Summer School on Variational Problems and Functional Inequalities (at OCAMI)

RIMS Research Project

organizers: Futoshi Takahashi (Osaka Metropolitan University)
Michinori Ishiwata (Osaka University)

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Date : 2022, September, 21, 9:50 – September, 22, 17:30

Venue : Science Building E 408
Department of Mathematics, Osaka Metropolitan University
3-3-138, Sugimoto-cho, Osaka-si, Osaka

21, September

9:50～10:00 Opening

10:00～11:00 Bernhard Ruf （Universitá di Milano）
Concentration on manifolds for solutions of polynomial elliptic equations on an annulus (I)

11:30～12:30 Luca Martinazzi（Universitá di Roma "La Sapienza"）
Variational methods for the Moser-Trudinger embedding (I)

14:00～14:45 Shoichi Hasegawa (Waseda University, PD)
Separation structure of radial solutions to the Lane-Emden equation on non-compact Riemannian manifolds

15:00～15:45 Ikkei Shimizu (Osaka University, Researcher)
Profile decomposition for the Schrödinger propagator on star graphs and its application to nonlinear problems

16:00～16:45 Nobuhito Miyake (The University of Tokyo, PD)
Effect of decay rates of initial data on the sign of solutions to Cauchy problems of polyharmonic heat equations

16:45～17:30 discussion

22, September

10:00∼11:00 Bernhard Ruf （Universitä di Milano）
Concentration on manifolds for solutions of polynomial elliptic equations on an annulus (II)

11:30∼12:30 Luca Martinazzi（Universitá di Roma "La Sapienza"）
Variational methods for the Moser-Trudinger embedding (II)

14:00∼14:45 Takeshi Suguro (RIMS, Project Fellow)
Shannon's inequality for a generalized entropy and its application

15:00∼15:45 Haoyu Li (Osaka University, Researcher)
Nodal Solutions to Coupled Schrödinger Systems

16:00∼16:45 Naoki Hamamoto (Osaka Metropolitan University, PD)
Best constants in some CKN type inequalities for test vector fields restricted by differential constraint

16:45∼17:30 discussion

Bernhard Ruf
(Universitá di Milano)

Concentration on manifolds for solutions of polynomial elliptic equations on an annulus I, II

Abstract: We consider elliptic equations with polynomial nonlinearities and with a small parameter ε (singular perturbation). It is known by the seminal work of Ni, Takagi and Wei that for small $\varepsilon > 0$ there exist solutions which concentrate in a single point. Furthermore, information on the asymptotic location of the concentration point can be given. A conjecture attributed to Ni states that when the problem is considered on an annulus in $\mathbb{R}^N$, then there will exist solutions which concentrate on k-dimensional spheres S^k, for any k with $0 \leq k \leq N-1$. In these lectures will describe recent progress which has been made towards this conjecture.

Luca Martinazzi
(Universitá di Roma “La Sapienza”)
Variational methods for the Moser-Trudinger embedding I, II

Abstract: We consider the embeddings of Sobolev spaces of functions defined on a 2-dimensional Euclidian domain or on a closed surface, having square-integrable gradient. Finding (constrained) critical points of such embeddings is a difficult problem strongly depending on the energy level (the squared Sobolev norm) at which we constrain the functional. For energies up to 4π, critical points are found as maximizers, as shown in celebrated results by Carleson-Chang, Struwe and Flucher. For energies slightly above 4π we will sketch Struwe's result on the existence of 2 critical points, based on ”local” compactness, a mountain pass structure and his famous monotonicity trick, crucial to overcome the failure of the Palais-Smale condition. This will be the content of the first lecture and will be inspired by a seminal paper of M. Struwe (AIHP 1988).
To find critical points with arbitrarily high energies, we will show how to employ the baricenter techniques of Djadli-Malchiodi to construct a variational structure that generalizes the mountain pass geometry. In order to use this structure to actually find critical points, we will again need the monotonicity trick of Struwe, and then some subtle compactness estimates, as pioneered by Mancini-Martinazzi and Druet-Thizy. This second lecture is based on a recent collaboration with F. De Marchis, A. Malchiodi and P-D. Thizy (Inventiones 2022).

