Workshop on Non-compact Variational Problems and Related Topics (at RIMS)

RIMS Research Project

organizers: Futoshi Takahashi (Osaka Metropolitan University)
Michinori Ishiwata (Osaka University)

Date : 2022, September, 26, 13:25 – September, 28, 12:10

Venue : Research Institute for Mathematical Sciences (RIMS) Room 420
Kyoto University
Kitashirakawaoiwake-cho, Sakyo-ku, Kyoto-shi

26, September

13:25∼13:30 Opening

13:30∼14:30 Bernhard Ruf （Universitá di Milano）
On an inequality of Bliss–Moser type

14:40∼15:40 Hiroaki Kikuchi (Tsuda College)
Uniqueness of ground states for double power nonlinear Schrödinger equations

15:50∼16:50 Asuka Takatsu (Tokyo Metropolitan University)
Hierarchy of entropies associated with deformed logarithmic functions

27, September

10:00∼11:00 Luca Martinazzi （Universitá di Roma “La Sapienza”）
Critical points of the Moser-Trudinger functional on closed surfaces

11:10∼12:10 Yohei Tsutsui (Kyoto University)
Rearrangements, medians and their maximal functions

13:30∼14:30 Masato Hashizume (Hiroshima University)
A power type approximation of Moser–Trudinger inequality

14:40∼15:40 Norihisa Ikoma (Keio University)
The existence and nonexistence of weak solutions to the Born-Infeld equation

15:50∼16:50 Michiaki Onodera (Tokyo Institute of Technology)
Quantitative stability in overdetermined problems

28, September

10:00∼11:00 Tastuya Miura (Tokyo Institute of Technology)
Li–Yau type inequality for curves and applications

11:10∼12:10 Yoshitsugu Kabeya (Osaka Metropolitan University)
Longitudinal bifurcating solutions to a nonlinear elliptic equation on a large spherical cap

Bernhard Ruf
(Universitá di Milano)

On an inequality of Bliss–Moser type

Abstract: We derive a limiting inequality for the integral inequalities by Bliss. We then consider a critical version of this inequality which is of Moser type, and discuss related non-compactness properties.

Hiroaki Kikuchi
(Tsuda College)

Uniqueness of ground states for double power nonlinear Schrödinger equations

Abstract: In this talk, we consider the uniqueness of ground state for double power nonlinear Schrödinger equations, which involving the Sobolev critical exponent in their nonlinearities. Akahori-Ibrahim-Ikoma-K-Nawa (‘19) showed that the ground state is unique in five and higher space dimensions for high frequencies. After that, Coles and Gustafson (‘20) obtained the corresponding result in three space dimensions by using the resolvent expansion and fixed point theorem. Here, combining the argument of our previous result and that of Coles and Gustafson (‘20), we can give an alternative proof of the uniqueness result in three space dimensions without employing the fixed point theorem.

Asuka Takatsu
(Tokyo Metropolitan University)

Hierarchy of entropies associated with deformed logarithmic functions

Abstract: One way to generalize the Boltzmann entropy is to use a deformed logarithmic function in information geometry. A typical example is a family of the Tsallis entropies associated with the porous medium equations. In this talk, I will explain the relation between the generalized entropies and evolution equations from the viewpoint of information geometry, and the hierarchy of the generalized entropy. This talk is based on the joint work with Kazuhiro Ishige (the University of Tokyo) and Paolo Salani (Universita di Firenze).

Luca Martinazzi
(Universitá di Roma "La Sapienza")

Critical points of the Moser-Trudinger functional on closed surfaces

Abstract: Given a 2-dimensional closed surface, we will show that the Moser-Trudinger functional has critical points of arbitrarily high energy. Since the functional is too critical to directly apply to it the known variational methods (in particular the Struwe monotonicity trick), we will approximate it by subcritical ones, which in fact interpolate it to the Liouville functional from conformal geometry. Hence our result will also unify and give common results for these two apparently unrelated problems. This is a joint work with F. De Marchis, A. Malchiodi and P-D. Thizy.

