**RIMS** Joint Research Activity

### Vertex algebras and related topics

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RIMS, Kyoto University

Organizer:

Ana Kontrec

ABSTRACTS OF TALKS

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#### On classification of irreducible modules of minimal unitary affine W-algebras

Dražen Adamović, University of Zagreb

We prove that any unitary highest weight module over a universal minimal affine W-algebra at non-critical level descends to its simple quotient and classify all irreducible positive energy modules for these W-algebras. We also classify all irreducible highest weight modules for the simple affine vertex algebras in the cases when the associated simple minimal W-algebra is unitary. We will also discuss the existence of irreducible modules with infinite-dimensional weight spaces for these W-algebras.

This talk is based on a recent joint paper with V. Kac, P. Möseneder Frajria and P. Papi.

### Generalized Grothendieck's simultaneous resolution and associated varieties of simple affine vertex algebra

#### Tomoyuki Arakawa, RIMS

The closure of a Diximier sheet is the image of a generalized Grothendieck's simultaneous resolution. We show that the associated variety of simple affine vertex algebras is contained in the closure of the Diximier sheet when a chiralization of generalized Grothendieck's simultaneous resolution exists. This generalizes in a conceptual manner the results obtained by Anne Moreau and the speaker. This is a joint work with Vyacheslav Futorny and Libor Krizka.

#### Irrational vertex operator algebras and graded pseudo-traces for strongly interlocked indecomposable non simple modules

#### Katrina Barron, University of Notre Dame

In 2004, Miyamoto extended much of the work of Zhu on the modular invariance of the space of the associated graded traces for  $C_2$ -cofinite rational VOAs, to  $C_2$ -cofinite irrational VOAs and the modular invariance of the space of the associated graded traces and pseudo-traces. However the machinery necessary to carry out Miyamoto's construction of graded pseudo-traces is very involved. In work with Batistelli, Orosz Hunziker, and Yamskulna, we develop a setting in which the  $C_2$ -cofiniteness of V is not necessary, and graded pseudo-traces are easily defined for certain V-modules. To do this, we introduce the notion of strongly interlocked modules for an irrational VOA, and we show that for such modules graded pseudo-traces are always well defined and, in particular, satisfy linearity, symmetry, and the logarithmic derivative property. We show that this setting has interesting applications to, for instance, the Heisenberg and Virasoro VOAs.

#### Connecting W-algebras and their representations

#### Justine Fasquel, University of Melbourne

W-algebras form a large family of vertex algebras associated to nilpotent orbits of simple Lie algebras. They provide an additional framework to study the representations of certain infinite dimensional Lie algebras. Indeed, W-algebras can be obtained from the latter by applying certain quantized Hamiltonian reductions. It is believed that most of the time this reduction procedure can be reversed to reconstruct the representations of the underling affine Lie algebra. In this talk, we will discuss inverse reductions as well as their consequences on the representation theory based on examples in small ranks. The talk report on recent papers with T. Creutzig, A. Linshaw and N. Nakatsuka and with C. Raymond and D. Ridout.

#### Quantum Manin Superspace, Quantum Grassmann superalgebra and beyond

Ge Feng, University of Shanghai for Science and Technology

Associated with the quantum general (special) linear superalgebra  $U_q(\mathfrak{gl}(m|n))$  ( $U_q(\mathfrak{sl}(m|n))$ ), we introduce the quantum Grassmann super-algebra  $\Omega_q(m|n)$  equipped with the quantum divided power superstructure. The quantum (restricted) Grassmann superalgebra  $\Omega_q$ , together with its Manin dual  $\Omega_q^!$ , is made into a  $U_q(\mathfrak{g})$ -module superalgebra by quantum differential operators, where  $\Omega_q = \Omega_q(m|n)$  for q generic, or  $\Omega_q(m|n, 1)$  for q root of unity, and  $\mathfrak{g} = \mathfrak{gl}(m|n)$  or  $\mathfrak{sl}(m|n)$ . The definition of quantum differential operators provides an explicit realization models for certain simple  $U_q(\mathfrak{g})$ -modules and their dimension-formulae, as well as the quantum super de Rham theory. This is a joint work with Naihong Hu et al.

