京都大学数理解析研究所，共同研究事業（公開型）

偏微分方程式と確率過程に現れる均質化理論

Homogenization in PDE and Stochastic Processes

Location: Room 111 (RIMS, Kyoto University)
Period: August 24 (Mon)–August 26 (Wed), 2026

Abstracts

- **August 24 (Mon) AM**

Quantitative Homogenization of PDEs with Neumann boundary conditions: a probabilistic approach

Zhen-Qing Chen (University of Washington)

In this talk, I will present a probabilistic framework for studying the quantitative homogenization of multi-scale, semilinear second-order PDEs on convex domains with Neumann boundary conditions. By linking these PDEs to backward stochastic differential equations (BSDEs) associated with slow-fast reflected SDE systems, we establish effective macroscopic models and quantify convergence rates. Based on joint work with Jing Wu.

Stochastic homogenization of viscous Hamilton-Jacobi equations in 1d: open problems

Elena Kosygina (Baruch College)

Stochastic homogenization of viscous Hamilton-Jacobi equations is well studied, but even in one spatial dimension many natural questions remain open. The one-dimensional setting is very special and should be more amenable to a quantitative study of convergence and properties of the effective Hamiltonian. The talk will describe some of these problems, both purely analytical and probabilistic. The latter have close connections to problems studied in a different context and might be of interest to specialists in both PDEs and probability.

Quantification of ergodicity for Hamilton–Jacobi equations in a dynamic random environment

Xiaoqin Guo (University of Cincinnati)

Motivated by the tensionless KPZ equation (a model of stochastic growth), we consider quantitative large-time averages for Hamilton–Jacobi equations in a dynamic random environment that is stationary ergodic and has finite- range dependence in time. In this talk, we establish, up to slowly varying factors, convergence rates with exponent 1/2 for the large-time averages of both the solutions and the associated metric problem toward their ergodic limits. Our proof introduces a new almost-Lipschitz regularity theory for the metric problem, which is of independent interest. Joint work with Wenjia Jing (Tsinghua), Hung V. Tran (Wisconsin), and Yuming Zhang (Auburn).

• **August 24 (Mon) PM**

A Liouville theorem for convex functions with periodic Monge-Ampere measure

Tianling Jin (Hong Kong University of Science and Technology)

We study global convex solutions of the Monge-Ampere equation with nonzero nonnegative locally finite periodic Borel measures in the whole Euclidean space. We prove a Liouville-type theorem showing that every such solution admits a unique decomposition, up to an additive constant, as the sum of a quadratic polynomial and a periodic function. This extends earlier results of Caffarelli-Li and Li-Lu, which required the measure to have a density with regular or bounded logarithm, to the full generality of periodic measures, allowing degeneracy and singularities. A key ingredient is a new dichotomous Harnack-type inequality for linearized Monge-Ampere equations with nonnegative periodic measures, which compensates for the failure of doubling and engulfing properties in the degenerate setting.

Birkhoff Average Estimates and Applications to Quasi-Periodic Homogenization

Son Tu (Baylor University)

For Diophantine frequency vectors, we establish quantitative estimates for the discrepancy between Birkhoff averages and spatial averages for observables with various regularity assumptions. These estimates lead, to the best of our knowledge, to the sharpest currently available convergence rates for Hölder continuous observables. The approach also yields new sharp estimates for discrete-time rotations on higher-dimensional tori. As applications, we derive improved quantitative homogenization results for Hamilton–Jacobi equations in spatially quasi-periodic media.

Emergence of an unsaturated Darcy-type law and a Richards-type equation via homogenization of a Stokes–Cahn–Hilliard system

Rong Lei (Tohoku University)

In this talk, we derive macroscopic unsaturated flow laws from a microscopic Stokes–Cahn–Hilliard system in a periodically perforated porous medium via homogenization. The resulting Darcy-type law contains an explicit capillary force represented by a Korteweg term and is generally not reducible to the classical Darcy–Buckingham form. Together with the homogenized Cahn–Hillard equation, it yields a generalized Richards-type equation. This is a joint work with Jun Masamune (Kyoto University).

