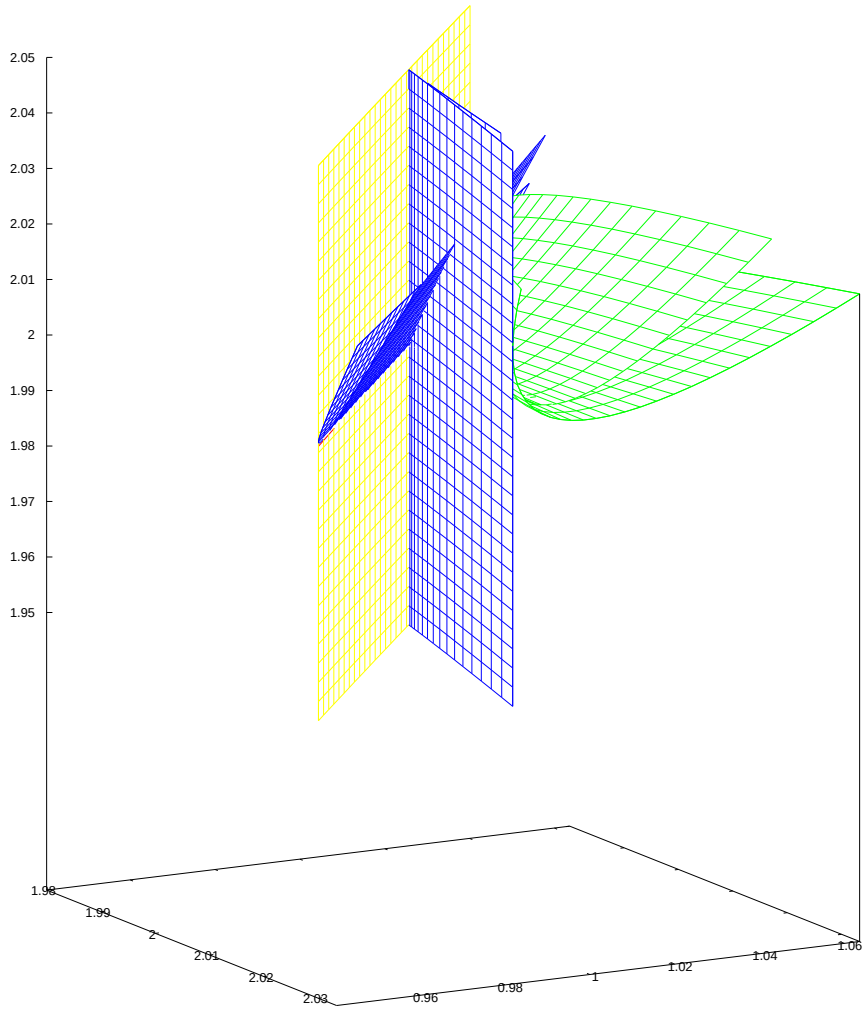

Figure 3.42. The ice-cream cones (blue), $3PT$ (yellow) and T_3 (green), $(a, b) = (1.0, 2.0)$, view$(81, 61)$

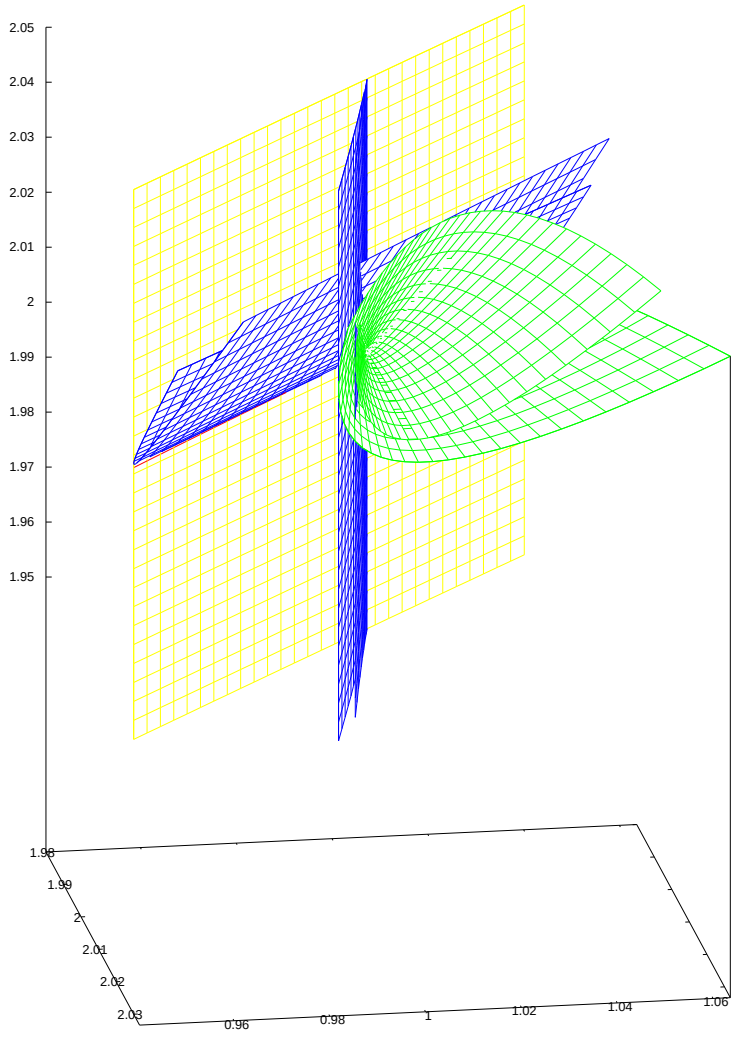

Figure 3.43. The ice-cream cones (blue), $3PT$ (yellow) and T_3 (green), $(a, b) = (1.0, 2.0)$, view$(78, 81)$