#### Twisted Zhu algebras

#### Naoki Genra, University of Tokyo

Let V be a freely generated pregraded vertex superalgebra, H a Hamiltonian operator of V, and g a diagonalizable automorphism of V commuting with H with modulus one eigenvalues. We prove that the (g, H)-twisted Zhu algebra of V has a PBW basis, is isomorphic to the universal enveloping algebra of some non-linear Lie superalgebra, and satisfies the commutativity of BRST cohomology functors, which are complete generalizations of results of De Sole and Kac. As applications, we can compute the twisted Zhu algebras of affine vertex algebras and W-algebras.

### Vector spaces of coinvariants and conformal blocks from representations of VOAs in families

#### Angela Gibney, University of Pennsylvania

To an *n*-pointed algebraic curve and a collection of n modules over V, a VOA of CFT-type, one may associate a vector space of coinvariants, and its dual space of conformal blocks. Properties of these spaces reflect aspects of V, and from them one may construct useful algebraic structures on the moduli space of stable *n*-pointed curves of genus g. For instance, when V is rational and  $C_2$ -cofinite, these vector spaces satisfy the factorization property, and give rise to vector bundles on the moduli space whose Chern classes lie in the tautological ring. In this talk I will briefly motivate our interest in these considerations, and highlight what is known in the challenging cases when V is not rational or not  $C_2$ -cofinite.

#### Integral forms for affine vertex operator superalgebras and their modules

#### Hongyan Guo, Central China Normal University

In this talk, we will first review some background about integral forms of vertex algebras, affine Lie superalgebras and associated vertex operator superalgebras. Then we present our recent results on integral forms of affine vertex operator superalgebras and their modules. An explicit construction of the integral form via Chevalley basis of classical Lie superalgebras will also be given. This is based on joint work in progress with Hongju Zhao.

#### Cluster realization of Weyl groups and q-characters of affine quantum groups

#### Rei Inoue, Chiba University

The cluster algebra is a commutative algebra introduced by Fomin and Zelevinsky around 2000. The characteristic concept of the algebra called 'mutation' is related to various notions in mathematics and mathematical physics.

In this talk, for a finite dimensional semisimple Lie algebra we introduce a realization of Weyl groups using the mutations. Further we discuss its application to the q-characters of quantum non-twisted affine Lie algebras introduced by Frenkel and Reshetikhin. This talk is based on joint works with Thomas Lam, Pavlo Pylyavskyy, Tsukasa Ishibashi, Hironori Oya, and Takao Yamazaki.

## On singular vectors of universal affine vertex operator algebras and varieties of $L_k(sl_n)$

Cuipo Jiang, Shanghai Jiao Tong University

We will talk about our recent progress on singular vectors of universal affine vertex operator algebras of types A and D, and varieties of the simple affine vertex operator algebras  $L_k(sl_n)$ .

### Rogers-Ramanujan exact sequences and free modules over free generalized vertex algebras

#### Kazuya Kawasetsu, Kumamoto University

In this talk, we introduce the notion of free modules over (generalized) vertex algebras and construct natural short exact sequences among those over singly generated free generalized vertex algebras. The construction naturally generalizes the interpretation of the Rogers-Ramanujan recursion by Capparelli, Lepowsky and Mias to a continuous family of recursions. We then apply the exact sequences to several topics in representation theory and related areas.

#### Categorical tools for rational orbifold theory

#### Hannes Knötzele, University of Hamburg

In orbifold theory, one tries to understand the representation theory of a vertex subalgebra  $V^G$  of a vertex algebra V fixed under the action of a finite group of automorphisms G. The category of representations of  $V^G$ can be obtained as an equivariantisation of the G-crossed category of twisted modules of V. In this talk, we sketch how the theory of extensions for fusion categories can constrain G-crossed categories and, therefore, their equivariantisation. Additional constraints can be obtained if we demand that the extended and the equivariantised categories are modular. We will recall categorical prerequisites, including module categories and Picard groups. Our main examples are Tambara-Yamagami-like, where the extended fusion category is pointed (in other words, every irreducible V-module is a simple current), and there is a unique irreducible g-twisted module for every g in G.