Shoichi Hasegawa
(Waseda University, PD)

Separation structure of radial solutions to the Lane-Emden equation on non-compact Riemannian manifolds

Abstract: We shall devote this talk to discussing separation phenomena of radial solutions to the Lane-Emden equation on non-compact Riemannian manifolds. Concerning the Lane-Emden equation in the Euclidean space, the separation structure of radial solutions has been well-investigated and the Joseph-Lundgren exponent is a critical exponent on the separation phenomena of radial solutions. On the other hands, on non-compact Riemannian manifolds, the separation structure of radial solutions to the Lane-Emden equation has not been clarified completely. In this talk, we shall prove the separation property of radial solutions for the supercritical case in the sense of the Joseph-Lundgren, and show that the Joseph-Lundgren exponent is a critical exponent on the separation phenomena of radial solutions to the Lane-Emden equation on non-compact Riemannian manifolds. Moreover, we also study the existence of a singular solution.

Ikkei Shimizu
(Osaka University)

Profile decomposition for the Schrödinger propagator on star graphs and its application to nonlinear problems

Abstract: The profile decomposition is a method to analyze non-compact factors in a normed space, where one decomposes given bounded sequence into a family of sequences moving along the non-compact directions with different speeds. In particular for the Schrödinger propagators, the non-compactness of the Strichartz estimate has been investigated so far via profile decomposition. In this talk, we consider the Schrödinger propagators on a star graph, and establish its profile decomposition by overcoming some technical difficulties. Moreover, we will apply this result to investigate the global-in-time dynamics for the nonlinear Schrödinger equations on a star graph. This is the joint work with M. Ikeda (RIKEN), T. Inui (Osaka University/UBC), and M. Hamano (Waseda University).

Nobuhito Miyake
(The University of Tokyo)

Effect of decay rates of initial data on the sign of solutions to Cauchy problems of polyharmonic heat equations

Abstract: In this talk, we consider the sign of solutions to Cauchy problems of polyharmonic heat equations. It is known that Cauchy problems of higher order parabolic equations have no positivity preserving property in general. On the other hand, it is expected that solutions to these Cauchy problems are eventually globally positive if initial data decay slowly enough. Our aim of this talk is to show the existence of the threshold for the decay rate of initial data which determines whether the corresponding solution to the Cauchy problem of the linear polyharmonic heat equation is eventually globally positive or not. As the application of this result, we construct eventually globally positive solutions to the Cauchy problem of semilinear polyharmonic heat equations.

Takeshi Suguro
(RIMS)

Shannon's inequality for a generalized entropy and its application

Abstract: Shannon's inequality is a functional inequality obtained from the entropy maximization problem for the probability density function under moment conditions. We show Shannon's inequality for a generalized entropy, an extension of the Boltzmann–Shannon entropy. We also consider the connection between the Shannon and logarithmic Sobolev inequalities and an application of these inequalities.

Li Haoyu
(Osaka University)

Nodal Solutions to Coupled Schrödinger Systems

Abstract: We consider the existence and multiplicity of the nodal solutions to

$$\begin{cases} -\Delta u_j + \lambda_j u_j = \mu_j u_j^3 + \sum_{i=1, i\neq j}^{N} \beta_{ij} u_j u_i^2 \quad \text{in } \Omega, \\ u_j \in H_r^1(\Omega), \quad j = 1, \ldots, N, \end{cases}$$

Here, Ω is a radial domain in $\mathbb{R}^2$ or $\mathbb{R}^3$. Combining a heat flow approach and the invariant sets method, we obtain multiple such solutions sharing the given nodal number. This report is based on *Li, H, Wang, Z.-Q, Multiple nodal solutions having shared componentwise nodal numbers for coupled Schrödinger equations. J. Funct. Anal. 280 (2021), no. 7, Paper No. 108872, 44 pp.*

Naoki Hamamoto
(Osaka Metropolitan University)

Best constants in some CKN type inequalities for test vector fields restricted by differential constraint

Abstract. We observe recent results about best constants and their improvements in a class of functional inequalities including Hardy ones and uncertainty principle ones for vector fields, which we collectively call CKN type inequalities. One of the points is that best values of the constants can be improved by restricting the test vector fields by differential constraint such as curl-free or divergence-free condition. The content of this talk contains some results obtained by the speaker's joint work with Professor F. Takahashi. This research is supported by JSPS KAKENHI Grant numbers JP21J00172 and JP22K13943, and is also partly supported by Osaka City University Advanced Mathematical Institute (MEXT Joint Usage/Research Center on Mathematics and Theoretical Physics JPMXP0619217849).