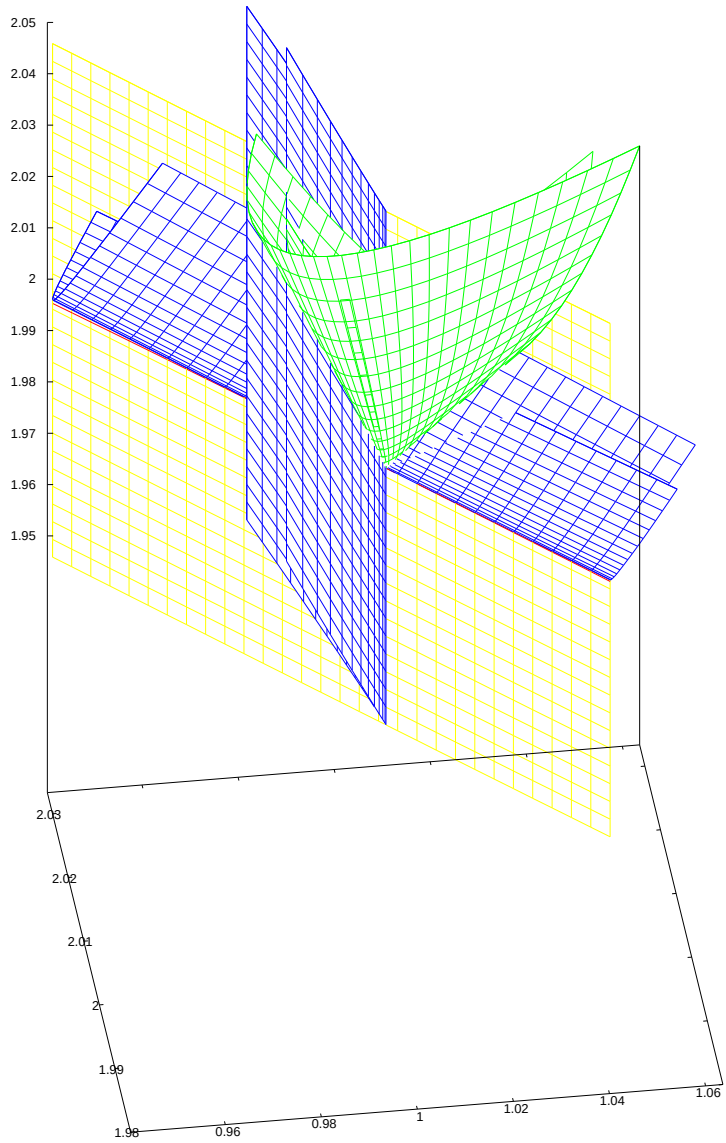

Figure 3.44. The ice-cream cones (blue), $3PT$ (yellow) and T_3 (green), $(a, b) = (1.0, 2.0)$, view$(114, 98)$

4. Enlarged figures of those in [HKS]

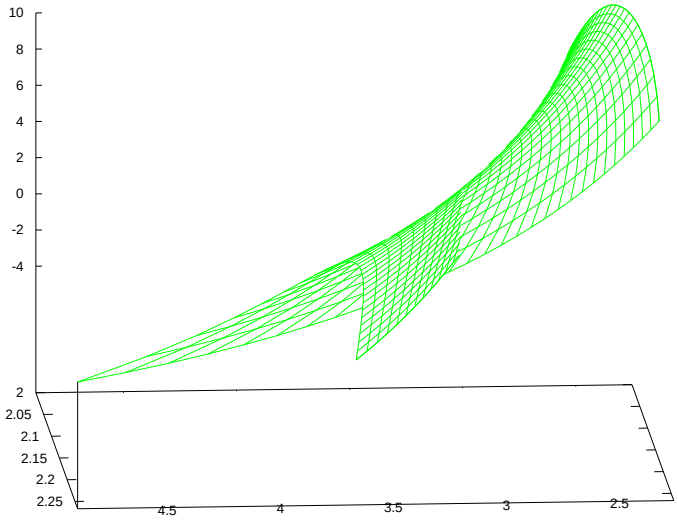

Figure 4.1. Figure 5.2 in [HKS]

