Heat Kernel Estimates, Potential Theory and Related Topics

11-13 May 2026

Research Institute for Mathematical Sciences, Kyoto University

Organisers: Naotaka Kajino (Kyoto University, Chair), David Croydon (Kyoto University), Ryosuke Shimizu (Kyoto University)

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A simplified characterization of stable-like heat kernel estimates

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Heat kernel estimates for Dirichlet forms degenerate at the boundary

15:10-15:50: Yuichi Shiozawa (Doshisha University)
Conservativeness of time changed processes and Liouville property for Schrödinger operators

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09:30-10:10: Kôhei Sasaya (University of Tokyo)
On p-energy measures: construction and properties inherited from associated p-energy forms

10:30-11:10: Aobo Chen (Tsinghua University)
Elliptic Harnack inequalities for mixed local and nonlocal p-energy form on metric measure spaces

11:20-12:00: Shiping Cao (Chinese University of Hong Kong)
Boundary trace theorems for reflected jump processes

13:30-14:10: Takashi Kumagai (Waseda University)
Quantitative homogenization on time-dependent random conductance models with stable-like jumps

14:20-15:00: Ecaterina Sava-Huss (University of Innsbruck)
Speed, displacement and entropy of branching random walks

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Quasi-isometric nonequivalence for random subsets in products of trees

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Abstracts

11 May 2026

13:20-14:00: Mathav Murugan (University of British Columbia/RIMS Visiting Associate Professor)

A simplified characterization of stable-like heat kernel estimates

We obtain a characterization of two-sided stable-like heat kernel estimates, modelled on symmetric stable processes in Euclidean space. Our characterization confirms a conjecture of Grigor'yan, E. Hu, and J. Hu, and is based on two-sided bounds on the jump kernel together with upper bounds on capacity across annuli.

14:10-14:50: Panki Kim (Seoul National University)

Heat kernel estimates for Dirichlet forms degenerate at the boundary

In this talk, we discuss estimates on the heat kernels of discontinuous symmetric Markov processes including ones with jump kernels degenerate at the boundary. Depending on the regions where the parameters belong, the heat kernels estimates have different forms. There are new forms qualitatively different from all previously known heat kernel estimates. We also discuss the processes killed either by a critical potential or upon hitting the boundary. Their heat kernel estimates have the approximate factorization property with survival probabilities decaying as a power of the distance to the boundary, where the power depends on the critical potential. This talk is based on joint papers with Soobin Cho, Renming Song and Zoran Vondraček.

15:10-15:50: Yuichi Shiozawa (Doshisha University)

Conservativeness of time changed processes and Liouville property for Schrödinger operators

This talk is based on a joint work with Masayoshi Takeda (Kansai University). We are concerned with the Liouville property for a Schrödinger operator of the form $-\mathcal{L}+\mu$, where $\mathcal{L}$ is a Markov generator without killing term and μ is a non-negative measure. Here the Liouville property means that any bounded solution h to the equation $(-\mathcal{L}+\mu)h=0$ must be zero. In this talk, we establish a criterion for the Liouville property via the conservativeness of the μ-time changed process. Using this criterion, we further obtain necessary and sufficient conditions for the Liouville property for some Schrödinger operators in terms of the decay rates of the potentials at infinity/boundary.

16:10-16:50: Xiangqian Meng (Kyoto University)

TBA

TBA

12 May 2026

09:30-10:10: Takanobu Hara (Tohoku University)

A two-weight Hardy inequality and its application to nonlocal problems

In this presentation, we generalize the proof of Hardy's inequality by Ancona (1986) and discuss the embedding from weighted Sobolev spaces into weighted Morrey spaces. Furthermore, by combining this result with the Caffarelli-Silvestre extension, we apply it to the existence problem of solutions for the fractional Poisson equation.

