# Random Geometry and Topology and Related Topics

29 September - 1 October 2025

Research Institute for Mathematical Sciences, Kyoto University

Organisers: Naotaka Kajino (Kyoto University, Chair), David Croydon (Kyoto University), Tomoyuki Shirai (Kyushu University), Satomi Watanabe (Kyoto University)

29 September 2025
13:30-14:10: Grégory MIERMONT (ENS Lyon/RIMS Visiting Professor)
14:20-15:00: Eleanor ARCHER (Université Paris-Dauphine)
15:20-16:00: Subhro GHOSH (National University of Singapore)
16:10-16:50: Shu KANAZAWA (Queen Mary University of London) Central limit theorem for linear eigenvalue statistics of the adjacency matrices of random simplicial complexes
30 September 2025
09:30-10:10: Takashi KUMAGAI (Waseda University)
10:20-11:00: Mo Dick WONG (Durham University)
11:15-11:40: Takumu OOI (Tokyo University of Science)
11:50-12:30: Jeremie BRIEUSSEL (Université de Montpellier)
14:00-14:40: Hélène GUÉRIN (Université du Québec à Montréal)
14:50-15:15: Kohki SAKAMOTO (University of Tokyo)
15:35-16:00: Noe KAWAMOTO (National Center for Theoretical Sciences, Taiwan)
16:10-16:50: Lu-Jing HUANG (Fujian Normal University)Polynomial lower bound on the effective resistance for the one-dimensional critical long-range percolation
1 October 2025
09:40-10:20: Ikkei HOTTA (Yamaguchi University)
10:30-11:10: Takashi OWADA (Purdue University)
11:30-12:10: Daisuke KAZUKAWA (Tokyo Metropolitan University)
13:40-14:20: Wei QIAN (City University of Hong Kong)
14:30-15:10: Yichao HUANG (Beijing Institute of Technology)
15:20-16:00: Pierre NOLIN (City University of Hong Kong)
https://www.kurims.kyoto-u.ac.jp/~croydon/RGTRT.html

## Abstracts

# 29 September 2025

### 13:30-14:10: Grégory MIERMONT (ENS Lyon/RIMS Visiting Professor)

Scaling limits of random planar maps with large faces

Many "local" models of random planar maps, obtained by gluing uniformly at random the edges of a collection of polygons with given degree, in order to obtain a topological sphere, are known to converge to the so-called Brownian sphere, a canonical model of random surface. However, it is possible to escape from the wide universality class of the Brownian sphere, either by considering models of random planar maps with long-range correlations, typically by endowing them with a statistical physics model at criticality, or by considering local models in which the variance of the face degrees distribution is infinite. In this talk, we investigate this second question by showing that random Boltzmann maps whose face degree distributions belong to a stable domain of attraction converge in the scaling limit to a random fractal object, which is called a stable carpet or a stable gasket depending on the value of the stable exponent. Indeed, a phase transition for the topology of these objects occur at the value 3/2 of the stable exponent. This completes earlier results by Le Gall-Miermont, who obtained a scaling limit result only up to extraction of subsequences. This is joint work with Nicolas Curien and Armand Riera.

#### 14:20-15:00: Eleanor ARCHER (Université Paris-Dauphine)

Scaling limits of stable quadrangulations

The Brownian sphere is a celebrated model in random planar maps and arises as a universal scaling limit of a large class of "finite variance" random planar map models. In this talk we introduce certain stable analogues obtained as scaling limits of quadrangulations with heavy-tailed vertex degrees, and discuss some of their properties. Based on joint work with Ariane Carrance and Laurent Ménard.

#### 15:20-16:00: Subhro GHOSH (National University of Singapore)

Fractal Gaussian Networks: A sparse random graph model based on Gaussian Multiplicative Chaos

We propose a novel stochastic network model, called Fractal Gaussian Network (FGN), that embodies well-defined and analytically tractable fractal structures. Such fractal structures have been empirically observed in diverse applications. FGNs interpolate between the popular purely random geometric graphs (a.k.a. the Poisson Boolean network), and random graphs with increasingly fractal behavior, forming a parametric family of sparse random geometric graphs that are parametrized by a fractality parameter which governs the strength of the fractal structure. FGNs are driven by the latent spatial geometry of Gaussian Multiplicative Chaos (GMC), a canonical model of fractality in mathematical physics. We investigate the asymptotics of various small motif counts in the FGN, the natural question of detecting fractality, the problem of parameter estimation and the fundamental properties of the FGN as a random graph model. We substantiate our model with phenomenological analysis of the FGN in the context of available scientific literature for fractality in networks, including applications to real-world massive network data. Based on joint work with K. Balasubramanian and X. Yang.