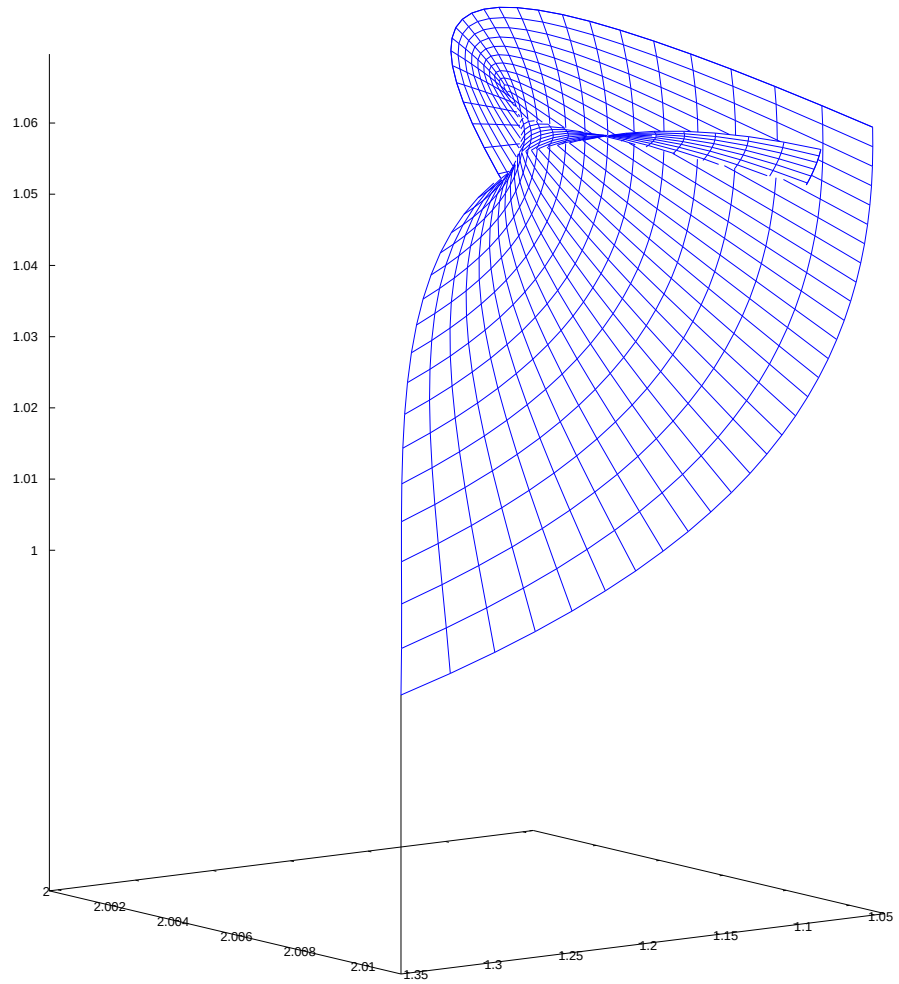

Figure 4.2. Figure 5.5 in [HKS]

