Winter School on Representation Theory

Date: Monday, January 19, 2015–Friday, January 23, 2015
Place: RIMS 420

	Monday 19	Tuesday 20	Wednesday 21	Thursday 22	Friday 23
10:30–11:30		Vasserot	Leclerc	Rouquier	Ben-Zvi
14:00–15:00	Henderson	Rouquier	Ben-Zvi	Henderson	Vasserot
15:20–16:20	Leclerc	Habiro	Iwaki	Naito	Wang
16:40–17:40	Ben-Zvi	Henderson	Vasserot	Leclerc	Rouquier

Monday, January 19

14:00–15:00 Anthony Henderson (Sydney)

Character sheaves and modular generalized Springer correspondence I
(click here for slides)

15:20–16:20 Bernard Leclerc (Caen)

Quivers with relations for symmetrizable Cartan matrices I

16:40–17:40 David Ben-Zvi (Texas)

Representation theory on the torus I

Tuesday, January 20

10:30–11:30 Eric Vasserot (Paris)

Representations on categories : quiver varieties, cherednik algebras and finite groups I

11:30–14:00 *Lunch break*

14:00–15:00 Raphael Rouquier (UCLA)

Calogero-Moser cells I

15:20–16:20 Kazuo Habiro (葉廣 和夫, RIMS)

Trace decategorification of categorified quantum sl(2)
(click here for slides)

16:40–17:40 Anthony Henderson (Sydney)

Character sheaves and modular generalized Springer correspondence II
(click here for slides)

18:30– *Banquet*

Wednesday, January 21

10:30–11:30 Bernard Leclerc (Caen)

Quivers with relations for symmetrizable Cartan matrices II

11:30–14:00 *Lunch break*

14:00–15:00 David Ben-Zvi (Texas)

Representation theory on the torus II

15:20–16:20 Kohei Iwaki (岩木 耕平, RIMS)

Exact WKB analysis and cluster algebras (click here for slides)

16:40–17:40 Eric Vasserot (Paris)

Representations on categories : quiver varieties, cherednik algebras and finite groups II

Thursday, January 22

10:30–11:30 Raphael Rouquier (UCLA)

Calogero-Moser cells II

11:30–14:00 *Lunch break*

14:00–15:00 Anthony Henderson (Sydney)

Character sheaves and modular generalized Springer correspondence III

15:20–16:20 Satoshi Naito (内藤 聡, Tokyo Institute of Technology)

Comparison of the two specializations of nonsymmetric Macdonald polynomials: at zero and at infinity (click here for slides)

16:40–17:40 Bernard Leclerc (Caen)

Quivers with relations for symmetrizable Cartan matrices III

Friday, January 23

10:30–11:30 David Ben-Zvi (Texas)

Representation theory on the torus III

11:30–14:00 *Lunch break*

14:00–15:00 Eric Vasserot (Paris)

Representations on categories : quiver varieties, cherednik algebras and finite groups III

15:20–16:20 Weiqiang Wang (Virginia)

Positivity of canonical bases of quantum coideal algebras and geometry of flag varieties (click here for slides)

16:40–17:40 Raphael Rouquier (UCLA)

Calogero-Moser cells III

Abstracts

David Ben-Zvi (University of Texas, Austin)

Representation theory on the torus (three lectures)

Topological field theory provides a powerful organizing principle for many objects in representation theory. In these talks we will explore some of the special features of gauge theories evaluated on the torus as a setting for diverse topics in representation theory.