• **August 25 (Tue) AM**

Equilibrium Fluctuations of the SSEP on Random Graphs with Conductances

Alessandra Faggionato (Sapienza University of Rome)

We study the symmetric simple exclusion process on a random weighted undirected graph whose vertices are given by a point process and whose edge weights (conductances) determine the jump rates. Under mild assumptions on the environment (stationarity, ergodicity, and suitable moment conditions), the empirical density converges, for almost every realization of the environment, to the solution of a heat equation with homogenized diffusivity. Here we analyze equilibrium fluctuations. For $d \geq 3$, under the same assumptions as for the hydrodynamic limit, the density fluctuation field converges, in the sense of finite-dimensional distributions and for almost every realization of the environment, to a generalized Ornstein–Uhlenbeck process. For d=2, the same result holds under additional regularity assumptions ensuring Hölder estimates for the associated parabolic equations. We also discuss applications to several classical random graph models. Joint work with Alberto Chiarini.

Quenched moderate deviations for random walk in random scenery

Ryoki Fukushima (University of Tsukuba)

Random walk in random scenery is a sum of values of random field along random walk path. Even if the random field has independent values, it is a sum of correlated random variables since the random walk may visit one site several times. This model naturally appears in some models of random walk in random environments such as random hopping dynamics or random conductance model with a layered structure. In this talk, I will report a moderate deviation type bound in the case when the random field is independent and has finite mean.

Scaling limit of additive functionals for reversible non-gradient exclusion process: critical cases

Chenlin Gu (Tsinghua University)

For the reversible speed-change exclusion process in $\mathbf{Z}^d$, we study the scaling limit of additive functionals. Concerning the local centered function f, the previous work (Commun. Math. Phys. 104, 1-19, 1986) by Kipnis and Varadhan and (Comm. Pure Appl. Math., 66: 649-677, 2013) by Goncalves and Jara respectively covered the cases $d \geq 3$ and $d = 1$. The present paper completes the missing part $d = 2$, and also develops the theory for functions with higher degree. The novelty is a quantitative homogenization of the resolvent, which allows to overcome the obstacle of correlation function in non-gradient models. This talk is based on a joint work with Linzhi Yang (Qiuzhen College) and Linjie Zhao (Huazhong University of Science and Technology).

• **August 25 (Tue) PM**

Navier's wall law for viscous flows in randomly rough domains

Mitsuo Higaki (Kobe University)

Wall laws have long been used in engineering as empirical effective boundary conditions for viscous flows over rough surfaces, classically exemplified by Nikuradse's experiments in 1933. In the laminar regime, such laws predict that flows in rough pipes can be approximated by solutions for a smooth pipe satisfying a Navier slip boundary condition, often called Navier's wall law. In this talk, I will present a rigorous justification of this prediction for steady Navier-Stokes flows in 3D randomly rough pipes. More precisely, for a probability space whose elements are rough domains, we prove optimal convergence rates under suitable quantitative ergodicity assumptions.

The proof relies on deterministic and probabilistic estimates for boundary layers, which describe the flow near the rough boundary. These estimates are inspired by methods from quantitative stochastic homogenization for elliptic equations, including ideas of Armstrong-Smart, Armstrong-Kuusi-Mourrat, Gloria-Neukamm-Otto, and Shen. Note, however, that we consider a probability space of rough domains rather than one of coefficient matrices. The quantitative ergodicity assumptions mentioned above are formulated through functional inequalities for random variables, such as the logarithmic Sobolev inequality and the spectral gap inequality. This talk is based on joint work with Jinping Zhuge (Morningside Center of Mathematics, China) and Yulong Lu (University of Minnesota, USA).

