NYU Shanghai-Kyoto-Waseda Young Probabilists' Meeting

8–9 May 2024

Research Institute for Mathematical Sciences, Kyoto University

Organisers: David Croydon (Kyoto University), Takashi Kumagai (Waseda University), Alejandro Ramirez (NYU Shanghai), Pierre Tarrès (NYU Shanghai)

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09:20–10:00: Alex DREWITZ (University of Cologne)
Random walk among moving traps

10:10–10:50: Makoto NAKASHIMA (Nagoya University)
Stochastic quantization of three dimensional Edwards' model

11:00–11:40: Wei-Min WANG (NYU Shanghai)
On the dynamics of nonlinear random and quasi-periodic systems

13:30–14:10: Satomi WATANABE (City University of Hong Kong)
Random walk on uniform spanning tree and loop-erased random walk

14:20–15:00: Wei WU (NYU Shanghai)
Continuous symmetry breaking for rotator models and dimers

15:20–15:40: Ryoichiro NODA (Kyoto University)
Convergence of local times of stochastic processes associated with resistance forms

15:50–16:10: Shuo QIN (NYU Shanghai)
Recurrence and transience of multidimensional elephant random walks

16:20–17:00: Stefan JUNK (Gakushuin University)
Strong disorder and very strong disorder are equivalent for directed polymers

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09:20–10:00: Xinyi LI (Peking University)
Sharp asymptotics of the disconnection time of large cylinders by simple and biased random walks

10:10–10:50: Tomohiro AYA (Kyoto University)
Quantitative stochastic homogenization of elliptic and parabolic equations with unbounded coefficients

11:00–11:40: Elie AIDEKON (Fudan University)
A connection between skew Brownian motion and BESQ flow

13:30–14:10: Eric ENDO (NYU Shanghai)
Local central limit theorem for long-range potentials with continuous spins

14:20–15:00: Takumu OOI (Tokyo University of Science)
Scaling limit of Liouville simple random walk on the 2-dimensional lattice

15:20–16:00: Benoit COLLINS (Kyoto University)
Strong freeness for tensor models

`https://www.kurims.kyoto-u.ac.jp/~croydon/NYU-Sh-Kyoto-Waseda/YoungProb2024.htm`

Abstracts

8 May 2024

09:20–10:00: Alex DREWITZ (University of Cologne)

Random walk among moving traps

Random walk and Brownian motion among static obstacles have been the subject of extensive research during the last couple of decades. Of particular interest have been the survival probability as well as the path behavior of the motion conditioned on survival. However, comparatively less attention has been given to scenarios involving moving traps. We survey some of the results obtained in the setting of a Poisson cloud of moving traps during the last decade and discuss a recently established functional central limit theorem in dimensions 6 and higher.

10:10–10:50: Makoto NAKASHIMA (Nagoya University)

Stochastic quantization of three dimensional Edwards' model

Edwards' model is a probabilistic model for long polymer chains that take into account their self-exclusive volume effects. It is defined by Edwards' measure ν_λ on $(X, \mathcal{B}(X))$, where $X = C([0,1], \mathbb{R}^d)$ is the set of continuous functions from $[0,1]$ to $\mathbb{R}^d$ with the topology of uniform convergence, $\mathcal{B}(X))$ is its Borel σ-algebra, and ν_λ $(\lambda > 0)$ is a probability measure that is formally defined by

$$\nu_\lambda(d\omega) = \frac{1}{Z_\lambda} \exp\left(-\lambda \int_0^1 \int_0^t \delta_0(\omega_t - \omega_s) ds dt\right) \nu_0(d\omega).$$

where ν_0 is a Wiener measure.

In this talk, we talk about the Dirichlet form approach for construction of an X-valued Markov process which has an invariant measure ν_λ.

This talk is based on the joint work with S. Albeverio (Univ. of Bonn), S. Kusuoka (Kyoto Univ.), S. Liang (Waseda Univ.).