#### Examples of the invariant Hilbert scheme of the Cox realization

#### Ayako Kubota, Saitama University

The invariant Hilbert scheme is a moduli space of affine schemes that are stable under an action of a reductive algebraic group. Under a suitable choice of the parameter, there exists a natural morphism, called the quotient-scheme map, from the invariant Hilbert scheme to an affine quotient variety, and it makes the invariant Hilbert scheme a candidate for a resolution of singularities of the quotient variety. As the quotient-scheme map depends on the quotient construction of the given singularity, we may ask if it has a good presentation. In this talk, we focus on a quotient presentation called the Cox realization and consider the associated invariant Hilbert scheme.

#### The applications of shift system

#### Hao Li

Feigin-Tipunin proposed the geometric method to construct higher rank logarithmic vertex operator algebras associated with the Lie algebras of ADE type. Sugimoto later rigorously showed the existence of such logarithmic VOAs. The study of these logarithmic VOAs has become increasingly important as there is a wide believe that there should be a ribbon tensor equivalence between the representation category of small quantum group and the representation category of the multiplet vertex operator at integral level. This logarithmic Kazhdan-Lusztig conjecture was proved by Gannon-Negron (and partially by Creutzig-Lentner-Rupert) in the rank 1 case. Continuing on Sugimotos talk, I will explain more details of shift system and their applications. In particular, we obtained the logarithmic VO(S)As associated with non-simply laced Lie algebras from general properties of the shift system. The talk is based on the joint work with Sugimoto and Shim.

#### Building blocks for W-algebras of classical types

#### Andrew Linshaw, University of Denver

There are three universal 2-parameter VOAs whose 1-parameter quotients are expected to be the building blocks for W-algebras in classical Lie types A, B, C, and D. These are  $\mathcal{W}_{\infty}, \mathcal{W}_{\infty}^{ev}$ , and  $\mathcal{W}_{\infty}^{\mathfrak{sp}}$ , which are freely generated of types  $\mathcal{W}(2,3,4,\ldots), \mathcal{W}(2,4,6,\ldots)$ , and  $\mathcal{W}(1^3,2,3^3,4,\ldots)$ , respectively. I will review these algebras and their expected role in the structure of W-algebras. I will also explain a conjecture that organizes all W-algebras of classical types into families that are governed by more complicated 2-parameter VOAs that are themselves glueings of copies of  $\mathcal{W}_{\infty}, \mathcal{W}_{\infty}^{ev}$ , and  $\mathcal{W}_{\infty}^{\mathfrak{sp}}$ . Finally, I will explain the construction of the first such example: it is freely generated of type  $\mathcal{W}(1, 2^3, 3, 4^3, \ldots)$ , and is a glueing of two copies of  $\mathcal{W}_{\infty}^{ev}$ . This is a joint work with Thomas Creutzig and Vlad Kovalchuk.

#### G-Crossed categories of lattice orbifolds

#### Sven Möller, University of Hamburg

G-crossed braided tensor categories (or their equivariantisations, which are often modular tensor categories) appear as categories of representations of fixed-point vertex algebras  $V^G$  under the action of some finite group G.

We develop techniques to determine these G-crossed categories in concrete examples, in particular when the original vertex algebras V has a pointed representations category (like a lattice or Heisenberg vertex algebra), i.e. when the fusion rules are characterised by an abelian group A.

Specifically, we generalise the Z/2Z-crossed Tambara-Yamagami categories, which have only one simple object in the twisted sector (and which are only non-degenerate for A odd), to define a class of Z/2Z-crossed categories whose untwisted and twisted sector are parametrised by A and A/(2A), respectively.