10:30-11:10: Eryan Hu (Tianjin University)

Regularity of jump-type Dirichlet forms on metric measure spaces

Let (M, d, μ) be a metric measure space satisfying the volume doubling condition. Given a *jump measure* $J(x, dy)$, consider the symmetric bilinear form $\mathcal{E}$ defined by its quadratic part

$$\mathcal{E}(u,u) := \int_M \int_M |u(x) - u(y)|^2 \, J(x, dy) \, d\mu(x),$$

where u is in the natural domain $\mathcal{F} := \{u \in L^2(M,\mu) : \mathcal{E}(u,u) < \infty\}$. The purpose of this paper is to provide conditions that ensure that $(\mathcal{E}, \mathcal{F})$ is a *regular* Dirichlet form. We prove that $(\mathcal{E}, \mathcal{F})$ is a regular Dirichlet form provided the jump measure satisfies the following three hypotheses: the *Andres-Barlow condition* $(\mathrm{AB})_{\mathrm{W}}$, the *Poincaré inequality* $(\mathrm{PI})_{\mathrm{W}}$ and the *tail estimate* $(\mathrm{TJ})_{\mathrm{W}}$, where $W = W(x, r)$ $(x \in M, r > 0)$ is a certain *scaling function*.
Combining with the known heat kernel estimates, we obtain the following result: the conjunction of the hypotheses $(\mathrm{AB})_{\mathrm{W}}$, $(\mathrm{PI})_{\mathrm{W}}$, and $(\mathrm{TJ})_{\mathrm{W}}$ is *equivalent* to the fact that $(\mathcal{E}, \mathcal{F})$ is a regular Dirichlet form and its heat kernel satisfies certain upper and lower estimates.

11:20-12:00: Minhyun Kim (Hanyang University)

Nonlocal potential theory

Nonlocal potential theory is the study of harmonic functions associated with nonlocal operators modeled on the fractional Laplacian. In this talk, I will present recent progress on local and boundary behavior of such harmonic functions. The main topics include Bôcher theorem, isolated singularity theorem, removability theorem, Wiener criterion, boundary regularity, Green function estimates and classification of regular boundary points. This talk is based on joint works with Anders Björn, Jana Björn, Ki-Ahm Lee, Se-Chan Lee, and Marvin Weidner.

13:30-14:10: Izumi Okada (University of Tokyo)

Phase transition on the fluctuation of the structure of random walk ranges

We investigate fluctuation phenomena for the graph distance and the number of cut points associated with random media arising from the range of a random walk. The behavior of the graph distance plays a central role in a variety of problems, including volume growth, metric scaling, and universality phenomena in random geometry. Our results demonstrate a sequence of dimension-dependent phase transitions in the scaling behavior of these fluctuations, leading to qualitatively different regimes across dimensions lower than six, equal to six, and higher than six. This is joint work with Arka Adhikari (University of Maryland).

14:20-15:00: Xinyi Li (Peking University)

Non-existence of several random fractals in Brownian motion and Brownian loop soup

In this talk, we will discuss a unified approach to establish the non-existence of three types of random fractals: (1) pioneer triple points of planar Brownian motion, (2) pioneer double cut points of planar and three-dimensional Brownian motions, and (3) double points on boundaries of clusters of planar Brownian loop soup at the critical intensity. These fractals have the common feature that they are associated with an intersection or disconnection exponent which yields a Hausdorff dimension "exactly zero". The non-existence of (1) and (3) answers open questions from Burdzy-Werner (1996) and Qian (2021). This is a joint work with Yifan Gao (Westlake), Runsheng Liu (PKU) and Wei Qian (HKU).

15:20-16:00: Jun Kigami (Ritsumeikan University)

Heat kernel lower bound estimates for symmetric pure jump processes via averaged jump kernels

We derive a heat kernel lower bound estimates for symmetric pure jump processes on general volume doubling metric measure spaces with possible degenerate and/or singular jump kernels using averaged jump kernels. As an application, the main result of this paper is applied to derive a lower bound estimate for the transition densify function of the trace of Brownian motions on Sierpinski gaskets on the bottom of the Sierpinski gasket.