Kazuo Habiro (RIMS)

Trace decategorification of categorified quantum $sl(2)$

Quantum groups are categorified by Khovanov, Lauda and Rouquier. In this talk, I consider Lauda's 2-categories U^* and U which categorifies the quantum $sl(2)$, using the extended graphical calculus introduced by Khovanov-Lauda-Mackaay-Stosic. The trace $Tr(C) = HH_0(C)$ of a linear category C is a module generated by the endomorphisms in C modulo the trace relation $[fg] = [gf]$. The notion of a trace is generalized to 2-categories: the trace of a linear 2-category C is a linear category $Tr(C)$ with the same object as C, and the hom space $Tr(C)(x, y)$ is the trace $Tr(C(x, y))$ of the hom category $C(x, y)$. Thus the traces of the 2-categories U^* and U are linear categories. It turns out that $Tr(U)$ is isomorphic to the split Grothendieck group $K_0(U)$, hence isomorphic to the idempotented integral form of $U_q(sl_2)$, and $Tr(U^*)$ is isomorphic to the idempotented integral form of the current algebra $U(sl_2[t])$. This is joint work with A. Beliakova, A. Lauda and M. Zivkovic.

Anthony Henderson (University of Sydney)

Character sheaves and modular generalized Springer correspondence (three lectures)

I will begin by reviewing Lusztig's classic theory of character sheaves on a connected reductive group G with characteristic-0 coefficients. This is a geometric version of the characteristic-0 representation theory of the finite group $G(\mathbb{F}_q)$, and captures the extent to which that representation theory is independent of q. Sheaf-theoretic analogues of the representation-theoretic induction and restriction functors give rise to geometric versions of cuspidal objects and induction series; in the simplified context of perverse sheaves on the unipotent variety, these constitute Lusztig's generalized Springer correspondence. The great achievement of Lusztig's theory is that it allows the computation of the ordinary irreducible characters of $G(\mathbb{F}_q)$.

A more recent trend in geometric representation theory has been the use of perverse sheaves with modular coefficients, as in the work of Soergel, Mirkovic-Vilonen and Juteau-Mautner-Williamson. In this spirit, one might hope for a theory of modular character sheaves on G, a geometric version of the modular representation theory of $G(\mathbb{F}_q)$. Conceivably, such a theory could provide greater insight into the difficult problem of determining decomposition numbers. I will explain a first step in this direction, the modular generalized Springer correspondence, developed jointly with Pramod Achar, Daniel Juteau and Simon Riche. The change to modular coefficients creates many difficulties and new features, but we do obtain some of the expected analogues of results in modular representation theory.

Kohei Iwaki (RIMS)

Exact WKB analysis and cluster algebras

Exact WKB analysis is an effective method for the global study of differential equations (containing a large parameter) defined on a complex domain. On the other hand, a cluster algebras is a particular class of commutative subalgebra of a field of rational functions with distinguished generators. I'll explain about a hidden cluster algebraic structure in exact WKB analysis.

Bernard Leclerc (Universit de Caen)

Quivers with relations for symmetrizable Cartan matrices (three lectures)

Quiver varieties have been used successfully in geometric representation theory to construct irreducible integrable representations of symmetric Kac-Moody algebras and l-integrable representations of their quantum affinizations [G, GV, N1, N2, N3]. In a recent joint work with Christof Geiss and Jan Schrer [GLS], we introduce new quiver algebras associated with symmetrizable Kac-Moody algebras and lay the foundations of their representation theory. The minicourse will explain the origin of these algebras (going back to [HL]), and their main structural properties generalizing classical theorems of Gabriel, Dlab-Ringel, Gelfand-Ponomarev for path algebras of quivers and preprojective algebras. The final part of the minicourse will sketch some work in progress on spaces of constructible functions over varieties of representations of these algebras, and how they should relate to Kac-Moody theory.

References :

[GLS] C. Geiss, B. Leclerc, J. Schrer, Quivers with relations for symmetrizable Cartan matrices, arXiv 1410.1403.

[G] V. Ginzburg, Lagrangian construction of the enveloping algebra $U(\mathfrak{sl}(n))$. C. R. Acad. Sci. Paris Math. 312 (1991), 907—912.

[GV] V. Ginzburg, E. Vasserot, Langlands reciprocity for affine quantum groups of type A_n . Internat. Math. Res. Notices 1993, 67—85.