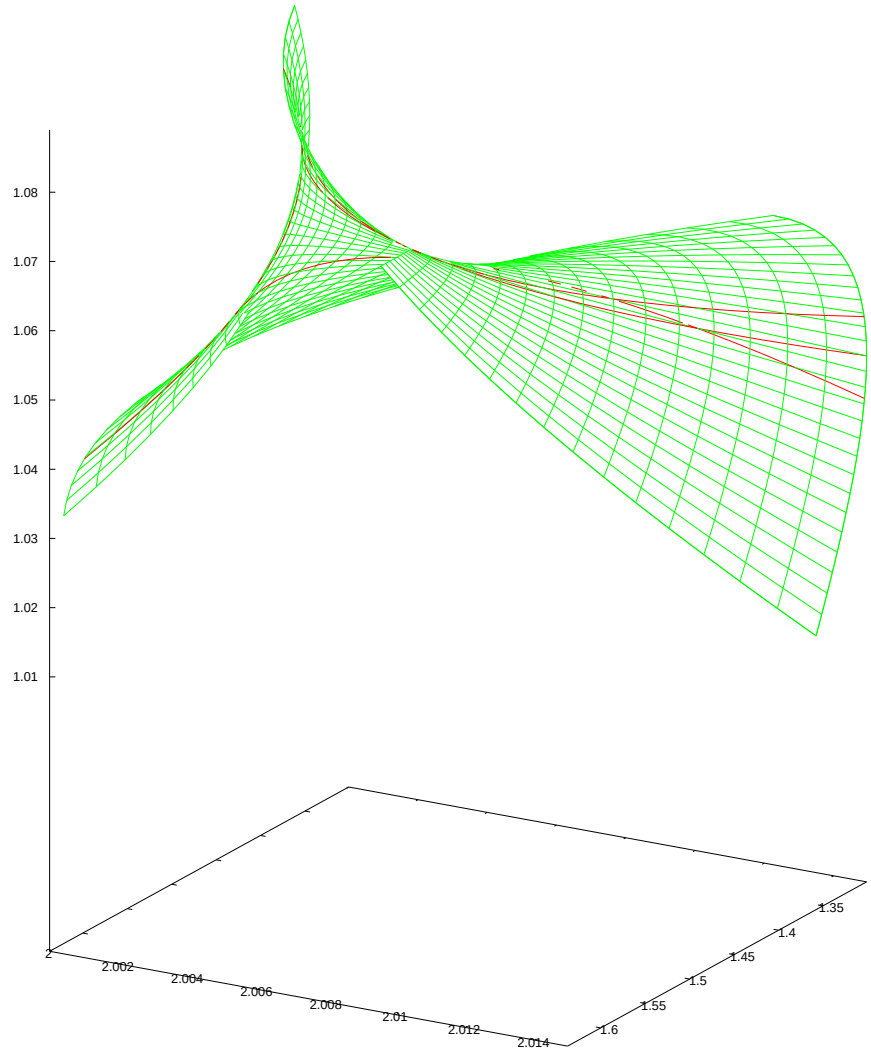

Figure 4.3. Figure 5.6 in [HKS]

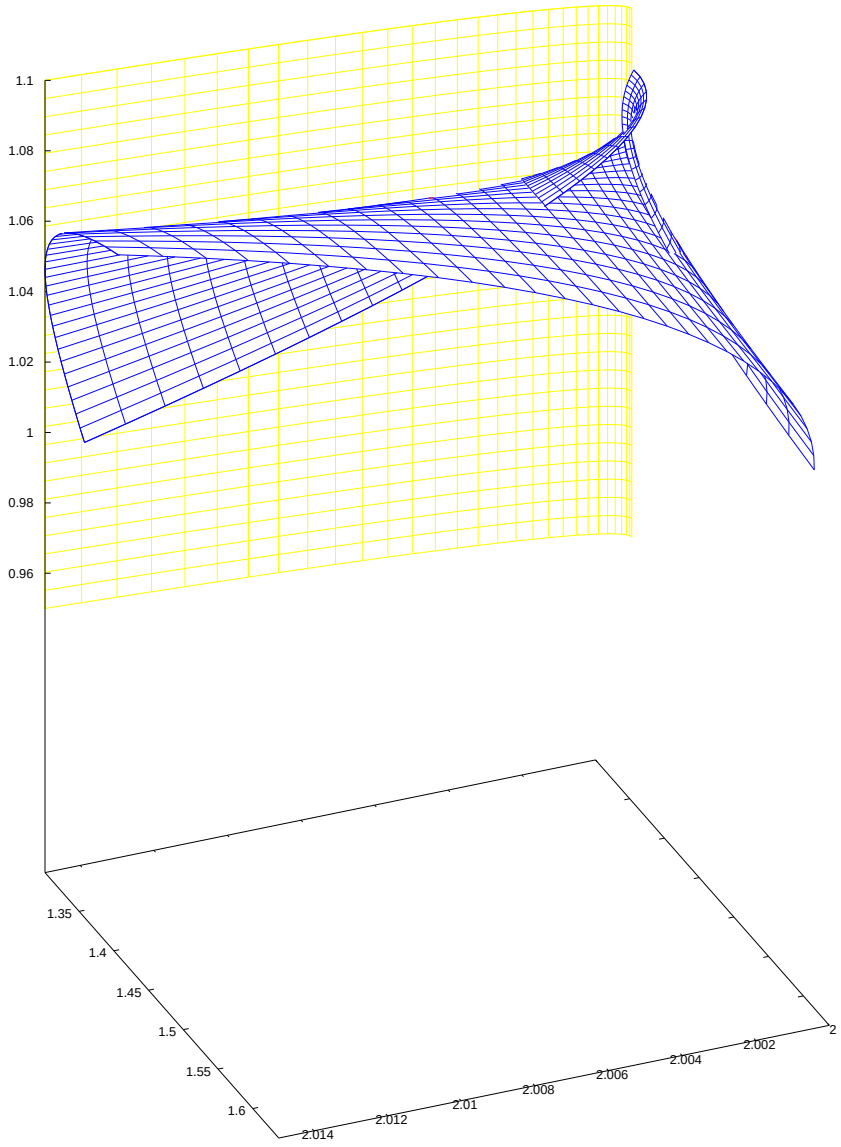

Figure 4.4. Figure 5.13 in [HKS]