16:10-16:50: Shouhei Honda (University of Tokyo)

Gel'fand's inverse problem under Ricci curvature bounds

The classical Gel'fand's inverse problem asks whether a Riemannian manifold is uniquely determined by the knowledge of the heat kernel on any open subset of the manifold. We study this inverse problem in the non-smooth setting in the framework of $\mathrm{RCD}(K,N)$ spaces, namely, metric-measure spaces with synthetic Riemannian Ricci curvature bounded below by K and dimension bounded above by N. We establish the unique solvability of Gel'fand's inverse problem for the class of compact $\mathrm{RCD}(K,N)$ spaces whose regular set admits C^1-Riemannian structure. As an application, we obtain the stability of Gel'fand's inverse problem in the class of closed Riemannian manifolds with bounded Ricci curvature, diameter and volume bounded from below. We note that the results are new even for Einstein orbifolds and (weighted) Riemannian manifolds with non-smooth boundary. This is a joint work with Jinpeng Lu (University of Helsinki), based on `arXiv:2602.14527`.

13 May 2026

09:30-10:10: Kôhei Sasaya (University of Tokyo)

On p-energy measures: construction and properties inherited from associated p-energy forms

In recent years, p-energy forms, which are L^p-counterparts of Dirichlet forms, have been extensively studied in analysis on fractals and metric spaces. In this talk, we present a result on the construction of canonical p-energy measures associated with strongly local, regular p-energy forms (S. 2026, JFA). We also discuss several properties of these measures that are inherited from the associated p-energy forms, based on joint work with Naotaka Kajino (RIMS).

10:30-11:10: Aobo Chen (Tsinghua University)

Elliptic Harnack inequalities for mixed local and nonlocal p-energy form on metric measure spaces

In the context of metric measure spaces, we introduce an axiomatic formulation of mixed local and nonlocal p-energy forms. Within this framework, we use the Poincaré inequality, the cutoff Sobolev inequality, and mild assumptions on the jump measure to establish the weak and strong elliptic Harnack inequalities for such mixed forms. Our approach is based on the De Giorgi–Nash–Moser method and extends the corresponding results for Dirichlet forms without the killing part, as well as for mixed energy forms on Euclidean spaces. This is joint work with Zhenyu Yu.

11:20-12:00: Shiping Cao (Chinese University of Hong Kong)

Boundary trace theorems for reflected jump processes

On domains in general metric measure spaces, we study the trace of reflected jump Dirichlet forms onto the boundary. We assume that the reflected Dirichlet form satisfies two-sided mixed stable-like heat kernel estimates, and the boundary satisfies a capacity density condition. In this talk, we will discuss the Besov space type characterization of the domain of the trace Dirichlet form, discuss the properties of harmonic measures, and finally provide estimates of the jump kernel of the trace Dirichlet form. This is a joint work with Zhen-Qing Chen and Takashi Kumagai.

13:30-14:10: Takashi Kumagai (Waseda University)

Quantitative homogenization on time-dependent random conductance models with stable-like jumps

Homogenization theory seeks effective equations (or limiting processes) that describe the averaged behavior of solutions to equations (or the scaling limits of processes) with heterogeneous coefficients. It has seen substantial development in both PDEs and probability. In this talk, I will present quantitative homogenization results for stable-like long range random walks in time-dependent random conductance models, where the conductances are bounded from above, but may be degenerate. This talk is based on joint work with X. Chen (Shanghai), Z.-Q. Chen (Seattle) and J. Wang (Fuzhou).

14:20-15:00: Ecaterina Sava-Huss (University of Innsbruck)

Speed, displacement and entropy of branching random walks

We consider supercritical branching random walks (BRW) on infinite graphs or groups, and we investigate the limit behavior of the population distributions. In particular, we focus the speed of the BRW, the maximal and the minimal displacements of the BRW, and we discuss shortly about the asymptotic entropy of such processes. This talk is based on works in collaboration R. Kaiser, M. Klötzer, and K. Kolesko.

15:20-16:00: Tianyi Zheng (University of California, San Diego)

Quasi-isometric nonequivalence for random subsets in products of trees

We study quasi-isometric embeddings from a random Bernoulli percolation sample on the product of two regular trees into the product itself, and show some rigidity properties that can be seen as an extension of quasi-isometric rigidity of higher rank non-uniform lattices. We also prove that two independent samples are almost surely not quasi-isometric equivalent, thus confirming that such a phenomenon occurs in the higher-rank setting, as conjectured by Abert. Joint work with Zhiqiang Li and Ranfeng Yu.