Quantitative homogenization of first-order ODEs

Panrui Ni (Waseda University)

In this talk, I will present my recent results on the quantitative homogenization of first-order ODEs. For single-scale scalar ODEs, we obtain a sharp $O(\epsilon)$ convergence rate and characterize the effective constant. In the multi-scale setting, our results match those of H. Ibrahim and R. Monneau for long times but improve the short-time error to $O(\epsilon)$. We also initiate the study of quasi-periodic homogenization in this context. The scalar framework is further extended to higher dimensions under a boundedness assumption on trajectories. For weakly coupled systems with fast switching rates, we obtain for the first time a convergence rate of order $O(\epsilon)$. These results have applications to linear transport equations and broader connections to PDEs and gradient systems.

Arm events of percolation with impurities

Satomi Watanabe (Kyoto University)

Recent developments in percolation with mesoscopic impurities have highlighted its significant connection to the forest fire model. In particular, the seminal work by van den Berg and Nolin bridged these domains, successfully elucidating the near-ctitical forest fire model (without recoveries) on the triangular lattice. In this talk, we review their achievements focusing on the stability of arm events. Furthermore, we discuss perspectives on future research directions, including a brief mention of our ongoing work.

• <u>**August 26 (Wed) AM**</u>

Anchored Nash inequalities and heat kernel bounds for random conductance models with long-range jumps

Sebastian Andres (Technische Universität Braunschweig)

In this talk we present an anchored version of the Nash inequality for discrete non-local divergence-form operators with degenerate weights. They allow to control the L^2-norm of a function in terms of Dirichlet forms associated with random walks having possibly unbounded jump rates. From those Nash inequalities we deduce on-diagonal upper heat kernel estimates for a class of random conductance models with degenerate jump rates that allow long-range jumps and are assumed to be ergodic and to satisfy a suitable moment condition. This talk is based on a joint work with Xin Chen (Shanghai), Martin Slowik (Mannheim) and Kun Yin (Shanghai).

Quantitative homogenization for obstacle problems of convex Hamilton-Jacobi equations

Takuya Sato (University of Tokyo)

We study the periodic homogenization of convex Hamilton-Jacobi equations with obstacle functions depending on microscopic spatial variables. Such obstacle problems arise as PDEs satisfied by value functions of optimal stopping problems in optimal control, as well as level-set equations describing interface motion whose propagation is partially blocked by obstacles. Based on the analysis of variational representation formulae, we establish the optimal convergence rate $O(\varepsilon)$ for the homogenization limit under the assumption that the obstacle function is Lipschitz continuous. Moreover, we show that this convergence rate may deteriorate due to the presence of the obstacle when the obstacle function is not Lipschitz continuous.

- **August 26 (Wed) PM**

Fluctuating hydrodynamics for a chain of nonlinear stochastic oscillators

Kohei Hayashi (University of Osaka)

We study macroscopic behavior of a chain of oscillators perturbed by some random noise, which admits three conservation laws. Particularly, we assume that the Hamiltonian dynamics of the oscillators is governed by a nonlinear potential that weakly perturbs the purely harmonic one. As a main result, we show that fluctuations of two phonon modes converge to the stochastic Burgers equations. This talk is based on a joint work with Stefano Olla.

Optimal rate of convergence to some degenerate asymptotic profiles for fast diffusion

Goro Akagi (Tohoku University)

In this talk, we discuss the optimal rate of convergence to asymptotic profiles of solutions, which vanish in finite time, to the Fast Diffusion Equation posed on bounded domains. Quantitative studies in this direction have developed significantly since the celebrated work of Bonforte and Figalli (2021). However, in most existing results, asymptotic profiles are assumed to be *nondegenerate*, meaning that the linearized operator has a trivial kernel. In fact, such nondegeneracy holds for generic domains. On the other hand, it is well known that least-energy solutions to the Emden–Fowler equation in thin annuli are nonradial. As a consequence, they are degenerate and thus fall outside the scope of previous works. In this talk, inspired by recent work of König and Yu (2024^+), we establish the optimal convergence rate to such degenerate asymptotic profiles in thin annuli for certain space dimensions. This talk is based on a recent joint work with Norihisa Ikoma (Keio University) and Yasunori Maekawa (Kyoto University).