These give the representation categories of certain lattice orbifold vertex algebras, but in contrast to earlier results, with all categorical data (and not just on the level of the fusion ring).

This is joint work with Csar Galindo and Simon Lentner.

## On a series of simple affine VOAs at non-admissible level arising from rank one 4D SCFTs

Anne Moreau, Universite Paris Saclay

It is known by the works of Adamović and Perše that the affine simple vertex algebras associated with  $G_2$  and  $B_3$  at level -2 can be conformally embedded into  $L_{-2}(D4)$ . In this talk, I will present a join work with Tomoyuki Arakawa, Xuanzhong Dai, Justine Fasquel, Bohan Li on the classification to the irreducible highest weight modules of these vertex algebras. I will also describe their associated varieties: it turns out that the associated variety of that corresponding to  $G_2$  is the orbifold of the associated variety of that corresponding to  $D_4$  by the symmetric group of degree 3 which is the Dynkin diagram automorphism group of  $D_4$ . This provides a new interesting example of associated variety satisfying a number of conjectures in the context of orbifold vertex algebras. It is interesting to notice that these vertex algebra also appear as the vertex operator algebras corresponding to rank one Argyres–Douglas theories in four dimension with flavour symmetry  $G_2$  and  $B_3$ .

#### Partial reductions among affine W-algebras of classical Lie types

#### Shigenori Nakatsuka

The webs of W-algebras introduced by Prochazka-Rapcak in physics provide rich perspectives on the "hidden hierarchy" among the type-A W-superalgebras with hook-type W-superalgebras as building blocks. One such perspective tells that W-superalgebras in type A should be obtained from the affine vertex superalgebras through reduction by stages (partial reductions) associated with hook-type partitions. As Morgan conjectured in the finite setting, there is a very good chance that two affine W-algebras are connected through partial reductions along nilpotent orbit degenerations in general. In this talk, we discuss how to see such a phenomenon through screening operators when nilpotent orbit degenerations are "basic" in the case of classical Lie types. The talk is based on joint works with Creutzig-Fasquel-Linshaw (arXiv:2408.08371), Fasquel-Fehily-Fursman (arXiv:2408.13785), and Fasquel-Kovalchuk (work in progress).

### Weight modules for affine vertex operator algebras with finite and infinite multiplicities

#### David Ridout, University of Melbourne

We investigate the representation theory of the simple affine vertex operator algebra  $L_k(\mathfrak{g})$  at admissible levels k. For  $\mathfrak{g} = \mathfrak{sl}_2$ , the irreducible weight modules all have finite multiplicities, while for higher ranks, infinite multiplicities are encountered for most k. This leads to generically irreducible families of infinite-multiplicity weight  $\mathfrak{g}$ -modules that are different from those constructed by other means (eg, as Gelfand–Tsetlin modules).

#### **On Kac-Wakimoto hypothesis**

Li Ren, Sichuan University

Motivated by the earlier work of Kac-Wakimoto on the coset constructions associated with affine vertex operator algebras, the categorial coset constructions are investigated and Kac-Wakimoto Hypothesis is proved under some mild conditions. In particular, the field identifications are obtained. These results are applied to the coset constructions in the theory of vertex operator algebras. This is a joint work with C. Dong and F. Xu.

#### Shift system and nested Feigin-Tipunin construction

#### Shoma Sugimoto, Tsinghua University

Giving rich examples and a unified research methodology of irrational VOA (logVOA) is an important problem. In this talk, I will introduce the nested Feigin-Tipunin (FT) construction as one of the guiding principles. In Part 1, after a brief review of the most famous logVOA, triplet Virasoro algebra, I will introduce a new purely Lie algebraic setting called shift system, and under such a setting, give a geometric construction called FT construction and its consequences. It allows us to study various generalizations of the triplet Virasoro algebra using the same geometric method (indeed, we consider three examples). In Part 2, (in the rank 1 case) I will explain that the FT construction can be "nested" by considering certain hypercubic structures. At each step of the nesting, the same arguments as for the original FT construction can be applied, in particular the Weyl-type character formula can be applied repeatedly. The resulting character almost coincides with the  $\hat{Z}$ -invariant of 3-manifolds introduced by S. Gukov et al. and suggest that the nested FT construction gives rich examples of logVOA corresponding to 3-manifolds. Part 1 is partially based on a joint work with Hao Li and Shim Myungbo (both YMSC).