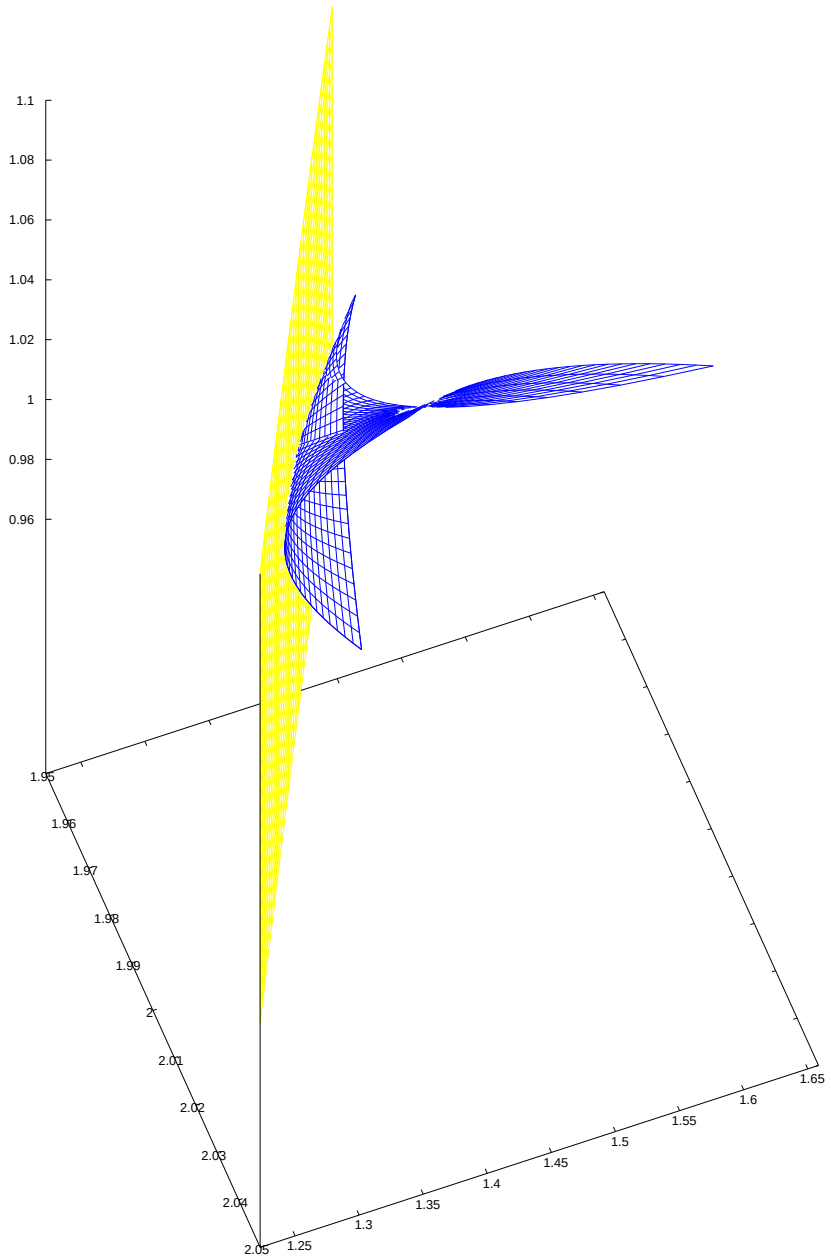

Figure 4.5. Figure 5.14 in [HKS]

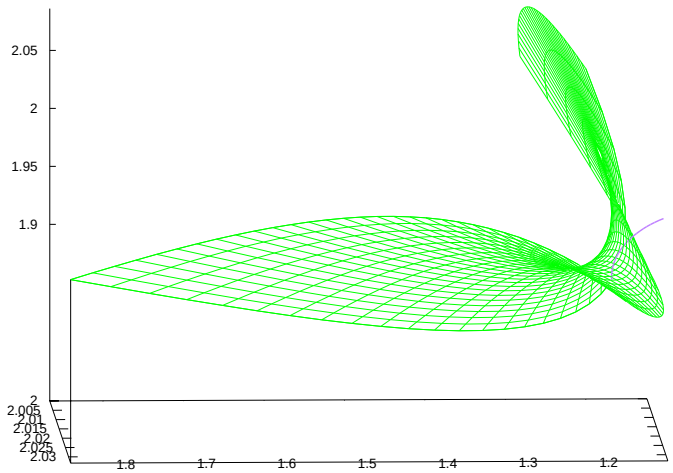

Figure 4.6. Figure 5.15 in [HKS]

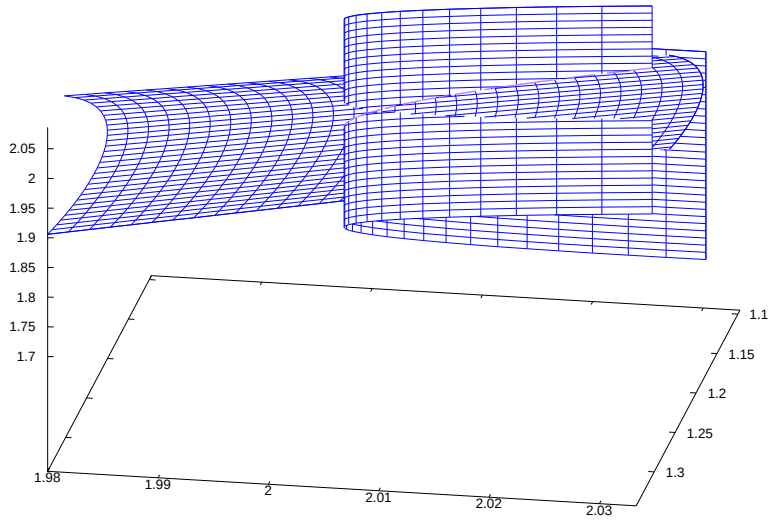

Figure 4.7. Figure 5.19 in [HKS]