11:00–11:40: Wei-Min WANG (NYU Shanghai)

On the dynamics of nonlinear random and quasi-periodic systems

We discuss time evolution of nonlinear random and quasi-periodic systems. Examples of such systems include the nonlinear random and the nonlinear quasi-periodic Schrödinger equations on the lattice. We shall elaborate on their similarities.

13:30–14:10: Satomi WATANABE (City University of Hong Kong)

Random walk on uniform spanning tree and loop-erased random walk

Random walks on random graphs are associated with diffusion phenomena in disordered media. This talk focuses on the simple random walks on uniform spanning trees (UST) and loop-erased random walks (LERW). We will first demonstrate log-logarithmic heat kernel fluctuations for the three-dimensional UST. Next, we will discuss the sub-Gaussian estimate for the annealed off-diagonal heat kernel of the simple random walk on high-dimensional LERWs.

14:20–15:00: Wei WU (NYU Shanghai)

Continuous symmetry breaking for rotator models and dimers

Abstract TBC

15:20–15:40: Ryoichiro NODA (Kyoto University)

Convergence of local times of stochastic processes associated with resistance forms

The class of resistance metrics is a generalization of effective resistance metrics on electrical networks. A resistance metric combined with measure determines a Dirichlet form and hence a strong Markov process on the space, which corresponds to a continuous-time Markov chain on a measured electrical network. Croydon-Hambly-Kumagai (2017) showed that if a sequence of spaces equipped with resistance metrics and measures converge with respect to the Gromov-Hausdorff-vague topology and a uniform volume doubling (UVD) condition is satisfied, then the associated stochastic processes and local times also converge. However, the UVD condition is too strong for many sequences of random graphs. In the subsequent work of Croydon (2018), the UVD condition was relaxed and the convergence of the processes was established under a weaker non-explosion condition. However, the convergence of local times was left open. In our results, it is established that if the spaces additionally satisfy a certain metric-entropy condition, which is weaker than the UVD condition, then the local times of the processes also converge. The metric-entropy condition can be checked using volume estimates of balls in the spaces, and we also present some example applications.

15:50–16:10: Shuo QIN (NYU Shanghai)

Recurrence and transience of multidimensional elephant random walks

We prove a conjecture by Bertoin that the multidimensional elephant random walk on the d-dimensional lattice is transient if $d \geq 3$. In dimensions $d = 1, 2$, we prove that phase transitions between recurrence and transience occur at $p = (2d + 1)/(4d)$.

16:20–17:00: Stefan JUNK (Gakushuin University)

Strong disorder and very strong disorder are equivalent for directed polymers

We consider the directed polymer model, which describes paths affected by a random space-time environment in spatial dimension $d > 2$. The model undergoes a phase transition between a high-temperature, weak disorder phase and a low-temperature, strong disorder phase, which is characterized by whether the associated (normalized) partition function converges to zero. From the physical point of view, it is more natural to consider a very-strong-disorder regime, characterized by whether the partition function converges to zero exponentially fast. It has been a long-standing conjecture that these two notions are equivalent and we now give a proof of this. Moreover, our proof reveals that weak disorder holds at the critical value. Joint work with Hubert Lacoin.

9 May 2024

09:20–10:00: Xinyi LI (Peking University)

Sharp asymptotics of the disconnection time of large cylinders by simple and biased random walks

We consider the asymptotic disconnection time of a discrete cylinder $(\mathbb{Z}/N\mathbb{Z})^d \times \mathbb{Z}$, $d \geq 2$ by simple and biased (in the $\mathbb{Z}$ direction) random walks. For simple random walk, we derive a sharp asymptotic lower bound that matches the upper bound from [A.-S. Sznitman, Ann. Probab., 2009] which allows us to identify the weak limit of the rescaled disconnection time. For the biased walk, we obtain bounds that asymptotically match in the principal order when the bias is not too strong, which greatly improves previous results from [D. Windisch, Ann. Appl. Probab., 2008]. Based on a joint work in progress with Yu Liu (PKU) and Yuanzheng Wang (PKU).