Yohei Tsutsui
(Kyoto University)

Rearrangements, medians and their maximal functions

Abstract: Aim of this talk is twofold. First, we give an exact expression of the set of all medians, of functions on $\mathbb{R}^n$, in terms of (non-) increasing rearrangements. Second, a version of Hardy-Littlewood-Sobolev inequality for fractional maximal operators, defined by rearrangements and medians, is given.

Masato Hashizume
(Hiroshima University)

A power type approximation of Moser–Trudinger inequality

Abstract: The Moser–Trudinger inequality is considered to be a limiting case of the Sobolev inequality in the framework of Orlicz spaces. However, the Moser–Trudinger inequality is not obtained via a direct limiting procedure for the Sobolev inequality. In this talk, we consider a Sobolev type inequality and some properties of the inequality. In particular, we show that the Carleson-Chang limit on the Moser–Trudinger inequality is derived as a limit of the concentration level of the Sobolev inequality. This talk is based on a joint work with Norisuke Ioku (Tohoku University).

Norihisa Ikoma
(Keio University)

The existence and nonexistence of weak solutions to the Born-Infeld equation

Abstract: The Born-Infeld equation was derived by M. Born and L. Infeld in 1930s as a modification of Maxwell's equations. Its principal part coincides with the mean curvature operator of spacelike surfaces in the Minkowski space and the equation has been studied from the geometrical point of view around 1970s and 1980s. Recently, the equation is studied from the analytical point of view and one of interesting problems is to reveal the relation between the class of force terms and the regularity of solutions. In this talk, we focus on the properties of the minimizer of the functional corresponding to the Born-Infeld equation. This talk is based on joint work with Jaeyoung Byeon (KAIST), Andrea Malchiodi (Scuola Normale Superiore) and Luciano Mari (Università degli Studi di Torino).

Michiaki Onodera
(Tokyo Institute of Technology)

Quantitative stability in overdetermined problems

Abstract: I will present recent work on quantitative stability estimates of the spherical symmetry in overdetermined problems when boundary data are slightly perturbed from constants. The main tool is a new implicit function theorem for a pair of Banach triplets, which allows one to handle a loss of derivatives and derive optimal quantitative estimates in a natural regularity framework.

Tastuya Miura
(Tokyo Institute of Technology)

Li–Yau type inequality for curves and applications

Abstract: In this talk I will present a geometric inequality of Li-Yau type involving the bending energy and multiplicity for curves in Euclidean space, with some applications to elastic networks and knots.

Longitudinal bifurcating solutions to a nonlinear elliptic equation on a large spherical cap

Yoshitsugu Kabeya*
Department of Mathematics, Osaka Metropolitan University

1 Introduction

In this talk, we consider the nonlinear elliptic problem

$$\begin{cases} \Lambda u + \lambda(-u + u_+^p) = 0 & \text{in } \Omega_\varepsilon \subset \mathbb{S}^2 \\ u = 0 & \text{on } \partial\Omega_\varepsilon \end{cases} \tag{1.1}$$

Here Λ denotes the Laplace-Beltrami operator on the standard unit sphere $\mathbb{S}^2 \subset \mathbb{R}^3$, $\varepsilon > 0$ is sufficiently small, $\lambda > 0$ is a parameter and the domain Ω_ε is expressed in the polar coordinates as

$$\Omega_\varepsilon = \Big\{ (\theta, \phi) \,\Big|\, \ 0 \le \theta < \pi - \varepsilon,\ 0 \le \phi < 2\pi \Big\}.$$

We take the usual polar coordinates: Let (y_1, y_2, y_3) be the Cartesian coordinates in $\mathbb{R}^3$. Then in the polar coordinates, we have

$$\begin{cases} y_1 = \sin\theta\cos\phi, \\ y_2 = \sin\theta\sin\phi, \\ y_3 = \cos\theta. \end{cases}$$

We consider the case when Ω_ε is close to the full sphere $\mathbb{S}^2$, and we will see what happens to the structure of solutions to (1.1). Obviously, $u \equiv 1$ is a solution to the equation, but it does not satisfy the boundary condition. However, we can expect the existence of a solution which is close to $u \equiv 1$.

In this talk, we discuss the existence of such solutions in the bifurcation-theoretic point of view. This talk is based on the joint work with Professor C. Bandle (University of Basel, Switzerland) and Professor Hirokazu Ninomiya (Meiji University, Japan).

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