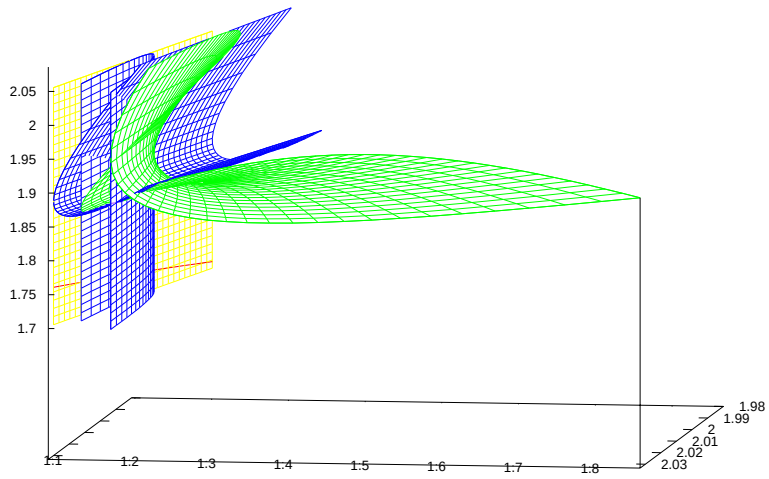

Figure 4.8. Figure 5.20 in [HKS]

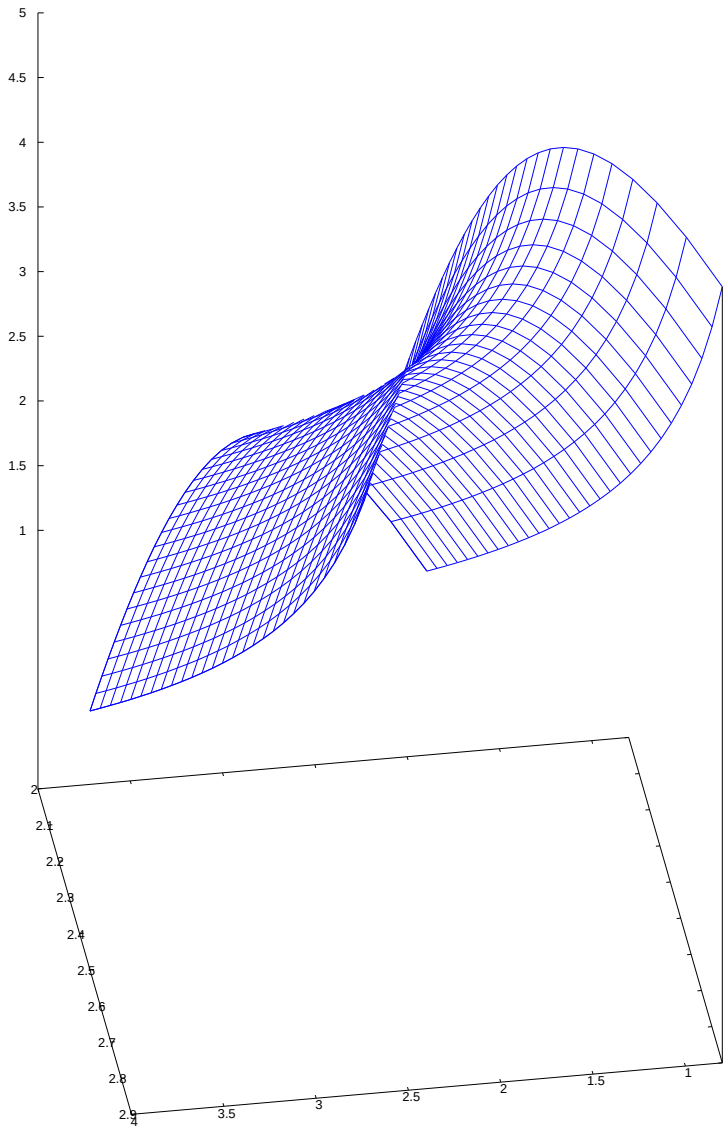

Figure 4.9. Figure A.2 in [HKS]