### 16:10-16:50: Shu KANAZAWA (Queen Mary University of London)

Central limit theorem for linear eigenvalue statistics of the adjacency matrices of random simplicial complexes

We consider the (higher-dimensional) adjacency matrix of the Linial–Meshulam complex model, which is a higher-dimensional generalization of the Erdős–Rényi random graph model. Recently, Knowles and Rosenthal proved that the empirical spectral distribution of the adjacency matrix is asymptotically given by Wigner's semicircle law in a diluted regime. In this talk, I will present a central limit theorem for the linear eigenvalue statistics for test functions of polynomial growth that are of class  $C^2$  on a closed interval. The proof is based on higher-dimensional combinatorial enumerations and concentration properties of random symmetric matrices. Furthermore, when the test function is a polynomial function, we obtain an explicit formula for the variance of the limiting Gaussian distribution. This is joint work with Khanh Duy Trinh (Waseda University).

# 30 September 2025

### 09:30-10:10: Takashi KUMAGAI (Waseda University)

Scaling limit for Brownian motions on the l-level Sierpinski gaskets

In two dimensions, the l-level Sierpinski gasket SG(l) is obtained by splitting an equilateral triangle into a collection of  $l^2$  equilateral triangles of equal size and with the same total area, retaining only the l(l+1)/2 triangles with the same orientation as the original triangle, and then iterating this procedure indefinitely. In this talk, we will present a result that the canonical diffusions on the spaces SG(l),  $l \geq 2$ , can be rescaled to yield Brownian motion on the initial triangle. Our argument also applies to the analogous higher-dimensional Sierpinski gaskets. Moreover, we derive a corresponding scaling limit for the associated transition densities. This is a joint work with B. M. Hambly (Oxford) and D. A. Croydon (Kyoto).

### 10:20-11:00: Mo Dick WONG (Durham University)

Spectral property of Liouville quantum gravity

The Liouville quantum gravity (LQG) surface, formally defined as a 2-dimensional Riemannian manifold with conformal factor being the exponentiation of a Gaussian free field, is closely related to random planar geometry as well as scaling limits of models from statistical mechanics. In this talk, I shall explain a Weyl law for subcritical LQG surfaces (joint work with Nathanael Berestycki) and also discuss some challenges and ongoing work in the critical setup.

### 11:15-11:40: Takumu OOI (Tokyo University of Science)

Characterization and continuity of the Revuz correspondence via killing of processes

It is known that there is a one-to-one correspondence, called the Revuz correspondence, between smooth measures and positive continuous additive functionals (PCAFs). For example, Liouville Brownian motion is a time-changed process of Brownian motion by the PCAF associated with the Liouville measure. In this talk, we present a new characterization of the Revuz correspondence using the killing measure for a killing inside and the energy functional for continuously escaping to the cemetery point. We also discuss its homeomorphism and continuity with respect to the topology defined through the killing measure and the functional. This talk is based in part on joint work with K. Tsuchida and T. Uemura.

### 11:50-12:30: Jeremie BRIEUSSEL (Université de Montpellier)

Furstenberg entropy spectrum of stationary actions of special linear groups

A measure on a space is invariant if it is preserved by the action of each element of the group. A measure is stationary if it is equal to the average, according to some probability measure, of its translates by group elements. The Furstenberg entropy is an invariant of stationary systems that measures the lack of invariance. In particular it vanishes for invariant measures. The aim of the talk is to give a description of all possible values taken by the Furstenberg entropy of stationary actions of  $SL(d,\mathbb{R})$  endowed with a reasonable probability measure. For  $SL(2,\mathbb{R})$ , this is related to some Markov chains on random non-compact surfaces. It is a joint work with Tianyi Zheng.