10:10–10:50: Tomohiro AYA (Kyoto University)

Quantitative stochastic homogenization of elliptic and parabolic equations with unbounded coefficients

Quantitative stochastic homogenization is a field that aims to obtain a rate of the convergence in stochastic homogenization under the assumption of quantitative mixing conditions in ergodicity. In this talk, we consider stochastic homogenization of elliptic and parabolic equations with unbounded and non-uniformly elliptic coefficients. Extending subadditive arguments, we get an estimate for the rate of the convergence of the solution of the Cauchy-Dirichlet problem under the condition that coefficients in the unit cube have a certain exponential integrability.

11:00–11:40: Elie AIDEKON (Fudan University)

A connection between skew Brownian motion and BESQ flow

The skew Brownian motion (Itô–McKean, 1965, Walsh, 1978) is a diffusion which behaves as a Brownian motion away from 0, but has a "local drift" every time it crosses 0. It can be constructed by assigning signs to Brownian excursions away from 0, each excursion being positive with probability p and negative with probability $1-p$. It can equivalently (Harrison–Shepp, 1981) be seen as the strong solution of the SDE $dX_t = dB_t + qdL_t(X)$ where $L_t(X)$ denotes the local time of the diffusion at 0, and $q = 2p - 1$. We show how this process naturally arises when exploring a flow of squared Bessel processes driven by a white noise.

13:30–14:10: Eric ENDO (NYU Shanghai)

Local central limit theorem for long-range potentials with continuous spins

The local central limit theorem is a result that is vastly studied for random variables generated by models coming from statistical mechanics. Several papers such as [1, 2, 3] give certain conditions for the potentials such that, if the central limit theorem holds for random fields defined on the lattice, then the local central limit theorem will hold as well.

We consider potentials with two-body long-range interactions of Ising type with spins in $\mathbb{R}$ on the lattice $\mathbb{Z}^d$ where the boundary conditions are tempered. We prove that for a sequence of Gibbs measures for which the central limit theorem is satisfied, then the local central limit theorem also holds for that sequence. Our result extends [1] that shows for short-range models.

Joint work with R. Fernández (NYU Shanghai, China), V. Margarint (University of North Carolina Charlotte, USA), and T.X. Nguyen (NYU Shanghai, China).

[1] M. Campanino, G. Del Grosso, and B. Tirozzi. Local limit theorem for Gibbs random fields of particles and unbounded spins. *Journal of Mathematical Physics* **20**(8): 1752–1758, 1979.

[2] E. O. Endo, and V. Margarint. Local central limit theorem for long-range two-body potentials at sufficiently high temperatures. *Journal of Statistical Physics* **189**(34), 2022.

[3] A. Procacci, B. Scoppola. On the local central limit theorem for interacting spin systems. *ArXiv 2308.06178*, 2023.

14:20–15:00: Takumu OOI (Tokyo University of Science)

Scaling limit of Liouville simple random walk on the 2-dimensional lattice

Liouville Brownian motion (LBM) is 2-dimensional Brownian motion time-changed by the Liouville measure. We introduce the Liouville simple random walk on the 2-dimensional lattice as the counterpart of LBM to the 2-dimensional lattice. We provide that the scaling limit of the Liouville simple random walk on the 2-dimensional lattice is LBM, and more general results. We also consider the convergence of their spectral structures.

15:20–16:00: Benoit COLLINS (Kyoto University)

Strong freeness for tensor models

We consider random GUE tensors arising in the modeling of epsilon freeness (in a joint work with Charlesworth) and prove that they are strongly asymptotically free under some natural conditions. Our result relies on techniques developed by Bandeira, Boedihardjo, and van Handel. This is a report on a joint work with Wangjun Yuan (Luxembourg).