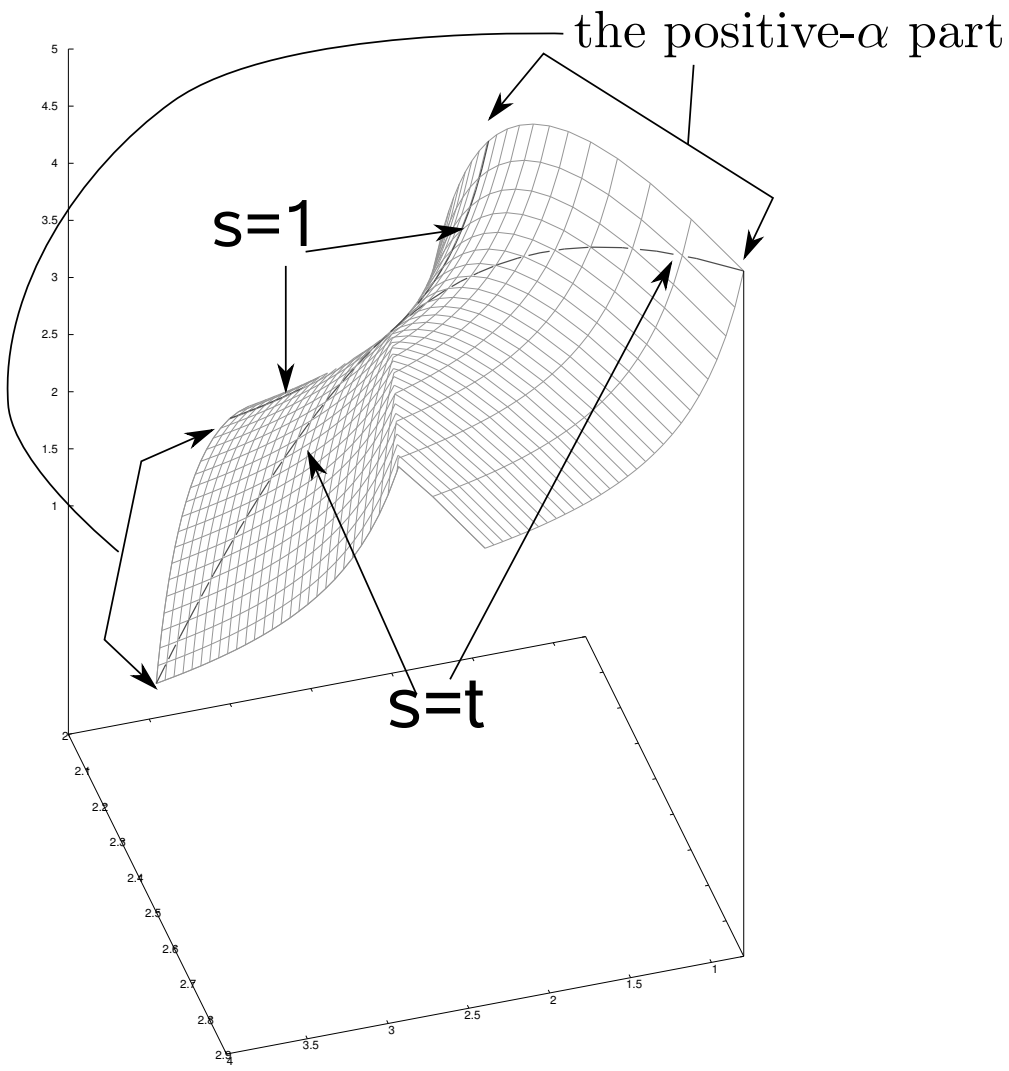

Figure 4.10. Figure A.3 in [HKS]

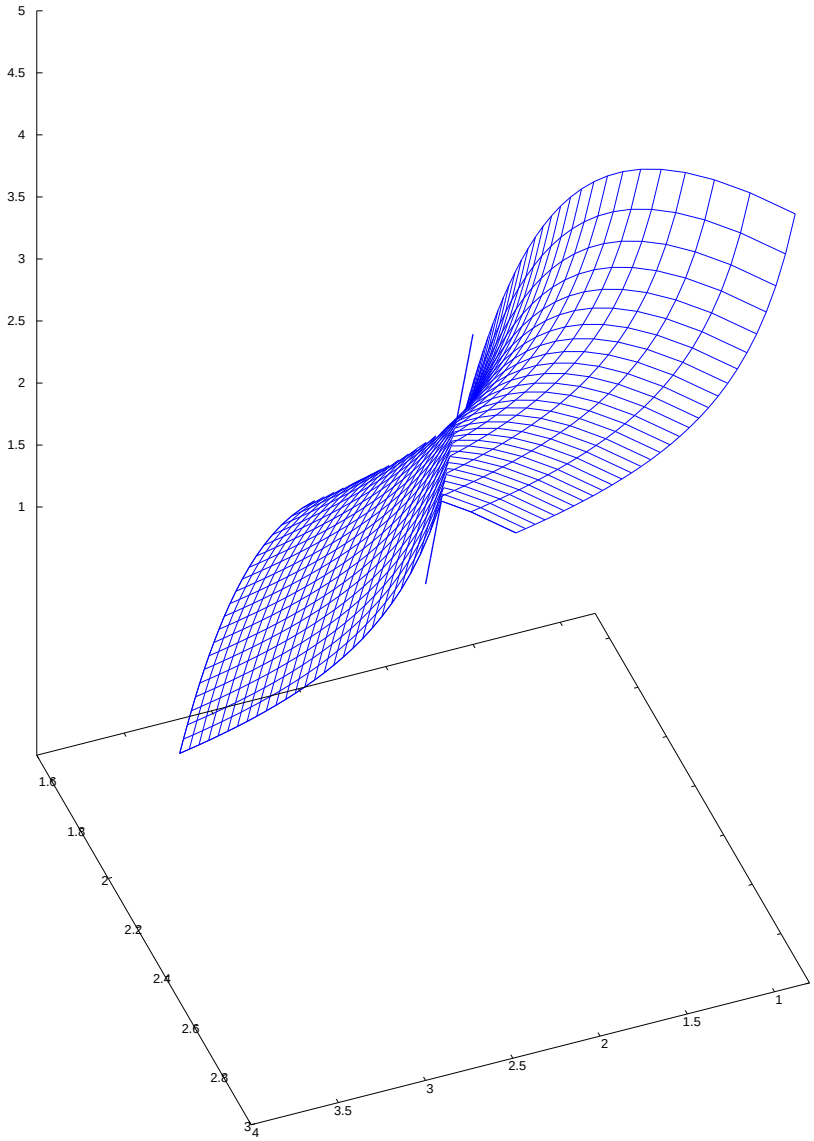

Figure 4.11. Figure A.5 in [HKS]