# 14:00-14:40: Hélène GUÉRIN (Université du Québec à Montréal)

Manifold explored by a diffusion process and estimation of its invariant measure

From the observation of a diffusion path on a compact, connected manifold without boundary, we consider the problem of estimating its invariant measure. As noted by Divol (2022), in the context of i.i.d. random variables sampled from the invariant measure on the manifold, the rate of convergence can be increased by smoothing the empirical measure. Therefore, we introduce a kernel estimator based on the occupation measure of a diffusion path, which improves the speed of convergence obtained by Wang and Zhu (2023) in the case of a Langevin diffusion. The resulting rate is indeed the minimax rate of estimation for this problem. This is a joint work with Vincent Divol, Dinh Toan Nguyen, and Viet Chi Tran.

### 14:50-15:15: Kohki SAKAMOTO (University of Tokyo)

Poisson-Voronoi tessellations on symmetric spaces

It is known from the work of Benjamini, Krauz, and Paquette that the simple random walk on a Poisson-Voronoi tessellation in hyperbolic spaces has a strictly positive linear drift, which allows us to study its limit distribution on the boundary. I will present a formula for the Hausdorff dimension of this limit distribution, with some background from random walks on hyperbolic groups and on random trees. If time permits, I will also give a brief survey on recent interactions between Poisson-Voronoi tessellations on symmetric spaces and measured group theory.

### 15:35-16:00: Noe KAWAMOTO (National Center for Theoretical Sciences, Taiwan)

Asymptotic expansion of the critical point for oriented percolation in high dimensions

We consider the nearest-neighbor oriented percolation, and study an asymptotic expansion of the critical point  $p_c$  in powers of  $d^{-1}$  as  $d \to \infty$ , in a way that  $p_c = 1 + C_1 d^{-2} + C_2 d^{-3} + C_3 d^{-4} + O(d^{-5})$ , where  $C_1$  to  $C_3$  are constants. The proof relies on the lace expansion, which is one of the most powerful tools to analyze the mean-field behavior of statistical-mechanical models in high dimensions. In this talk, we focus on the background of this topic and the potential problem that may arise from the results.

### 16:10-16:50: Lu-Jing HUANG (Fujian Normal University)

Polynomial lower bound on the effective resistance for the one-dimensional critical long-range percolation

In this work, we study the critical long-range percolation on  $\mathbb{Z}$ , where an edge connects i and j independently with probability 1 for |i-j|=1 and with probability  $1-\exp\{-\beta\int_i^{i+1}\int_j^{j+1}|u-v|^{-2}dudv\}$  for some fixed  $\beta>0$ . Viewing this as a random electric network where each edge has a unit conductance, we show that with high probability the effective resistances from the origin 0 to  $[-N,N]^c$  and from the interval [-N,N] to  $[-2N,2N]^c$  (conditioned on no edge joining [-N,N] and  $[-2N,2N]^c$ ) both have a polynomial lower bound in N. Our bound holds for all  $\beta>0$  and thus rules out a potential phase transition (around  $\beta=1$ ) which seemed to be a reasonable possibility. This is based on a joint work with Jian Ding and Zherui Fan.

### 1 October 2025

#### 09:40-10:20: Ikkei HOTTA (Yamaguchi University)

 $In finite-slit\ limits\ of\ multiple\ radial/chordal\ SLEs$ 

Since its introduction in 2000, the Schramm-Loewner Evolution (SLE) has provided scaling limits for various models in two-dimensional statistical physics. SLE describes random curves in the upper half-plane  $\mathbb{H} := \{z \in \mathbb{C} : \operatorname{Im}(z) > 0\}$ . While classical SLE focuses on a single curve, its generalization to multiple curves, known as multiple SLE, has also been studied. This talk presents results concerning the limit as the number of curves  $n \to \infty$ , in both the radial and chordal settings. Connections to non-commutative (in particular free) probability will also be discussed if time permits.

#### 10:30-11:10: Takashi OWADA (Purdue University)

Limit theorems under heavy-tailed scenario in the age-dependent random connection models

We consider limit theorems associated with subgraph counts in the age-dependent random connection model. First, we identify regimes where the count of sub-trees converges weakly to a stable random variable under suitable assumptions on the shape of trees. The proof relies on an intermediate result on weak convergence of associated point processes towards a Poisson point process. Additionally, we prove the same type of results for the clique counts. Here, a crucial ingredient includes the expectation asymptotics for clique counts, which itself is a result of independent interest. This is joint work with Christian Hirsch at Aarhus University.