[HL] D. Hernandez, B. Leclerc, A cluster algebra approach to q-characters of Kirillov-Reshetikhin modules, JEMS (to appear), arXiv 1303.0744.

[N1] H. Nakajima, Instantons on ALE spaces, quiver varieties, and Kac-Moody algebras. Duke Math. J. 76 (1994), 365—416.

[N2] H. Nakajima, Quiver varieties and Kac-Moody algebras. Duke Math. J. 91 (1998), 515—560.

[N3] H. Nakajima, Quiver varieties and finite-dimensional representations of quantum affine algebras. J. Amer. Math. Soc. 14 (2001), 145—238.

Satoshi Naito (Tokyo Insitute of Technology)

Comparison of the two specializations of nonsymmetric Macdonald polynomials: at zero and at infinity

In this talk, I first show that the specialization $E_{w_0\lambda}(x, q^{-1}, 0)$ at $t = 0$ of the nonsymmetric Macdonald polynomial $E_{w_0\lambda}(x, q^{-1}, t)$ is identical to a graded character of a canonical quotient (often called a local Weyl module) $W(\lambda)$ of a special Demazure submodule of the level-zero extremal weight module of extremal weight λ over a quantum affine algebra, where w_0 is the longest element of the finite Weyl group; here we note that the crystal basis of the local Weyl module $W(\lambda)$ is realized as the set of quantum Lakshmibai-Seshadri paths of shape λ. Second, I show that the specialization $E_{w_0\lambda}(x; q, \infty)$ at $t = \infty$ of the nonsymmetric Macdonald polynomial $E_{w_0\lambda}(x; q, t)$ can also be described as another graded character of the (same) local Weyl module $W(\lambda)$, with a curious grading different from the above one; this grading can be described explicitly in terms of the quantum Bruhat graph. Finally, I would like to examine the difference between these two gradings on the (same) local Weyl module $W(\lambda)$.

Rapahel Rouquier (UCLA)

Calogero-Moser cells (three lectures)

I will discuss rational Cherednik algebras associated to a complex reflection group W (at t=0), and the associated Calogero-Moser spaces of Etingof-Ginzurg. Ramification theory leads to partitions of W into left, right and two-sided cells. Conjecturally, these coincide with Kazhdan-Lusztig cells, in the case of Coxeter groups (joint work with C.Bonnafe).

References:

P.Etingof and V.Ginzburg, "Symplectic reflection algebras, Calogero-Moser space and deformed Harish-Chandra homomorphism", Inv. Math. 147 (2002), 243—348.

C.Bonnafe and R.Rouquier, "Calogero-Moser versus Kazhdan-Lusztig cells, Pacific Journal of Mathematics 261 (2013), 45-51".

C.Bonnafe and R.Rouquier, "Cellules de Calogero Moser", arXiv:1302.2720.

Eric Vasserot (Universit de Paris 7)

Representations on categories : quiver varieties, cherednik algebras and finite groups (three lectures)

We'll first review a few basic facts on representations in categories. The n we'll explain a relation with quiver varieties and Weyl modules of current algebras. In particular we'll explain how it gives the mu ltiplicative structure of the cohomology of the Gieseker moduli space. Finally, we will recall briefly the relation with the category of Cherednik algebra of wreath products and, if there is enough time, some new application to modular representations of finite unitary groups.

Weiqiang Wang (University of Virginia)

Positivity of canonical bases of quantum coideal algebras and geometry of flag varieties

In a recent work with Huanchen Bao, new canonical bases arise from quantum coideal algebras in a new approach to the Kazhdan-Lusztig theory of (super) type B/C. In this talk, we will give a construction of canonical bases of modified quantum coideal algebras and connection to partial flag varieties of classical type, generalizing the work of Beilinson-Lusztig-MacPherson (joint work with Bao, Yiqiang Li, and Jon Kujawa). We further determine the negativity vs positivity of the canonical bases of modified quantum gl_n, quantum sl_n, as well as their coideal subalgebras (joint work with Li).