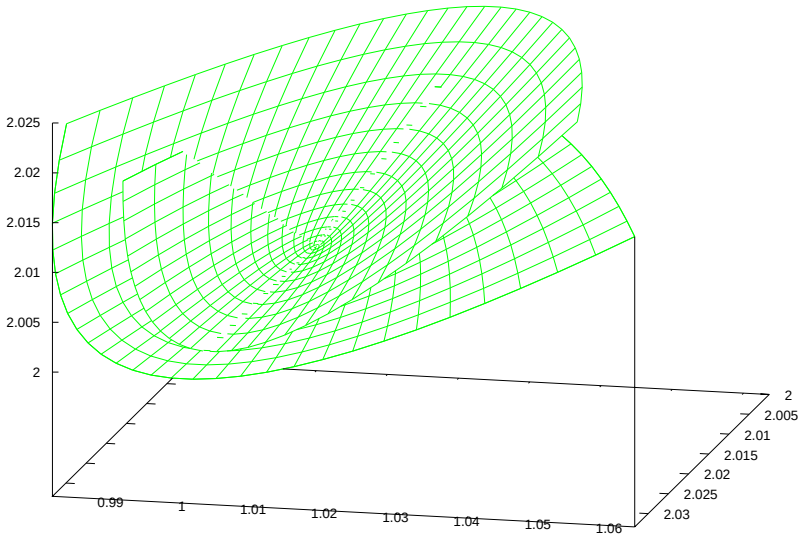

Figure 4.12. Figure B.1 in [HKS]

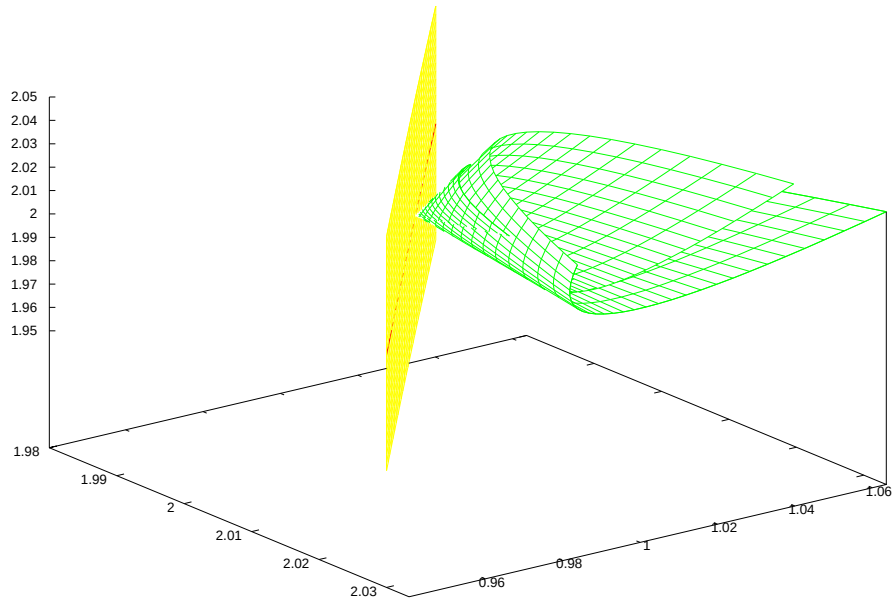

Figure 4.13. Figure B.2 in [HKS]

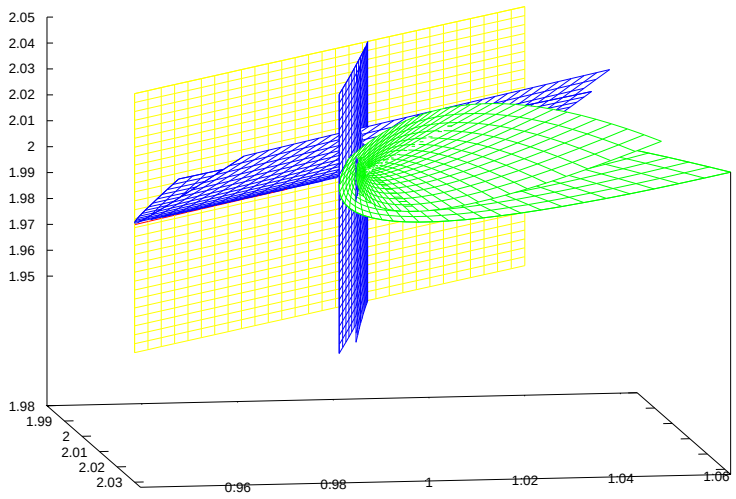

Figure 4.14. Figure B.3 in [HKS]