### 11:30-12:10: Daisuke KAZUKAWA (Tokyo Metropolitan University)

Concentration of Gaussian and other stable distributions with  $l_p$ -norm

The concentration of measure phenomenon has been attracting attention in recent years. This originated from Lévy and Milman, and has recently become known through the work of Talagrand et. al. One way to describe the concentration is through concentration inequalities, which measure the difference between 1-Lipschitz functions and their medians quantitatively. The Gaussian concentration inequality follows from the Gaussian isoperimetric inequality by Borell and Sudakov-Tsirel'son. Concentration inequalities for other stable distributions have been studied by Houdré-Marchal. In this talk, we will establish the geometrical part of this phenomenon. The geometric interpretation of concentration is given by Gromov's convergence theory for metric measure spaces and is actively being researched. In this perspective, it is important to find the correct scale that the sequence of spaces concentrates and to determine the limit at that scale. We will answer this about high-dimensional stable distributions equipped with  $l_p$ -norm. Our results provide optimality or improvements in concentration inequalities. This talk is based on a joint work with S. Esaki (Oita University) and A. Mitsuishi (Fukuoka University).

#### 13:40-14:20: Wei QIAN (City University of Hong Kong)

No double points on the boundaries of Brownian loop-soup clusters at the critical intensity

We show that there do not exist double points on the boundaries of Brownian loop-soup clusters at the critical intensity. Such double points are closely related to a question of rewiring excursions into loops in the critical Brownian loop soup. In previous works, we have established that the double points on the boundaries of critical Brownian loop-soup clusters have Hausdorff dimension exactly zero.

In a recent work, we develop a unified approach to establish the non-existence of several types of random fractals in the Brownian motion which have dimension zero: Apart from the aforementioned double points on the boundaries of loop-soup clusters, we also show the non-existence of pioneer triple points in the planar Brownian motion, the pioneer double cut points of the planar and three-dimensional Brownian motions.

This talk is based on several works, including a recent joint work with Gao, Li and Liu.

### 14:30-15:10: Yichao HUANG (Beijing Institute of Technology)

Reinforced loop soup revisited

The vertex reinforced counterpart of the Markovian loop soup developed by Le Jan is introduced. I will review some basic notions about reinforced processes and susy hyperbolic sigma models, then explain a concrete construction of the so-called reinforced loop soup via a variant of Wilson's algorithm with reinforcement mechanism. As a corollary we obtain the susy hyperbolic version of isomorphism theorems for the reinforced loop soup. This is joint work with Yinshan Chang (Sichuan University), Dang-Zheng Liu (University of Science and Technology of China), and Xiaolin Zeng (Universit de Strasbourg).

### 15:20-16:00: Pierre NOLIN (City University of Hong Kong)

Percolation of discrete GFF in dimension two

We study percolation of two-sided level sets for the discrete Gaussian free field (DGFF) in dimension two. For a DGFF  $\varphi$  defined in a box with side length N, we show that with probability tending to 1 polynomially fast in N, there exist "low" crossings, along which  $|\varphi| \leq \varepsilon \sqrt{\log N}$ , for any  $\varepsilon > 0$  (while the average and the maximum of  $\varphi$  are of order  $\sqrt{\log N}$  and  $\log N$ , respectively). Our method also strongly suggests the existence of such crossings below  $C\sqrt{\log\log N}$ , for C large enough. As a consequence, we obtain connectivity properties for the set of thick points of a random walk.

We rely on an isomorphism between the DGFF and the random walk loop soup (RWLS) with critical intensity  $\alpha = 1/2$ . We further extend our study to the occupation field of the RWLS for all subcritical intensities  $\alpha \in (0, 1/2)$ , and in that case we uncover a non-trivial phase transition. This work relies heavily on new tools and techniques that we developed for the RWLS, especially surgery arguments on loops, which were made possible by a separation result for random walks in a loop soup. This allowed us to obtain a precise upper bound for the probability that two large connected components of loops "almost touch", which is instrumental here.

This talk is based on the three preprints https://arxiv.org/abs/2409.16230, https://arxiv.org/abs/2409.16273, and https://arxiv.org/abs/2504.06202, joint with Yifan Gao and Wei Qian (City University of Hong Kong).