#### The center of the modular affine vertex algebra

#### Juan Villarreal, University of Colorado Boulder

We review some properties of the center of the universal enveloping algebra of finite dimensional simple Lie algebras in positive and zero characteristic. Then, we review the case of affine vertex algebras. Finally, I will mention some results of the center of the affine vertex algebra in positive characteristic. This is based on a joint work with T. Arakawa and L. Topley.

# Classification of irreducible modules for the affine vertex algebra $L_{-\frac{2n+1}{2}}(\mathfrak{sl}_{2n})$ in certain categories

Ivana Vukorepa, University of Split

We investigate the representation theory of simple affine vertex algebra  $L_k(\mathfrak{g})$  at special non-admissible levels  $k = -\frac{2n+1}{2}$  for  $\mathfrak{g} = \mathfrak{sl}_{2n}$ . We classify irreducible  $L_k(\mathfrak{sl}_{2n})$ -modules in category  $KL_k(\mathfrak{sl}_{2n})$  and prove that  $KL_k(\mathfrak{sl}_{2n})$  is a semi-simple, rigid braided tensor category. Moreover, all modules in  $KL_k(\mathfrak{sl}_{2n})$  are simple-currents and they appear in the decomposition of conformal embeddings  $\mathfrak{gl}_{2n} \hookrightarrow \mathfrak{sl}_{2n+1}$  at level  $k = -\frac{2n+1}{2}$ .

In the second part of the talk we present a new method for proving simplicity of quotients of universal affine vertex algebras  $V^k(\mathfrak{sl}_{2n})$ . We use this result to prove that in the case n = 3 a maximal ideal is generated by one singular vector of conformal weight 4. As a byproduct, we classify irreducible modules in the category  $\mathcal{O}$  for  $L_{-7/2}(\mathfrak{sl}_6)$ . The talk is based on joint papers with D. Adamović, T. Creutzig and O. Perše.

### Affine vertex operator superalgebra $L_{osp(1|2)}(l,0)$ at admissible level

Qing Wang, Xiamen University

We present our recent results on affine vertex operator superalgebra  $L_{osp(1|2)}(l,0)$  at admissible level l. We prove that the category of weak  $L_{osp(1|2)}(l,0)$ -modules on which the positive part of osp(1|2) acts locally nilpotently is semisimple. Then we prove that Q-graded vertex operator superalgebras  $(L_{osp(1|2)}(l,0),\omega_{\xi})$  with a new Virasoro element  $\omega_{\xi}$  are rational and the irreducible modules are exactly the admissible modules for osp(1|2), where  $0 < \xi < 1$  is a rational number. Furthermore, we determine the Zhu's algebras  $A(L_{osp(1|2)}(l,0))$  and their bimodules A(L(l,j)) for  $(L_{osp(1|2)}(l,0),\omega_{\xi})$ , where j is the admissible weight. As an application, we calculate the fusion rules among the irreducible ordinary modules of  $(L_{osp(1|2)}(l,0),\omega_{\xi})$ . This is a joint work with Huaimin Li.

### On fusion products of twisted modules in pemutation orbifolds

#### Nina Yu, Xiamen University

Orbifold theory explores the structure of vertex operator algebras (VOAs) when finite groups act on them, with a particular focus on understanding the representation theory of the fixed point subalgebra. A key aspect of this field is permutation orbifolds, which study how the symmetric group acts on the n-fold tensor product of a VOA. In this talk, I will present our recent findings on the fusion product of twisted modules in permutation orbifolds. This work is a collaboration with C. Dong and F. Xu.